Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Spring · 🧮 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester senior propulsion course for aerospace engineers. It is modeled on the topics and depth of MIT 16.50 Introduction to Propulsion Systems and MIT 16.512 Rocket Propulsion (free on MIT OpenCourseWare), using NASA Glenn's free Beginner's Guide to Propulsion, which we link where it helps. None of those authors, schools or agencies endorses this page.
>
> What we add: every unit ties to real work with a cited public source (NASA's RS-25 engines, NASA engine-core research and the U.S. Department of Energy's sustainable aviation fuel work), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a spreadsheet.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Propulsion is where aviation's fuel burn and emissions are decided: every point of engine efficiency is fuel not burned, and new fuels can cut the carbon in what is burned.

Rockets carry Earth-observing satellites to orbit, and cleaner, more efficient engines make that access more sustainable. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample or textbook values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (AERO 410).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Thermodynamics: the first and second laws, ideal gases and cycles.
- Fluid Mechanics: control volumes, compressible flow basics.
- Aerodynamics I and Aircraft Performance: drag, thrust required and range.
- Orbital Mechanics: the rocket equation and Δv.

How to study

- Draw a control volume around the engine and mark inflow and outflow before writing any equation.
- Keep temperatures in kelvin and use stagnation (total) values where the flow is fast.
- Separate thermal efficiency (heat to kinetic energy) from propulsive efficiency (kinetic energy to useful thrust power).
- Check every answer against a real engine’s order of magnitude.
- Track units: thrust in newtons, mass flow in kg/s, specific impulse in seconds.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free PhET Gas Properties and Energy Skate Park simulations.
- Spreadsheet software (any free one works), or Python.
- NASA Glenn’s Beginner’s Guide to Propulsion pages in a web browser.
- A calculator.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Thrust, Momentum and Propulsive Efficiency	Thrust and propulsive efficiency; Lab A	NASA Glenn
2	The Brayton Cycle and Thermal Efficiency	Brayton cycle; Lab B	NASA Glenn
3	Compressible Flow: Inlets and Nozzles	Stagnation properties and choking	MIT 16.50
4	Compressible Flow: Inlets and Nozzles	Nozzles and inlets; Lab C	MIT 16.50
5	Turbojets: Components and Cycle Analysis	Turbojet stations and work balance	MIT 16.50
6	Turbojets: Components and Cycle Analysis	Component efficiency and afterburners	NASA Glenn
7	Turbofans and Turboprops	Turbofans; Lab D; midterm practice	NASA Glenn
8	Fuel Burn, TSFC and Emissions	Fuel burn, TSFC and CO_2	NASA HyTEC
9	Rocket Fundamentals and Specific Impulse	Rocket thrust and Isp	NASA Glenn
10	Rocket Fundamentals and Specific Impulse	RS-25 case study	NASA RS-25
11	Rocket Nozzles, Propellants and Throttling	Rocket nozzles and throttling; Lab E	MIT 16.512
12	The Rocket Equation and Electric Propulsion	Rocket equation and electric propulsion	MIT 16.512
13	Sustainable Propulsion: SAF, Hydrogen and Electric Flight	SAF, hydrogen and batteries; Lab F	DOE SAF
14	Sustainable Propulsion: SAF, Hydrogen and Electric Flight	Electric aircraft and review	DOE SAF
15	Sustainable Propulsion: SAF, Hydrogen and Electric Flight	Review; final practice	MIT 16.50

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Thrust, Momentum and Propulsive Efficiency

The control-volume thrust equation, pressure thrust, specific thrust, thrust power, jet power, propulsive efficiency, and why big slow jets are efficient.

Learning objectives

By the end of this unit you can:

- Derive thrust from momentum flux.
- Compute specific thrust and thrust power.
- Compute propulsive efficiency.
- Explain why high bypass saves fuel.

Key concepts

- Thrust is the net momentum flux out of the engine plus a pressure term: F = ṁ_eV_e − ṁ_0V_0 + (p_e − p_0)A_e.
- For an air-breathing engine with small fuel flow and a matched nozzle, F ≈ ṁ(V_e − V_0).
- Specific thrust F/ṁ (N·s/kg) tells you how much thrust each kilogram per second of air gives.
- Propulsive efficiency η_p = 2/(1 + V_e/V_0) is the share of jet kinetic power that becomes useful thrust power F·V_0.
- Accelerating a lot of air a little (V_e close to V_0) is efficient but needs a big engine; that is the turbofan's trade.

Key equations

$$F = \dot{m}_e V_e - \dot{m}_0 V_0 + (p_e - p_0) A_e$$

```
F= m_eV_e- m_0V_0+(p_e-p_0)A_e
```

Thrust equation

$$F \approx \dot{m}(V_e - V_0)$$

Simple thrust

$$\eta_p = \frac{2}{1 + V_e/V_0}$$

```
_p=(2)/(1+V_e/V_0)
```

Propulsive efficiency

$$P_{thrust} = FV_0$$

`P_thrust=FV_0`

Thrust power

Worked examples, step by step

Example 1.1 An engine moves 100 kg/s of air from 250 m/s to 600 m/s (sample values). Find the thrust.

1. F = 100 × (600 − 250).

Answer: F = 35 kN.

Example 1.2 Find its propulsive efficiency.

1. η_p = 2/(1 + 600/250).

Answer: η_p = 0.588.

Example 1.3 Check that efficiency with powers.

1. Thrust power = 35,000 × 250 = 8.75 MW.
2. Jet power = ½ × 100 × (600² − 250²) = 14.875 MW.

Answer: 8.75/14.875 = 0.588.

Practice problems

Work each on paper first, then check.

1. Specific thrust of that engine?

   350 N·s/kg.

2. η_p if V_e = 300 m/s at V_0 = 250 m/s?

   0.909.

3. Why do fighters use small, fast jets?

High specific thrust at the cost of efficiency.

4. When does pressure thrust matter?

   When the nozzle exit pressure differs from ambient, as in many rockets.

Common misconceptions

Myth: More exhaust speed always means more efficiency.

Actually: Propulsive efficiency falls as V_e/V_0 grows.

Myth: Thrust pushes on the air behind.

Actually: It is the reaction to accelerating the flow.

Myth: Pressure thrust is always zero.

Actually: Only for a perfectly expanded nozzle.

Flashcards

Thrust

Net momentum flux + pressure term.

Specific thrust

$F/\dot{m}$.

η_p

$2/(1 + V_e/V_0)$.

Thrust power

$F \cdot V_0$.

Jet power

$\frac{1}{2}\dot{m}(V_e^2 - V_0^2)$.

Matched nozzle

$p_e = p_0$.

Self-check quiz

1. η_p = ...

$2/(1 + V_e/V_0)$

V_e/V_0

$1 - V_0/V_e$

FV_e

$2/(1 + V_e/V_0)$. Froude.

2. Specific thrust units...

N·s/kg

N

kg/s

s

N·s/kg. $F/\dot{m}$.

3. Efficient engines move...

lots of air slowly

little air fast

no air

only fuel

lots of air slowly. Low V_e/V_0.

4. Thrust power is...

FV_0

FV_e

$\dot{m}V_e^2$

$F/\dot{m}$

FV_0. Useful power.

Try this quiz again

How this helps save the planet

NASA Glenn's guide explains that most modern airliners use turbofans because of their high thrust and good fuel efficiency, and that high-bypass turbofans are nearly as fuel efficient as turboprops.

That efficiency comes straight from this unit's equation: moving more air at a speed closer to flight speed raises propulsive efficiency, which means less fuel burned and less CO_2 for the same trip.

On Saving Planets: Aircraft Performance · Solutions

Sources: NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

Free learning resources

- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT lectures.
- NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html): Free NASA explainer.

☐ **I finished this unit**

The Brayton Cycle and Thermal Efficiency

The ideal Brayton cycle, isentropic compression and expansion, thermal efficiency versus pressure ratio, compressor and turbine work, component efficiencies, and overall efficiency.

Learning objectives

By the end of this unit you can:

- Compute ideal Brayton efficiency from pressure ratio.
- Find temperatures after isentropic compression.
- Compute compressor work.
- Combine thermal and propulsive efficiency.

Key concepts

- A gas turbine runs the Brayton cycle: isentropic compression, constant-pressure heat addition, isentropic expansion and heat rejection.
- Ideal thermal efficiency depends only on the pressure ratio: η_th = 1 − π^(−(γ−1)/γ).
- Isentropic compression raises temperature by T_2/T_1 = π^((γ−1)/γ).
- Compressor work per kg is c_p(T_2 − T_1); the turbine must supply it before any is left for the jet.
- Overall efficiency η_0 = η_th × η_p: heat to useful thrust power.

Key equations

$$\eta_{th} = 1 - \pi^{-(\gamma-1)/\gamma}$$

_th=1-π^-(γ-1)/γ

Ideal Brayton efficiency

$$\frac{T_2}{T_1} = \pi^{(\gamma-1)/\gamma}$$

(T_2)/(T_1)=π^(γ-1)/γ

Isentropic compression

$$w_c = c_p(T_2 - T_1)$$

w_c=c_p(T_2-T_1)

Compressor work

$$\eta_0 = \eta_{th}\eta_p$$

`_0=_th_p`

Overall efficiency

Worked examples, step by step

Example 2.1 Find the ideal Brayton efficiency for pressure ratios 10 and 30 (γ = 1.4).

1. 1 − 10^(−0.2857); 1 − 30^(−0.2857).

Answer: 48.2% and 62.2%.

Example 2.2 Air at 288 K is compressed by a ratio of 30. Find T_2.

1. T_2 = 288 × 30^0.2857 = 288 × 2.643.

Answer: T_2 = 761 K.

Example 2.3 Find the compressor work per kg.

1. 1005 × (761 − 288).

Answer: w_c = 475 kJ/kg.

Practice problems

Work each on paper first, then check.

1. Efficiency at π = 20?

   57.5%.

2. Efficiency at π = 40?

   65.1%.

3. Overall efficiency for η_th = 0.5 and η_p = 0.8?

   0.40.

4. Why not raise π forever?

   Compressor exit gets too hot and heavy; turbine limits.

Common misconceptions

Myth: Brayton efficiency depends on peak temperature.

Actually: Ideal efficiency depends only on π.

Myth: All compressor work is lost.

Actually: The turbine recovers it from the hot gas.

Myth: Thermal efficiency is the whole story.

Actually: Multiply by propulsive efficiency.

Flashcards

Brayton cycle

Compress, burn, expand.

Pressure ratio π

p_2/p_1.

η_th

1 − π^(−0.2857).

Compressor work

c_pΔT.

Overall efficiency

η_th η_p.

Isentropic

Reversible and adiabatic.

Self-check quiz

1. Ideal Brayton η depends on...

- pressure ratio
- fuel
- size
- altitude only

pressure ratio. Formula.

2. Heat is added at...

constant pressure

constant volume

constant T

zero pressure

constant pressure. Burner.

3. η_0 = ...

η_th × η_p

η_th + η_p

η_p/η_th

1

η_th × η_p. Chain.

4. Compressing air...

heats it

cools it

nothing

liquefies it

heats it. Isentropic.

Try this quiz again

How this helps save the planet

NASA's Hybrid Thermally Efficient Core (HyTEC) project worked with industry to develop small-core engine technologies that enable fuel-burn reductions, more electric power extraction for aircraft

systems, and better compatibility with sustainable aviation fuels. NASA notes the project has concluded.

Smaller, hotter, higher-pressure cores are this unit's efficiency formula in hardware: raising the pressure ratio raises thermal efficiency, and every percent saves fuel on every flight.

On Saving Planets: Thermodynamics · Solutions

Sources: NASA Aeronautics: Hybrid Thermally Efficient Core (HyTEC) project (https://www.nasa.gov/directorates/armd/aavp/hytec/)

Free learning resources

- NASA Glenn Beginner's Guide: Brayton cycle (https://www.grc.nasa.gov/www/k-12/airplane/brayton.html): Free NASA explainer.
- PhET: Gas Properties (https://phet.colorado.edu/en/simulations/gas-properties): Compression heating interactively.

☐ **I finished this unit**

Compressible Flow: Inlets and Nozzles

Stagnation temperature and pressure, isentropic relations with Mach number, choked flow, area–Mach relations, converging and converging-diverging nozzles, and inlet pressure recovery.

Learning objectives

By the end of this unit you can:

- Use stagnation-to-static ratios.
- Find the critical (sonic) conditions.
- Use the area–Mach relation.
- Compute ideal nozzle exit velocity.

Key concepts

- Stagnation temperature $T_0 = T(1 + (\gamma - 1)M^2/2)$; stagnation pressure follows by the isentropic power $\gamma/(\gamma - 1)$.
- At the throat of a choked nozzle M = 1, with $T^*/T_0 = 0.833$ and $p^*/p_0 = 0.528$ for air.
- Subsonic flow speeds up in a converging duct; supersonic flow speeds up in a diverging one. A/A* gives the area needed for each Mach number.
- Ideal exit velocity: $V_e = \sqrt{(2c_pT_0(1 - (p_e/p_0)^{((\gamma-1)/\gamma)}))}$.
- Inlets slow air to subsonic speed for the compressor; losses in stagnation pressure cut thrust and efficiency.

Key equations

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

`(T_0)/(T)=1+(γ-1)/(2)M^2`

Stagnation temperature

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

`(p_0)/(p)=((T_0)/(T))^γ/(γ-1)`

Stagnation pressure

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

`(A)/(A^*)=1M[(2)/(γ+1)(1+(γ-1)/(2)M^2)]^(γ+1)/(2(γ-1))`

Area–Mach relation

$$V_e = \sqrt{2 c_p T_0 \left[1 - (p_e/p_0)^{(\gamma-1)/\gamma}\right]}$$

`V_e=2c_pT_0[1-(p_e/p_0)^(γ-1)/γ]`

Ideal exit velocity

Worked examples, step by step

Example 3.1 Find T_0/T and p_0/p at Mach 0.8.

1. $T_0/T = 1 + 0.2 \times 0.64$.
2. $p_0/p = 1.128^{3.5}$.

Answer: $T_0/T = 1.128$; $p_0/p = 1.524$.

Example 3.2 Find A/A* for Mach 2.

1. Area–Mach relation with $\gamma = 1.4$.

Answer: A/A* = 1.6875.

Example 3.3 Gas at $T_0 = 1000$ K expands to half its stagnation pressure (sample values). Find V_e.

1. $V_e = \sqrt{(2 \times 1005 \times 1000 \times (1 - 0.5^{0.2857}))}$.

Answer: V_e = 601 m/s.

Practice problems

Work each on paper first, then check.

1. Critical pressure ratio for air?

   $p^*/p_0 = 0.528$.

2. T^*/T_0?

   0.833.

3. A/A* at Mach 1.5?

1.176.

4. A/A* at Mach 3?

   4.235.

Common misconceptions

Myth: Nozzles always converge.

Actually: Supersonic exits need a diverging section.

Myth: Static and stagnation values are the same.

Actually: They differ at speed.

Myth: Choking stops all flow.

Actually: It caps the mass flow for given upstream conditions.

Flashcards

Stagnation temperature

$T(1 + 0.2M^2)$ for air.

Choked

M = 1 at the throat.

A/A*

Area for a Mach number.

p^*/p_0

0.528.

Converging–diverging

For supersonic exits.

Pressure recovery

Inlet p_0 kept.

Self-check quiz

1. At a choked throat M = …

1

0

2

0.5

1. Sonic.

2. Supersonic flow accelerates in a…

diverging duct

converging duct

constant duct

none

diverging duct. Area–Mach.

3. p^*/p_0 for air…

0.528

0.833

1

0.1

0.528. $\gamma = 1.4$.

4. T_0/T at M = 0.8…

1.128

1.8

1

0.8

1.128. Formula.

Try this quiz again

How this helps save the planet

NASA's RS-25 page notes that during Artemis I, engine thrust and mixture-ratio control valves were within 0.5 percent of pre-flight predicted values and internal pressures and temperatures were within two percent.

Predictions that close come from the isentropic-flow relations in this unit, refined by test data: getting nozzle and chamber conditions right is what makes an engine perform as designed.

On Saving Planets: Fluid Mechanics · Artemis II

Sources: NASA: SLS (Space Launch System) RS-25 Core Stage Engine (https://www.nasa.gov/reference/space-launch-system-rs-25-core-stage-engine/)

Free learning resources

- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT lectures.
- NASA Glenn Beginner's Guide: Rocket thrust summary (https://www.grc.nasa.gov/www/k-12/rocket/rktthsum.html): Free NASA explainer.

- [ ] **I finished this unit**

Turbojets: Components and Cycle Analysis

Inlet, compressor, burner, turbine and nozzle; work balance between compressor and turbine; turbine inlet temperature limits; afterburners; and real component efficiencies.

Learning objectives

By the end of this unit you can:

- Trace stations through a turbojet.
- Balance compressor and turbine work.
- Explain turbine temperature limits.
- Describe how afterburning trades fuel for thrust.

Key concepts

- Stations: free stream, inlet, compressor exit, burner exit (turbine inlet), turbine exit and nozzle exit.
- The turbine drives the compressor on a shared shaft: $c_p(T_4 - T_5) = c_p(T_3 - T_2)$ for ideal flow with equal c_p.
- Higher turbine inlet temperature gives more specific thrust and efficiency, limited by blade materials and cooling.
- Real compressors and turbines have isentropic efficiencies below 1, so more work is needed and less is recovered.
- Afterburners add fuel after the turbine for extra thrust at a large fuel cost.

Key equations

$$T_4 - T_5 = T_3 - T_2$$

T_4-T_5=T_3-T_2

Shaft work balance

$$\eta_c = \frac{T_{3s} - T_2}{T_3 - T_2}$$

_c=_3s-T_2T_3-T_2

Compressor efficiency

$$\eta_t = \frac{T_4 - T_5}{T_4 - T_{5s}}$$

_t=_4-T_5T_4-T_5s

Turbine efficiency

$$F = \dot{m}(V_e - V_0)$$

`F= m(V_e-V_0)`

Thrust

Worked examples, step by step

Example 4.1 With a 475 kJ/kg compressor (Unit 2) and turbine inlet 1600 K (sample value), find the turbine exit temperature.

1. ΔT = 475,440/1005 = 473 K.

Answer: T_5 = 1127 K.

Example 4.2 A real compressor with η_c = 0.85 needs how much work for the same pressure ratio?

1. 475/0.85.

Answer: 559 kJ/kg.

Example 4.3 Find the thrust of a turbojet moving 100 kg/s from 250 to 450 m/s.

1. 100 × 200.

Answer: 20 kN.

Practice problems

Work each on paper first, then check.

1. Temperature rise of that real compressor?

   559,341/1005 = 557 K.

2. Why cool turbine blades?

   Gas is hotter than the metal can survive.

3. What does an afterburner raise?

   V_e and thrust.

4. What does it lower?

   Propulsive efficiency and range.

Common misconceptions

Myth: The turbine makes the thrust.

Actually: It mainly drives the compressor; the nozzle makes thrust.

Myth: Hotter is always possible.

Actually: Materials limit T_4.

Myth: Afterburners are efficient.

Actually: They burn fuel at low pressure.

Flashcards

Turbojet

Compressor–burner–turbine–nozzle.

T_4

Turbine inlet temperature.

Work balance

Turbine = compressor.

η_c

Ideal/actual ΔT.

Afterburner

Burn after the turbine.

Station numbers

0 free stream to 9 nozzle exit.

Self-check quiz

1. Turbine work equals...

compressor work

thrust

fuel energy

zero

compressor work. Shared shaft.

2. T_4 is limited by...

materials

Mach

fuel color

altitude

materials. Blades.

3. η_c < 1 means...

more work needed

less work

no change

free work

more work needed. Losses.

4. Afterburners raise...

thrust

efficiency

range

bypass

thrust. Extra heat.

Try this quiz again

How this helps save the planet

NASA Glenn's guide notes that low-bypass turbofans are still more fuel efficient than basic turbojets, and that many modern fighters use low-bypass turbofans with afterburners so they can cruise efficiently but still have high thrust when needed.

The pure turbojet survives mainly as a teaching model: its simple work balance shows why later engines added a fan to cut fuel burn.

On Saving Planets: Aerodynamics I · Thermodynamics

Sources: NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

Free learning resources

- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT lectures.
- NASA Glenn Beginner's Guide: Brayton cycle (https://www.grc.nasa.gov/www/k-12/airplane/brayton.html): Free NASA explainer.

- [ ] **I finished this unit**

Turbofans and Turboprops

Bypass ratio, core and fan streams, the turbofan thrust equation, propulsive efficiency of the fan stream, turboprops and open rotors, and the size and weight trade.

Learning objectives

By the end of this unit you can:

- Apply the turbofan thrust equation.
- Compute bypass ratio and total flow.
- Compare turbofan and turbojet thrust for the same core.
- Explain the limits of very high bypass.

Key concepts

- Bypass ratio bpr = ṁ_fan/ṁ_core; total flow $\dot{m}_0$ = ṁ_fan + ṁ_core.
- NASA Glenn's turbofan thrust: F = (ṁV)_e + bpr·ṁ_c·V_f − $\dot{m}_0 V_0$.
- The fan accelerates lots of air a little, so its stream has high propulsive efficiency.
- Turboprops and open rotors push bypass even higher, efficient at lower speeds.
- Bigger fans add weight, drag and noise and need slower fan speeds, often with a gearbox.

Key equations

$$bpr = \frac{\dot{m}_f}{\dot{m}_c}$$

```
bpr=( m_f)/( m_c)
```

Bypass ratio

$$\dot{m}_0 = \dot{m}_f + \dot{m}_c$$

```
m_0= m_f+ m_c
```

Total flow

$$F = \dot{m}_c V_e + bpr\, \dot{m}_c V_f - \dot{m}_0 V_0$$

```
F= m_cV_e+bpr m_cV_f- m_0V_0
```

Turbofan thrust

$$\eta_{p,f} = \frac{2}{1 + V_f/V_0}$$

`_p,f=(2)/(1+V_f/V_0)`

Fan-stream efficiency

Worked examples, step by step

Example 5.1 A turbofan has core flow 100 kg/s and bpr = 10, with V_e = 450 m/s, V_f = 300 m/s and V_0 = 250 m/s (sample values). Find the total flow and thrust.

1. $\dot{m}_0$ = 100 + 1000 = 1100 kg/s.
2. F = 100 × 450 + 1000 × 300 − 1100 × 250.

Answer: F = 70 kN.

Example 5.2 Compare with a turbojet using the same core flow and exit speed.

1. F = 100 × (450 − 250).

Answer: 20 kN: the fan multiplies thrust by 3.5.

Example 5.3 Find the fan stream's propulsive efficiency.

1. 2/(1 + 300/250).

Answer: η_p = 0.909.

Practice problems

Work each on paper first, then check.

1. Core-stream η_p (450 at 250 m/s)?

   0.714.

2. Thrust ratio turbofan/turbojet?

   3.5.

3. Why gear the fan?

   So the fan turns slower than the low-pressure turbine.

4. Why are props used on slow aircraft?

Efficient at low speed; tips go supersonic at high speed.

Common misconceptions

Myth: The fan makes little thrust.

Actually: In high-bypass engines it makes most of it.

Myth: Higher bypass is always better.

Actually: Weight and drag grow with fan size.

Myth: Turbofans burn more fuel than turbojets.

Actually: They burn less for the same thrust.

Flashcards

Bypass ratio

Fan flow/core flow.

Turbofan thrust

Core + fan − inlet momentum.

Fan stream

High η_p.

Turboprop

Very high bypass via propeller.

Geared fan

Slower fan, faster turbine.

Open rotor

Unducted fan.

Self-check quiz

1. bpr = …

ṁ_f/ṁ_c

ṁ_c/ṁ_f

V_f/V_e

F/ṁ

ṁ_f/ṁ_c. Definition.

2. High-bypass fans...

move lots of air slowly

are supersonic

add no thrust

burn fuel

move lots of air slowly. Efficiency.

3. Turboprops suit...

low speeds

Mach 2

rockets

space

low speeds. Prop limits.

4. Same core, add a fan: thrust...

rises

falls

same

zero

rises. More air.

Try this quiz again

How this helps save the planet

NASA Glenn explains that because the core's fuel flow changes only a small amount when a fan is added, a turbofan generates more thrust for nearly the same fuel, which is why it is very fuel

efficient.

This unit's example shows the effect: the same 100 kg/s core makes 20 kN as a turbojet and 70 kN with a bypass-ratio-10 fan, for nearly the same core fuel flow.

On Saving Planets: Aircraft Performance · Solutions

Sources: NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

Free learning resources

- NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html): Free NASA explainer.
- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT lectures.

☐ **I finished this unit**

Fuel Burn, TSFC and Emissions

Fuel heating value, overall efficiency, thrust-specific fuel consumption, CO_2 from burning jet fuel, and how engine efficiency links to range and climate impact.

Learning objectives

By the end of this unit you can:

- Compute overall efficiency from fuel flow.
- Compute TSFC.
- Estimate CO_2 emissions from fuel burn.
- Link TSFC to range.

Key concepts

- Fuel power is ṁ_f × LHV; jet fuel's lower heating value is about 43 MJ/kg (textbook value).
- Overall efficiency η_0 = F·V_0/(ṁ_f·LHV).
- Thrust-specific fuel consumption TSFC = ṁ_f/F; lower is better.
- Burning 1 kg of jet fuel makes about 3.16 kg of CO_2 (textbook value), since carbon picks up oxygen from the air.
- In the Breguet range equation, range grows with η_0 × LHV, so a better engine flies farther or burns less fuel.

Key equations

$$\eta_0 = \frac{F V_0}{\dot{m}_f \, LHV}$$

`_0=(FV_0)/( m_f LHV)`

Overall efficiency

$$TSFC = \frac{\dot{m}_f}{F}$$

`TSFC=( m_f)/(F)`

TSFC

$$\dot{m}_{CO_2} = 3.16 \, \dot{m}_f$$

`m_CO_2=3.16 m_f`

CO_2 rate (textbook)

$$R = \frac{\eta_0 LHV}{g} \frac{L}{D} \ln \frac{W_i}{W_f}$$

R=(_0 LHV)/(g)L/D(W_i)/(W_f)

Breguet range

Worked examples, step by step

Example 6.1 The Unit 5 turbofan (70 kN at 250 m/s) burns 1.2 kg/s of fuel with LHV 43 MJ/kg (sample values). Find η_0.

1. η_0 = 70,000 × 250/(1.2 × 43 × 10^6).

Answer: η_0 = 0.339.

Example 6.2 Find its TSFC.

1. 1.2/70,000 kg/(N·s).

Answer: TSFC = 17.14 mg/(N·s).

Example 6.3 Find CO_2 per hour.

1. Fuel = 1.2 × 3600 = 4320 kg.
2. CO_2 = 4320 × 3.16.

Answer: 13,651 kg of CO_2 per hour.

Practice problems

Work each on paper first, then check.

1. CO_2 rate per second?

   3.792 kg/s.

2. Thermal efficiency if η_0 = 0.339 and η_p = 0.8?

   0.424.

3. Fuel saved per hour by a 10% lower TSFC at the same thrust?

432 kg.

4. Does cutting fuel cut CO_2 in proportion?

   Yes, for the same fuel.

Common misconceptions

Myth: Fuel weighs more than its CO_2.
Actually: CO_2 is about 3.16 times the fuel mass.

Myth: TSFC higher is better.
Actually: Lower is better.

Myth: Efficiency only affects cost.
Actually: It cuts emissions too.

Flashcards

LHV

Fuel energy per kg.

η_0

$FV_0/(\dot{m}_f \text{ LHV})$.

TSFC

Fuel per thrust.

CO_2 factor

≈3.16 (textbook).

Breguet

Range equation.

Fuel power

$\dot{m}_f \times$ LHV.

Self-check quiz

1. TSFC = ...

$\dot{m}_f/F$

$F/\dot{m}_f$

FV_0

LHV/F

ṁ_f/F. Definition.

2. CO_2 per kg jet fuel (textbook)…

≈3.16 kg

1 kg

0.3 kg

10 kg

≈3.16 kg. Chemistry.

3. Range grows with…

η_0 × LHV

TSFC

weight

drag

η_0 × LHV. Breguet.

4. η_0 example…

0.339

0.9

0.05

1.2

0.339. Worked.

Try this quiz again

How this helps save the planet

NASA's HyTEC project aimed at small-core engine technologies to enable fuel-burn reductions and better compatibility with sustainable aviation fuels.

This unit's numbers show why that matters: one large engine at cruise in our example emits about 13.7 tonnes of CO_2 per hour, so even small TSFC gains add up across a fleet.

On Saving Planets: Solutions · Earth Observation

Sources: NASA Aeronautics: Hybrid Thermally Efficient Core (HyTEC) project (https://www.nasa.gov/directorates/armd/aavp/hytec/)

Free learning resources

- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT lectures.
- NASA Aeronautics: Hybrid Thermally Efficient Core (HyTEC) project (https://www.nasa.gov/directorates/armd/aavp/hytec/): NASA project page.

☐ **I finished this unit**

Rocket Fundamentals and Specific Impulse

Rocket thrust, effective exhaust velocity, specific impulse, total impulse, mass flow, and real numbers from NASA's RS-25 engine.

Learning objectives

By the end of this unit you can:

- Use F = ṁV_eq.
- Convert between Isp and exhaust velocity.
- Compute total impulse.
- Work with RS-25 thrust data.

Key concepts

- A rocket carries all its propellant, so thrust is F = ṁV_e + (p_e − p_0)A_e = ṁV_eq.
- NASA Glenn defines specific impulse Isp = F/(ṁg_0), in seconds; V_eq = Isp × g_0.
- Total impulse I = F × Δt measures what a motor delivers overall.
- Higher Isp means less propellant for the same impulse.
- Thrust is often quoted as a percent of a rated level, as with the RS-25.

Key equations

$$F = \dot{m} V_{eq}$$

```
F= mV_eq
```

Rocket thrust

$$I_{sp} = \frac{F}{\dot{m} g_0}$$

```
I_sp=(F)/( m g_0)
```

Specific impulse

$$V_{eq} = I_{sp} g_0$$

```
V_eq=I_spg_0
```

Equivalent velocity

$$I = F \Delta t$$

$I = F\Delta t$

Total impulse

Worked examples, step by step

Example 7.1 NASA says shuttle RS-25s ran at 104.5% of rated thrust, about 491,000 lbf. Convert to newtons and find 100% thrust.

1. 491,000 × 4.448 N.
2. 491,000/1.045.

Answer: 2.184 MN; about 469,856 lbf.

Example 7.2 Using a textbook vacuum Isp of 452 s, find V_eq.

1. 452 × 9.80665.

Answer: V_eq = 4433 m/s.

Example 7.3 Find the propellant flow at 2.184 MN.

1. $\dot{m} = 2.184 \times 10^6/4433$.

Answer: $\dot{m}$ = 492.7 kg/s (textbook-based).

Practice problems

Work each on paper first, then check.

1. Four engines at 491,000 lbf?

   1.964 million lbf.

2. Four at 109% of rated?

   About 2.049 million lbf.

3. Total impulse of a 1000 N motor for 20 s?

   20,000 N·s.

4. V_eq for Isp 300 s?

2942 m/s.

Common misconceptions

Myth: Isp is a time the engine runs.

Actually: It is thrust per weight flow of propellant.

Myth: Rockets need air to push on.

Actually: They work best in vacuum.

Myth: Percent thrust above 100 is impossible.

Actually: It is relative to a historic rated level.

Flashcards

Isp

F/(ṁg_0), seconds.

V_eq

Isp × g_0.

Total impulse

F × Δt.

RS-25

SLS core stage engine.

Rated thrust

100% reference level.

g_0

9.80665 m/s².

Self-check quiz

1. Isp = ...

F/(ṁg_0)

ṁ/F

Fg_0

V/F

F/(ṁg_0). Glenn.

2. V_eq for Isp 452 s...

4433 m/s

452 m/s

9.8 km/s

300 m/s

4433 m/s. Multiply.

3. Total impulse units...

N·s

N

s

kg

N·s. F × t.

4. RS-25 shuttle level...

104.5%

50%

200%

10%

104.5%. NASA.

Try this quiz again

How this helps save the planet

NASA's RS-25 page reports that the engines flew 135 Space Shuttle missions, with more than 3,000 starts and over 1 million seconds of total ground and flight firing; the SLS Block 1 rocket uses four

RS-25s and two solid rocket boosters to send 27 metric tons to the Moon.

The same page notes that during Artemis I in November 2022 the engines produced over 2 million pounds of vacuum thrust. Reusing proven engines is one way NASA builds Earth-observing and exploration capability.

On Saving Planets: Artemis II · Orbital Mechanics

Sources: NASA: SLS (Space Launch System) RS-25 Core Stage Engine (https://www.nasa.gov/reference/space-launch-system-rs-25-core-stage-engine/) · NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html)

Free learning resources

- NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html): Free NASA explainer.
- MIT OpenCourseWare 16.512 Rocket Propulsion (Fall 2005) (https://ocw.mit.edu/courses/16-512-rocket-propulsion-fall-2005/): Free MIT rocket course.

☐ **I finished this unit**

Rocket Nozzles, Propellants and Throttling

Chamber conditions, nozzle expansion, exit velocity from the isentropic relations, expansion ratio and altitude, propellant choices and throttling.

Learning objectives

By the end of this unit you can:

- Compute ideal rocket exhaust velocity.
- Explain over- and under-expansion.
- Compare propellant properties.
- Relate throttle settings to thrust.

Key concepts

- Ideal exit velocity: V_e = √(2γRT$_0$/(γ − 1) × (1 − (p_e/p$_0$)^((γ−1)/γ))), with R the gas constant per kg of exhaust.
- Hot, light exhaust (high T$_0$/M) gives high V_e: hydrogen–oxygen engines lead chemical Isp.
- Bigger expansion ratios lower p_e and raise V_e, but a nozzle sized for vacuum overexpands at sea level.
- Thrust coefficient and chamber pressure link nozzle size to thrust.
- Throttling varies propellant flow and chamber pressure; NASA quotes RS-25 power levels as percent of rated thrust.

Key equations

$$V_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{R_u T_0}{\mathcal{M}}\left[1 - (p_e/p_0)^{(\gamma-1)/\gamma}\right]}$$

```
V_e=(2γ)/(γ-1)(R_uT_0)/( M)[1-(p_e/p_0)^(γ-1)/γ]
```

Ideal rocket exit velocity

$$I_{sp} = V_e / g_0$$

```
I_sp=V_e/g_0
```

Isp (matched)

$$F = \dot{m} V_e + (p_e - p_a) A_e$$

```
F= mV_e+(p_e-p_a)A_e
```

Thrust with pressure term

$$F_{\%} = \frac{F}{F_{100}}$$

F_\%=_100

Power level

Worked examples, step by step

Example 8.1 Exhaust with γ = 1.2, molar mass 13 g/mol and T_0 = 3500 K (sample values) expands to p_e/p_0 = 0.001. Find V_e and Isp.

1. V_e = √(12 × 8.314 × 3500/0.013 × (1 − 0.001^0.1667)).

Answer: V_e = 4286 m/s; Isp = 437 s.

Example 8.2 Repeat for p_e/p_0 = 0.01.

1. Smaller expansion.

Answer: V_e = 3794 m/s.

Example 8.3 NASA says new RS-25s run at 111% and were tested up to 113%. Find 113% in lbf from 469,856 lbf rated.

1. 469,856 × 1.13.

Answer: 530,937 lbf.

Practice problems

Work each on paper first, then check.

1. 111% of rated?

   About 521,540 lbf (NASA: about 522,000).

2. Isp at p_e/p_0 = 0.01?

   387 s.

3. Why does T_0/M matter?

V_e scales with $\sqrt{(T_0/M)}$.

4. What happens to a vacuum nozzle at sea level?

   Overexpansion; lower thrust and flow separation risk.

Common misconceptions

Myth: Heavier exhaust is better.

Actually: Lighter exhaust gives higher V_e.

Myth: Bigger nozzles always help.

Actually: Not at sea level.

Myth: Throttle only changes flow.

Actually: Chamber pressure changes too.

Flashcards

Expansion ratio

A_e/A*.

Overexpanded

p_e < p_a.

Underexpanded

p_e > p_a.

Molar mass

Lighter exhaust, faster jet.

Power level

Percent of rated thrust.

Chamber pressure

Sets thrust for a nozzle size.

Self-check quiz

1. V_e scales with...

$\sqrt{(T_0/M)}$

M

T_0^2

$1/T_0$

$\sqrt{(T_0/M)}$. Formula.

2. Highest chemical Isp...

H_2/O_2

gunpowder

water

kerosene only

H_2/O_2. Light exhaust.

3. Lower p_e/p_0 gives...

higher V_e

lower V_e

same

zero

higher V_e. More expansion.

4. RS-25 tests reached...

113%

50%

150%

200%

113%. NASA.

Try this quiz again

How this helps save the planet

NASA reports that RS-25s flew at 104.5% on the shuttle and were tested to 111%; SLS uses 109% for its first four flights, new production engines operate at 111%, and testing has reached 113%. L3Harris is building 24 new engines with modern methods such as selective laser melting.

Each added percent of thrust follows from chamber and nozzle physics in this unit, and lets the same engine lift more mass to orbit.

On Saving Planets: Artemis II · GOES-19

Sources: NASA: SLS (Space Launch System) RS-25 Core Stage Engine (https://www.nasa.gov/reference/space-launch-system-rs-25-core-stage-engine/)

Free learning resources

- MIT OpenCourseWare 16.512 Rocket Propulsion (Fall 2005) (https://ocw.mit.edu/courses/16-512-rocket-propulsion-fall-2005/): Free MIT rocket course.
- NASA Glenn Beginner's Guide: Rocket thrust summary (https://www.grc.nasa.gov/www/k-12/rocket/rktthsum.html): Free NASA explainer.

☐ **I finished this unit**

The Rocket Equation and Electric Propulsion

The Tsiolkovsky equation, mass ratio, staging, electric thrusters, power-limited thrust, and the trade between Isp and thrust.

Learning objectives

By the end of this unit you can:

- Use the rocket equation.
- Compare chemical and electric Δv for the same propellant.
- Compute power-limited thrust.
- Explain why electric thrusters are low thrust.

Key concepts

- Tsiolkovsky: Δv = V_e ln(m_0/m_f).
- Staging drops empty mass so later stages see better mass ratios.
- Electric thrusters (ion, Hall) reach very high Isp but are limited by power.
- Jet power ½ṁV_e² = ηP, so thrust F = 2ηP/V_e: higher V_e means less thrust for the same power.
- Electric propulsion suits long, patient missions; chemical rockets suit launch and quick maneuvers.

Key equations

$$\Delta v = V_e \ln \frac{m_0}{m_f}$$

`Δ v=V_e(m_0)/(m_f)`

Rocket equation

$$F = \frac{2\eta P}{V_e}$$

`F=(2 P)/(V_e)`

Power-limited thrust

$$V_e = I_{sp} g_0$$

`V_e=I_spg_0`

Exhaust velocity

$$\frac{m_0}{m_f} = e^{\Delta v / V_e}$$

`(m_0)/(m_f)=e^Δ v/V_e`

Mass ratio

Worked examples, step by step

Example 9.1 A Hall thruster has Isp 1800 s (sample value). Find V_e.

1. 1800 × 9.80665.

Answer: V_e = 17,652 m/s.

Example 9.2 With 4.5 kW and efficiency 0.5 (sample values), find thrust.

1. F = 2 × 0.5 × 4500/17,652.

Answer: F = 0.255 N.

Example 9.3 A 1000 kg craft uses 100 kg of propellant. Compare Δv with this thruster and with a 300 s chemical engine.

1. 17,652 ln(1000/900); 2942 ln(1000/900).

Answer: About 1860 m/s versus 310 m/s.

Practice problems

Work each on paper first, then check.

1. Mass ratio for Δv = V_e?

   e = 2.718.

2. Thrust if power doubles?

   0.51 N.

3. Why not launch with Hall thrusters?

   Thrust is far below weight.

4. Why stage?

   Drop dead mass.

Common misconceptions

Myth: High Isp means high thrust.

Actually: Power limits thrust.

Myth: Δv is linear in propellant.

Actually: It is logarithmic.

Myth: Electric engines need no propellant.

Actually: They still expel mass.

Flashcards

Rocket equation

$\Delta v = V_e \ln(m_0/m_f)$.

Mass ratio

m_0/m_f.

Staging

Discard empty tanks.

Hall thruster

Electric, high Isp.

Power-limited

$F = 2\eta P/V_e$.

Δv

Velocity change budget.

Self-check quiz

1. Δv = …

$V_e \ln(m_0/m_f)$

$V_e m_0$

F/m

Isp

$V_e \ln(m_0/m_f)$. Tsiolkovsky.

2. Power-limited F = ...

$2\eta P/V_e$

PV_e

P/η

$\dot{m}g$

$2\eta P/V_e$. Energy.

3. Hall V_e at 1800 s...

17.65 km/s

1.8 km/s

300 m/s

100 km/s

17.65 km/s. Multiply.

4. Staging helps by...

dropping empty mass

adding fuel

raising g

nothing

dropping empty mass. Mass ratio.

Try this quiz again

How this helps save the planet

NASA's DART mission, a planetary-defense test, launched on November 24, 2021 aboard a Falcon 9 from Vandenberg Space Force Base and impacted the asteroid moonlet Dimorphos, about 160 meters across, on September 26, 2022.

Getting there took the rocket-equation budgeting of this unit: launch Δv from a chemical rocket, then precise in-space propulsion to steer into a small target.

On Saving Planets: Orbital Mechanics · Solutions

Sources: NASA Science: Double Asteroid Redirection Test (DART) (https://science.nasa.gov/mission/dart/)

Free learning resources

- MIT OpenCourseWare 16.512 Rocket Propulsion (Fall 2005) (https://ocw.mit.edu/courses/16-512-rocket-propulsion-fall-2005/): Free MIT rocket course.
- NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html): Free NASA explainer.

☐ **I finished this unit**

Sustainable Propulsion: SAF, Hydrogen and Electric Flight

Sustainable aviation fuels, energy density per kilogram, hydrogen and batteries compared with jet fuel, and what the numbers mean for future aircraft.

Learning objectives

By the end of this unit you can:

- Compare specific energies of fuels and batteries.
- Describe SAF feedstocks and pathways.
- Estimate fuel energy changes.
- Weigh hydrogen’s mass and volume trade.

Key concepts

- Specific energy (MJ/kg) sets how far an aircraft can fly per kilogram carried; jet fuel ≈ 43 MJ/kg, liquid hydrogen ≈ 120 MJ/kg and good batteries ≈ 0.25 MJ/kg (textbook values).
- SAF is made from biomass, wastes or captured carbon and can drop into existing engines with blending limits.
- Hydrogen has high energy per kilogram but very low density, so tanks are large and heavy.
- Batteries suit short flights and small aircraft; their low specific energy limits range.
- Life-cycle emissions depend on how the fuel is made, not just what comes out of the engine.

Key equations

$$\frac{e_{fuel}}{e_{batt}} = \frac{43}{0.25}$$

```
_fuele_batt=(43)/(0.25)
```

Fuel vs battery (textbook)

$$\frac{e_{H_2}}{e_{jet}} = \frac{120}{43}$$

```
_H_2e_jet=120/43
```

Hydrogen vs jet fuel (textbook)

$$m_{fuel} = \frac{E}{e}$$

```
m_fuel=E/e
```

Mass for an energy

$$E = m\,LHV$$

`E=m LHV`

Fuel energy

Worked examples, step by step

Example 10.1 How many times more energy per kg does jet fuel hold than a battery (textbook values)?

1. 43/0.25.

Answer: 172 times.

Example 10.2 How does liquid hydrogen compare with jet fuel per kg?

1. 120/43.

Answer: 2.79 times.

Example 10.3 DOE says a LANL bio-acetone SAF has 12% more energy than conventional jet fuel. Using 43 MJ/kg as the base (textbook value), estimate its energy.

1. 1.12 × 43.

Answer: About 48.2 MJ/kg.

Practice problems

Work each on paper first, then check.

1. Battery mass to store the energy in 1 kg of jet fuel?

   172 kg.

2. Hydrogen mass for the energy of 1000 kg of jet fuel?

   358 kg.

3. Name a SAF feedstock.

Biomass, wet waste or waste CO.

4. Why is hydrogen hard to store?

   Very low density; cryogenic tanks.

Common misconceptions

Myth: Hydrogen is energy-poor.
Actually: Per kilogram it beats jet fuel; per liter it does not.

Myth: Batteries will soon match jet fuel.
Actually: The gap is over 100×.

Myth: SAF needs new engines.
Actually: It is designed to work in existing ones.

Flashcards

SAF

Sustainable aviation fuel.

Specific energy

MJ/kg.

LH_2

Liquid hydrogen.

Drop-in fuel

Works in current engines.

Life cycle

Production to combustion.

Billion-Ton Report

DOE biomass study.

Self-check quiz

1. Jet fuel vs battery (textbook)...

172×

2×

10×

1000×

172×. 43/0.25.

2. Hydrogen’s weak point...

volume

mass

energy

color

volume. Density.

3. DOE biomass potential...

>1 billion tons a year

1 ton

1 million tons

none

>1 billion tons a year. Billion-Ton Report.

4. LANL SAF energy vs jet fuel...

+12%

−50%

same

+100%

+12%. DOE.

Try this quiz again

How this helps save the planet

The U.S. Department of Energy says its 2023 Billion-Ton Report found the United States could produce more than 1 billion tons of biomass a year, enough for about 60 billion gallons of biofuels, and that a LANL bio-acetone SAF has 12% more energy than conventional jet fuel.

DOE also describes SAF from wet waste and from waste CO through alcohol-to-jet routes. Combined with efficient engines from earlier units, cleaner fuel is a direct route to lower aviation emissions.

On Saving Planets: Solutions · Earth Observation

Sources: U.S. Department of Energy: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels)

Free learning resources

- U.S. Department of Energy: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels): DOE explainer.
- NASA Aeronautics: Hybrid Thermally Efficient Core (HyTEC) project (https://www.nasa.gov/directorates/armd/aavp/hytec/): NASA project page.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Thrust, Momentum and Propulsive Efficiency
   4 problems
2. The Brayton Cycle and Thermal Efficiency
   4 problems
3. Compressible Flow: Inlets and Nozzles
   4 problems
4. Turbojets: Components and Cycle Analysis
   4 problems
5. Turbofans and Turboprops
   4 problems
6. Fuel Burn, TSFC and Emissions
   4 problems
7. Rocket Fundamentals and Specific Impulse
   4 problems
8. Rocket Nozzles, Propellants and Throttling
   4 problems
9. The Rocket Equation and Electric Propulsion
   4 problems

10. Sustainable Propulsion: SAF, Hydrogen and Electric Flight
    4 problems

The lab (Propulsion lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab runs in a free simulation or a spreadsheet, so there are no physical hazards. Do not mix, make or burn propellants or build engines for this course. Take screen breaks, sit with good posture, and keep cables out of walkways.

Safety reference: OSHA: Walking-working surfaces (slip, trip and fall hazards) (https://www.osha.gov/walking-working-surfaces)

Lab A: Thrust and propulsive efficiency sweep

WEEK 1

Goal: Build a spreadsheet that sweeps exhaust velocity at fixed flight speed and finds thrust, specific thrust and propulsive efficiency.

$$\eta_p = \frac{2}{1 + V_e/V_0}$$

```
_p=(2)/(1+V_e/V_0)
```

Propulsive efficiency

Procedure

1. Set ṁ = 100 kg/s and V_0 = 250 m/s.
2. Sweep V_e from 300 to 900 m/s.
3. Compute F, F/ṁ and η_p in each row.
4. Plot η_p against specific thrust and describe the trade.

Data table

V_e (m/s)	F (kN)	η_p
300	5	0.909
450	20	0.714
600	35	0.588
900	65	0.435

Computed sample values.

Sample calculation

1. F = 100 × (600 − 250) = 35 kN.
2. η_p = 2/(1 + 900/250) = 0.435.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

Planet link: NASA Glenn notes high-bypass turbofans are nearly as fuel efficient as turboprops.
NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

☐ **I completed this lab (wet or virtual)**

Lab B: Compression heating and Brayton efficiency

WEEK 2

Goal: Watch gas heat when compressed in PhET Gas Properties, then tabulate ideal Brayton efficiency versus pressure ratio.

$$\eta_{th} = 1 - \pi^{-0.2857}$$

```
_th=1-π^-0.2857
```

Ideal Brayton efficiency (γ = 1.4)

Procedure

1. In PhET Gas Properties, insulate the box (hold heat constant) and push the wall in quickly; note the temperature rise.
2. In a spreadsheet, list π = 10, 20, 30, 40.
3. Compute η_th and T_2 from 288 K.
4. Plot η_th against π and note diminishing returns.

Data table

π	η_th	T_2 (K)
10	0.482	556
20	0.575	678
30	0.622	761
40	0.651	825

Computed ideal values.

Sample calculation

1. η = 1 − 30^(−0.2857) = 0.622.
2. T_2 = 288 × 40^0.2857 = 825 K.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any browser and spreadsheet work.

PhET: Gas Properties (https://phet.colorado.edu/en/simulations/gas-properties) · NASA Glenn Beginner's Guide: Brayton cycle (https://www.grc.nasa.gov/www/k-12/airplane/brayton.html)

Planet link: NASA's HyTEC project developed small-core engine technologies for fuel-burn reductions. NASA Aeronautics: Hybrid Thermally Efficient Core (HyTEC) project (https://www.nasa.gov/directorates/armd/aavp/hytec/)

- [ ] **I completed this lab (wet or virtual)**

Lab C: Building isentropic flow tables

WEEK 4

Goal: Compute T_0/T, p_0/p and A/A^* versus Mach number for air and find where the area is smallest.

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^3$$

`(A)/(A^*)=1M[(2)/(γ+1)(1+(γ-1)/(2)M^2)]^3`

Area–Mach relation (γ = 1.4)

Procedure

1. List M from 0.2 to 3 in steps of 0.1.
2. Compute T_0/T, p_0/p and A/A^*.
3. Plot A/A^* and confirm its minimum of 1 at M = 1.
4. Read off the area ratio a Mach 2 nozzle needs.

Data table

M	p_0/p	A/A^*
0.8	1.524	1.038
1.0	1.893	1.000
2.0	7.824	1.688
3.0	36.73	4.235

Computed values for γ = 1.4.

Sample calculation

1. p_0/p at M = 2: 1.8^3.5 = 7.824.
2. A/A^* at M = 0.8: (1/0.8)(1.128/1.2)³ = 1.038.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/)

Planet link: NASA reports Artemis I RS-25 valves within 0.5% of predicted values. NASA: SLS (Space Launch System) RS-25 Core Stage Engine (https://www.nasa.gov/reference/space-launch-system-rs-25-core-stage-engine/)

- [ ] **I completed this lab (wet or virtual)**

Lab D: The turbofan bypass trade

WEEK 7

Goal: Sweep bypass ratio for a fixed core and fan speed and compare thrust and fan-stream efficiency.

$$F = \dot{m}_c V_e + bpr\, \dot{m}_c V_f - \dot{m}_0 V_0$$

```
F= m_cV_e+bpr m_cV_f- m_0V_0
```

Turbofan thrust

Procedure

1. Fix ṁ_c = 100 kg/s, V_e = 450, V_f = 300, V_0 = 250 m/s.
2. Sweep bpr from 0 to 12.
3. Compute F and the thrust ratio to bpr = 0.
4. Discuss why real engines stop near today's bypass ratios.

Data table

bpr	F (kN)	Ratio
0	20	1.0
5	45	2.25
10	70	3.5
12	80	4.0

Computed sample values.

Sample calculation

1. F(5) = 45,000 + 150,000 − 150,000 = 45 kN.
2. Each unit of bpr adds 100 × 50 = 5 kN.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

Planet link: NASA Glenn: a turbofan makes more thrust for nearly the same core fuel flow. NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Rocket nozzle expansion and Isp

WEEK 11

Goal: Compute ideal rocket exhaust velocity and Isp as the nozzle pressure ratio changes, then relate RS-25 power levels to thrust.

$$V_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{R_u T_0}{\mathcal{M}}\left[1 - (p_e/p_0)^{(\gamma-1)/\gamma}\right]}$$

```
V_e=(2γ)/(γ-1)(R_uT_0)/( M)[1-(p_e/p_0)^(γ-1)/γ]
```

Ideal exit velocity

Procedure

1. Use γ = 1.2, molar mass 13 g/mol and T_0 = 3500 K (sample values).
2. Sweep p_e/p_0 from 0.1 to 0.0001.
3. Compute V_e and Isp.
4. Tabulate RS-25 thrust at 104.5%, 109%, 111% and 113% of 469,856 lbf.

Data table

p_e/p_0	V_e (m/s)	Isp (s)
0.01	3794	387
0.001	4286	437

Computed sample values; RS-25 levels from NASA.

Sample calculation

1. V_e = √(12 × 639.5 × 3500 × (1 − 0.001^0.1667)) = 4286 m/s.
2. 109% × 469,856 = 512,143 lbf.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn Beginner's Guide: Rocket thrust summary (https://www.grc.nasa.gov/www/k-12/rocket/rktthsum.html) · NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html)

Planet link: NASA says new RS-25s operate at 111% and testing reached 113%. NASA: SLS (Space Launch System) RS-25 Core Stage Engine (https://www.nasa.gov/reference/space-launch-system-rs-25-core-stage-engine/)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Energy density and the SAF question

WEEK 13

Goal: Compare the mass needed to carry the same energy as jet fuel, liquid hydrogen, a battery and a higher-energy SAF.

$$m = E/e$$

`m=E/e`

Mass for an energy

Procedure

1. Set E = 43,000 MJ (1000 kg of jet fuel, textbook value).
2. Compute mass for each energy carrier.
3. Plot the masses on a log axis.
4. Write a paragraph on where each option fits.

Data table

Carrier	MJ/kg	Mass (kg)
Jet fuel	43	1000
Liquid hydrogen	120	358
Battery	0.25	172,000
SAF (+12%)	48.16	893

Textbook specific energies; SAF energy gain from DOE.

Sample calculation

1. 43,000/0.25 = 172,000 kg.
2. 43,000/48.16 = 893 kg.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

U.S. Department of Energy: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels)

Planet link: DOE says the US could produce more than 1 billion tons of biomass a year. U.S. Department of Energy: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels)

☐ **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. An engine moves 80 kg/s from 200 to 500 m/s. Find thrust and η_p.

    1. F = 80 × 300.
    2. η_p = 2/(1 + 2.5).

    Answer: 24 kN; 0.571.

2. Find ideal Brayton efficiency at π = 25.

    1. 1 − 25^(−0.2857).

    Answer: 60.1%.

3. Find T_0/T at Mach 2.

    1. 1 + 0.2 × 4.

    Answer: 1.8.

4. Find the compressor exit temperature for π = 20 from 288 K.

    1. 288 × 20^0.2857.

    Answer: 678 K.

5. A turbofan has ṁ_c = 50 kg/s, bpr = 8, V_e = 400, V_f = 280, V_0 = 240 m/s. Find thrust.

    1. 50 × 400 + 400 × 280 − 450 × 240.

    Answer: 24 kN.

Final practice set (Units 1–10)

1. An engine makes 50 kN at 240 m/s burning 0.9 kg/s of 43 MJ/kg fuel. Find η_0 and TSFC.

    1. 50,000 × 240/(0.9 × 43 × 10^6).
    2. 0.9/50,000.

    Answer: 0.310; 18 mg/(N·s).

2. Find CO_2 per hour for 0.9 kg/s of fuel (3.16 kg/kg, textbook).

    1. 0.9 × 3600 × 3.16.

    Answer: 10,238 kg.

3. Find V_eq for Isp 350 s.

    1. 350 × 9.80665.

    Answer: 3432 m/s.

4. Find Δv for V_e = 3432 m/s and mass ratio 5.

    1. 3432 ln 5.

    Answer: 5524 m/s.

5. A 2 kW ion thruster (η = 0.6) has V_e = 30 km/s. Find thrust.

    1. 2 × 0.6 × 2000/30,000.

    Answer: 0.08 N.

6. How much liquid hydrogen (120 MJ/kg) matches 500 kg of jet fuel (43 MJ/kg)?

    1. 500 × 43/120.

    Answer: 179 kg.

Formula sheet

$$F = \dot{m}(V_e - V_0) + (p_e - p_0)A_e,\ \eta_p = 2/(1 + V_e/V_0$$

$$\eta_{th} = 1 - \pi^{-(\gamma-1)/\gamma},\ T_2/T_1 = \pi^{(\gamma-1)/\gamma}$$

$$T_0/T = 1 + \frac{\gamma - 1}{2}M^2,\ p_0/p = (T_0/T)^{\gamma/(\gamma-1)}$$

$$w_c = c_p(T_3 - T_2) = c_p(T_4 - T_5)$$

$$F = \dot{m}_c V_e + bpr\,\dot{m}_c V_f - \dot{m}_0 V_0$$

$$\eta_0 = FV_0/(\dot{m}_f LHV),\ TSFC = \dot{m}_f/F$$

$$I_{sp} = F/(\dot{m} g_0),\ I = F\Delta t$$

$$V_e = \sqrt{\frac{2\gamma}{\gamma - 1}\frac{R_u T_0}{\mathcal{M}}\left[1 - (p_e/p_0)^{(\gamma-1)/\gamma}\right]}$$

$$\Delta v = V_e \ln(m_0/m_f),\ F = 2\eta P/V_e$$

$$m = E/e$$

Constants

Constant	Value
Air (textbook values)	γ = 1.4, c_p = 1005 J/(kg·K)
Standard gravity	g_0 = 9.80665 m/s²
1 lbf	4.44822 N
Jet fuel (textbook values)	heating value 43 MJ/kg; 3.16 kg CO_2 per kg burned
Liquid hydrogen (textbook value)	heating value 120 MJ/kg
Universal gas constant	8314 J/(kmol·K)

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, NASA guides and simulations alongside the course.

- MIT OpenCourseWare 16.50 Introduction to Propulsion Systems (Spring 2012) (https://ocw.mit.edu/courses/16-50-introduction-to-propulsion-systems-spring-2012/): Free MIT propulsion lectures and problem sets.
- MIT OpenCourseWare 16.512 Rocket Propulsion (Fall 2005) (https://ocw.mit.edu/courses/16-512-rocket-propulsion-fall-2005/): Free MIT rocket propulsion course.
- NASA Glenn Beginner's Guide: Turbofan thrust (https://www.grc.nasa.gov/www/k-12/airplane/turbfan.html): Free NASA explainer.
- NASA Glenn Beginner's Guide: Specific impulse (https://www.grc.nasa.gov/www/k-12/airplane/specimp.html): Free NASA explainer.
- NASA Glenn Beginner's Guide: Brayton cycle (https://www.grc.nasa.gov/www/k-12/airplane/brayton.html): Free NASA explainer.
- PhET: Gas Properties (https://phet.colorado.edu/en/simulations/gas-properties): Gas laws interactively.

Glossary

Afterburner
Burner after the turbine for extra thrust.

Area ratio

A/A*, nozzle area over throat area.

Brayton cycle
Gas-turbine thermodynamic cycle.

Bypass ratio
Fan flow over core flow.

Choked flow
Sonic flow at a throat that caps mass flow.

Compressor
Raises air pressure.

Hall thruster
Electric thruster using crossed fields.

Isentropic
Reversible and adiabatic.

Mach number
Speed over sound speed.

Overall efficiency
Thrust power over fuel power.

Pressure ratio
Compressor exit over inlet pressure.

Propulsive efficiency
Thrust power over jet power.

Rocket equation
$\Delta v = V_e \ln(m_0/m_f)$.

SAF
Sustainable aviation fuel.

Specific impulse
Thrust per weight flow of propellant.

Specific thrust
Thrust per unit air flow.

Stagnation temperature
Temperature if flow is brought to rest adiabatically.

TSFC
Fuel flow per thrust.

Turbine
Extracts work to drive the compressor.

Turbofan
Gas turbine with a bypass fan.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Sets up control volumes and stations, picks the right relations, checks units and explains efficiency and emissions trades in plain words.
3 · Proficient	Solves the practice problems with occasional slips in units or stations.
2 · Developing	Follows worked examples but mixes stagnation and static values or Isp and exhaust velocity.
1 · Beginning	Plugs numbers into formulas without a station diagram; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Probability & Statistics MATH 350
- Flight Dynamics & Controls AERO 420
- Spacecraft Design AERO 430
- All courses