Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 2, Spring

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester sophomore mechanics of materials (strength of materials) course. It is modeled on the topics and depth of MIT 2.001 Mechanics and Materials I (on MIT OpenCourseWare), Purdue ME 323 Mechanics of Materials and Georgia Tech COE 3001 Mechanics of Deformable Bodies, and on the free OpenStax University Physics, which we link where it helps. None of those authors or schools endorses this page.
>
> What we add: every unit ties to real clean-energy, infrastructure or space work with a cited public source (bridges, lighter cars, wind turbine towers, shafts and blades, mass timber, hydrogen tanks, NASA rocket structures), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a cheap home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Mechanics of materials decides how much material a structure needs. Using just enough steel, aluminum, composite or wood, and no more, makes cars lighter and more efficient, wind turbines taller, buildings lower in carbon, and rockets able to carry more to orbit.

It is also how we keep what we have built safe: bridges, towers and tanks must carry their loads for decades without yielding, cracking from fatigue or buckling. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values, and material properties are typical textbook values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (ENGR 213).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Statics: free-body diagrams, equilibrium, centroids and area moments of inertia.
- Calculus I and II: derivatives and integrals; Unit 9 integrates the beam equation twice.
- Physics I helps with springs, energy and units.
- Dynamics is not required.

How to study

- Start every problem with statics: a free-body diagram and the internal force, torque or moment at the cut.
- Then ask how the part deforms: stretch, twist or bend, and which formula fits that deformation.
- Check that the formula applies: linear elastic, small deformations, thin walls or long slender members where required.
- Solve with symbols first, then put in SI numbers (N, m, Pa), and check units: 1 MPa = 1 N/mm^2.
- Compare the stress with the material's allowable stress and state a factor of safety.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free PhET simulations (they run in a browser).
- Rubber bands, a bag for coins or washers, and a kitchen scale (to know each mass).

- A 30 cm plastic ruler, a second ruler or tape measure, and tape or a clamp to hold it to a table edge.
- A foam rod or pool-noodle offcut and a marker for torsion lines.
- Dry spaghetti and small weights for the buckling lab.
- Safety glasses, a smartphone camera and spreadsheet software for graphs and fits.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety	Internal forces, normal and shear stress	MIT 2.001; OpenStax UP1 12.3
2	Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety	Bearing stress, allowable stress, factor of safety; Lab A	MIT 2.001
3	Strain and Material Behavior: Hooke’s Law, Poisson’s Ratio and the Stress–Strain Curve	Strain, stress–strain curve, Hooke’s law, Poisson’s ratio; Lab B	UP1 12.3–12.4
4	Axially Loaded Members: Deformation, Indeterminate Bars and Thermal Stress	Axial deformation, indeterminate bars, thermal stress	UP2 1.3
5	Torsion of Circular Shafts and Power Transmission	Torsion formula, polar moment, angle of twist	MIT 2.001
6	Torsion of Circular Shafts and Power Transmission	Power transmission, shaft design; Lab C	MIT 2.001
7	Shear and Moment Diagrams and Bending Stress	Shear and moment diagrams	MIT 2.001; UP1 12.2
8	Shear and Moment Diagrams and Bending Stress	Flexure formula, section modulus; Lab D; midterm practice	MIT 2.001
9	Transverse Shear in Beams, Shear Flow and Composite Sections	Transverse shear, shear flow, built-up beams	MIT 2.001
10	Stress Transformation, Mohr’s Circle, Failure Theories and Fatigue	Stress transformation, Mohr’s circle, failure theories, fatigue	MIT 2.001, 2.002
11	Thin-Walled Pressure Vessels and Combined Loading	Pressure vessels and combined loading	MIT 2.001

Week	Unit	Topics	Reading
12	Deflection of Beams	Elastic curve, integration, standard formulas; Lab E	MIT 2.001
13	Deflection of Beams	Superposition and indeterminate beams	MIT 2.001
14	Buckling of Columns	Column buckling and knockdown factors; Lab F	MIT 2.001
15	Review	Review and final practice set	All units

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety

Internal forces found by cutting a member, average normal stress in rods, average shear stress in pins and bolts (single and double shear), bearing stress, allowable stress design and the factor of safety.

Learning objectives

By the end of this unit you can:

- Find internal forces with a free-body diagram of a cut section.
- Compute average normal, shear and bearing stresses.
- Tell single shear from double shear in connections.
- Size a member from an allowable stress and a factor of safety.

Key concepts

- Stress is internal force per unit area. Its SI unit is the pascal (1 Pa = 1 N/m²); engineers use MPa, and 1 MPa = 1 N/mm².
- Normal stress σ = P/A acts perpendicular to a cut; tension is positive and compression negative.
- Average shear stress τ = V/A acts in the plane of the cut. A pin in double shear has two cut planes, so each carries V = P/2.
- Bearing stress σ_b = P/(td) is the contact pressure between a bolt and the hole in a plate of thickness t.
- Factor of safety FS = failure load/allowable load (or failure stress/allowable stress). Design keeps working stress at or below σ_allow = σ_fail/FS.

Key equations

$$\sigma = \frac{P}{A}$$

Average normal stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

`τ_avg=V/A`

Average shear stress

$$\sigma_b = \frac{P}{t\,d}$$

σ_b=(P)/(t d)

Bearing stress

$$FS = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

FS=σ_failσ_allow

Factor of safety

Worked examples, step by step

Example 1.1 A steel rod of 20 mm diameter carries a tensile load of 50 kN. Find the normal stress.

1. A = πd²/4 = π × 20²/4 = 314.2 mm².
2. σ = P/A = 50,000 N/314.2 mm².

Answer: 159.2 MPa (tension).

Example 1.2 A 12 mm pin in double shear carries a 20 kN load. Find the average shear stress in the pin.

1. Each of the two shear planes carries V = 10 kN.
2. A = π × 12²/4 = 113.1 mm²; τ = 10,000/113.1.

Answer: 88.4 MPa.

Example 1.3 A round steel hanger must carry 100 kN. The steel yields at 250 MPa and the factor of safety is 2. Find the smallest diameter.

1. σ_allow = 250/2 = 125 MPa.
2. A = P/σ_allow = 100,000/125 = 800 mm².
3. d = √(4A/π).

Answer: d = 31.9 mm (use 32 mm).

Practice problems

Work each on paper first, then check.

1. A 100 mm × 100 mm square post carries 200 kN in compression. Find the stress.

σ = 200,000/10,000 = 20 MPa (compression).

2. A 10 mm bolt in single shear carries 8 kN. Find the average shear stress.

   $A = 78.54\ mm^2$; τ = 101.9 MPa.

3. The same 10 mm bolt passes through a 6 mm thick plate. Find the bearing stress.

   $\sigma_b = 8000/(6 \times 10)$ = 133 MPa.

4. A link fails at 90 kN and works at 30 kN. Find the factor of safety.

   FS = 90/30 = 3.

Common misconceptions

Myth: Stress is the same as force.

Actually: Stress is force per area; a thin wire and a thick bar with the same force have very different stresses.

Myth: A pin in double shear carries the full load on each plane.

Actually: Each of the two planes carries half.

Myth: A higher factor of safety is always better.

Actually: It also means heavier, costlier parts; designers choose FS from codes and the uncertainty in loads and materials.

Flashcards

Normal stress

$\sigma = P/A$.

Shear stress

$\tau = V/A$.

Double shear

Each plane carries P/2.

Bearing stress

$\sigma_b = P/(td)$.

Factor of safety

FS = failure/allowable.

MPa

1 MPa = 1 N/mm^2.

Self-check quiz

1. 1 MPa equals…

1 N/m^2

1 N/mm^2

1 kN/m^2

1 N/cm^2

1 N/mm^2. 10^6 N/m^2 = 1 N/mm^2.

2. A pin in double shear has how many shear planes?

1

2

3

4

2. Two cut planes share the load.

3. Doubling a rod's diameter changes its axial stress (same load) by a factor of…

½

¼

2

4

¼. A grows with d^2.

4. Bearing stress is…

P/(td)

P/A of the bolt

V/A

Mc/I

P/(td). Contact pressure on the projected area td.

Try this quiz again

How this helps save the planet

Stress limits decide which bridges trucks may cross. The ASCE Report Card for America's Infrastructure counts 623,218 bridges in the United States, and 63,085 of them were posted for load in 2024, meaning they restrict the weight of vehicles that can safely cross, which is about 10.1% of all bridges. ASCE warns that trucks heavier than bridges were designed for threaten to overstress bridge elements and cause metal fatigue and cracking. Keeping working stress below the allowable stress, as in this unit, is how engineers rate and repair them, and preserving a bridge costs less than replacing it.

On Saving Planets: Statics

Sources: ASCE Report Card for America's Infrastructure: Bridges (https://infrastructurereportcard.org/cat-item/bridges-infrastructure/) · OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus)

Free learning resources

- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lecture notes on stress.
- PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law): Interactive.

☐ **I finished this unit**

Strain and Material Behavior: Hooke’s Law, Poisson’s Ratio and the Stress–Strain Curve

Normal and shear strain, the tensile test and the stress–strain diagram, Young’s modulus, yield and ultimate strength, ductility, Poisson’s ratio, the shear modulus, strain energy and specific stiffness.

Learning objectives

By the end of this unit you can:

- Compute normal and shear strain from deformations.
- Read E, yield strength, ultimate strength and ductility from a stress–strain curve.
- Use Hooke’s law, Poisson’s ratio and G = E/(2(1 + ν)).
- Compare materials by stiffness and strength per unit mass.

Key concepts

- Normal strain ε = δ/L is a stretch per unit length (dimensionless, often in μm/m). Shear strain γ is the change in a right angle, in radians.
- In the linear-elastic range, Hooke’s law holds: σ = Eε and τ = Gγ. E is Young’s modulus, the slope of the stress–strain curve.
- Beyond the yield strength, the material deforms plastically and does not spring back; the ultimate strength is the peak engineering stress, and ductile metals neck before they break.
- Stretching a bar makes it thinner: lateral strain = −ν × axial strain, with Poisson’s ratio ν about 0.3 for metals. For isotropic materials G = E/(2(1 + ν)).
- Specific stiffness E/ρ and specific strength σ/ρ compare materials per kilogram; that is why aluminum, magnesium and carbon fiber can replace steel in light structures.

Key equations

$$\varepsilon = \frac{\delta}{L}, \quad \sigma = E\varepsilon$$

Strain and Hooke’s law

$$\varepsilon_{\text{lat}} = -\nu\,\varepsilon_{\text{axial}}$$

_lat=-ν _axial

Poisson’s ratio

$$G = \frac{E}{2(1+\nu)}$$

`G=(E)/(2(1+ν))`

Shear modulus

$$u_r = \frac{\sigma_Y^2}{2E}$$

`u_r=(σ_Y^2)/(2E)`

Modulus of resilience

Worked examples, step by step

Example 2.1 A 2 m steel bar stretches 1.2 mm when the stress is 120 MPa. Find the strain and Young's modulus.

1. ε = 1.2/2000 = 6 × 10⁻⁴.
2. E = σ/ε = 120 MPa/0.0006.

Answer: ε = 600 μm/m; E = 200 GPa.

Example 2.2 An aluminum alloy has E = 70 GPa and ν = 0.33. Find its shear modulus.

1. G = E/(2(1 + ν)) = 70/2.66.

Answer: G = 26.3 GPa.

Example 2.3 A 20 mm steel rod (ν = 0.3) has an axial strain of 0.001. How much does its diameter change?

1. Lateral strain = −0.3 × 0.001 = −0.0003.
2. Δd = −0.0003 × 20 mm.

Answer: It shrinks by 0.006 mm.

Practice problems

Work each on paper first, then check.

1. Typical values: steel has E = 200 GPa and ρ = 7850 kg/m³; aluminum has E = 70 GPa and ρ = 2700 kg/m³. Compare E/ρ.

   Steel 25.5 × 10⁶ m²/s², aluminum 25.9 × 10⁶ m²/s²: almost the same stiffness per kilogram.

2. A tensile specimen with a 50 mm gauge length measures 62 mm at fracture. Find the elongation at break.

   (62 − 50)/50 = 24%: a ductile metal.

3. Find the modulus of resilience of steel with σ_Y = 250 MPa and E = 200 GPa.

   u_r = 250²/(2 × 200,000) = 0.156 MPa = 156 kJ/m³.

4. Sample values: a car that gets 30 mpg is made 10% lighter. Using DOE's 6% to 8% figure, estimate its new fuel economy.

   30 × 1.06 = 31.8 to 30 × 1.08 = 32.4 mpg (sample car).

Common misconceptions

Myth: Stronger means stiffer.

Actually: Strength (yield) and stiffness (E) are different; most steels share E = 200 GPa whatever their strength.

Myth: Strain has units of length.

Actually: It is length per length, dimensionless.

Myth: A material breaks at its yield strength.

Actually: Yield starts permanent deformation; ductile metals carry more load up to the ultimate strength.

Flashcards

Normal strain

ε = δ/L.

Hooke's law

σ = Eε, τ = Gγ.

Poisson's ratio

ε_lat = −νε.

Shear modulus

G = E/(2(1 + ν)).

Yield vs ultimate

Specific stiffness

Start of plastic flow vs peak stress.	E/ρ.

Self-check quiz

1. Young’s modulus is the slope of…

load vs time

stress vs strain (elastic part)

strain vs temperature

force vs area

stress vs strain (elastic part). σ = Eε.

2. Poisson’s ratio for most metals is about…

0.03

0.3

3

30

0.3. About 0.3.

3. G for E = 200 GPa and ν = 0.25 is…

50 GPa

80 GPa

100 GPa

160 GPa

80 GPa. 200/2.5 = 80 GPa.

4. Permanent deformation starts at the…

ultimate strength

yield strength

fracture point

proportional limit only for wood

yield strength. Yield.

Try this quiz again

How this helps save the planet

Choosing materials by stiffness and strength per kilogram saves fuel. The U.S. Department of Energy says a 10% reduction in vehicle weight can result in a 6% to 8% fuel economy improvement, and that replacing cast iron and traditional steel with high-strength steel, magnesium and aluminum alloys, carbon fiber and polymer composites can cut the weight of a vehicle's body and chassis by up to 50 percent. DOE adds that lightweight materials are especially important for electric vehicles, where they offset the weight of batteries and motors and can increase all-electric range.

On Saving Planets: General Chemistry I

Sources: U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks) · OpenStax UP1, 12.4 Elasticity and plasticity (https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity)

Free learning resources

- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- OpenStax UP1, 12.4 Elasticity and plasticity (https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity): Stress–strain curves.
- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures.

☐ **I finished this unit**

Axially Loaded Members: Deformation, Indeterminate Bars and Thermal Stress

Elongation of bars δ = PL/(AE), bars with several segments, statically indeterminate bars solved with compatibility, thermal strain and thermal stress, and stress concentrations.

Learning objectives

By the end of this unit you can:

- Compute the elongation of uniform and stepped bars.
- Solve statically indeterminate axial problems with equilibrium plus compatibility.
- Compute thermal expansion and the stress in a restrained bar.
- Explain stress concentrations at holes and fillets.

Key concepts

- A uniform bar under axial load stretches δ = PL/(AE). For several segments, add the δ of each: δ = Σ $P_iL_i/(A_iE_i)$.
- If equilibrium alone cannot give the forces (statically indeterminate), add a compatibility equation: how the deformations must fit together.
- A temperature change makes a free bar grow δ_T = αΔT L with no stress. If the bar is held, the growth is pushed back and σ = −EαΔT appears.
- Rails, pipelines, bridges and pavements need expansion gaps or joints for this reason.
- Holes, notches and sharp corners raise local stress: σ_max = K σ_avg, where K is the stress concentration factor (often 2 to 3 for a small hole).

Key equations

$$\delta = \frac{PL}{AE}$$

`=PL/AE`

Axial deformation

$$\delta = \sum \frac{P_i L_i}{A_i E_i}$$

`=(P_iL_i)/(A_iE_i)`

Stepped bar

$$\delta_T = \alpha \, \Delta T \, L$$

_T=α Δ T L

Thermal expansion

$$\sigma = -E\alpha\,\Delta T$$

Fully restrained bar

Worked examples, step by step

Example 3.1 A 3 m steel rod (E = 200 GPa) with A = 500 mm² carries 60 kN. Find its elongation.

1. δ = PL/(AE) = 60,000 × 3000/(500 × 200,000) in N and mm.

Answer: 1.8 mm.

Example 3.2 A steel bar (E = 200 GPa, α = 12 × 10⁻⁶/°C) is fixed between two rigid walls and heated by 40 °C. Find the stress.

1. Free growth would be αΔT L; the walls push it back.
2. σ = −EαΔT = −200,000 × 12 × 10⁻⁶ × 40 MPa.

Answer: 96 MPa compression.

Example 3.3 A uniform bar fixed at both ends carries an axial load P = 30 kN at a point 1 m from end A and 2 m from end B. Find both wall reactions.

1. Equilibrium: R_A + R_B = 30 kN.
2. Compatibility: the stretch of segment A equals the shortening of segment B, so R_A × 1 = R_B × 2.

Answer: R_A = 20 kN, R_B = 10 kN.

Practice problems

Work each on paper first, then check.

1. A 25 m steel rail warms by 30 °C with nothing holding it. How much does it grow?

   δ = 12 × 10⁻⁶ × 30 × 25,000 = 9 mm.

2. A bar has an aluminum segment (E = 70 GPa, A = 400 mm², L = 1 m) and a steel segment (A = 200 mm², L = 0.5 m) in series, carrying 20 kN. Find the total stretch.

   0.714 + 0.250 = 0.964 mm.

3. Find the strain in aluminum at 100 MPa (E = 70 GPa).

   $\varepsilon = 100/70{,}000 = 1.43 \times 10^{-3}$.

4. DOE says land-based hub heights reached about 103.4 m in 2023 after growing 83% since 1998–1999. What was the earlier average?

   103.4/1.83 = 56.5 m.

Common misconceptions

Myth: Thermal expansion always causes stress.

Actually: Only when the expansion is restrained.

Myth: An indeterminate bar can be solved with equilibrium alone.

Actually: You need compatibility (deformation) equations too.

Myth: A small hole barely matters.

Actually: It can triple the local stress, which is where fatigue cracks start.

Flashcards

Axial deformation

$\delta = PL/(AE)$.

Stepped bar

Add $PL/(AE)$ for each segment.

Compatibility

Deformations must fit together.

Thermal strain

$\alpha\Delta T$.

Thermal stress

$\sigma = -E\alpha\Delta T$ if fully held.

Stress concentration

$\sigma_{max} = K\sigma_{avg}$.

Self-check quiz

1. Doubling the length of a bar (same load and area) makes δ...

half

the same

double

four times

double. $\delta \propto L$.

2. A restrained bar that is heated is in...

tension

compression

no stress

shear only

compression. The supports push back.

3. Statically indeterminate problems need...

only equilibrium

compatibility of deformations

dynamics

no free-body diagram

compatibility of deformations. More unknowns than equilibrium equations.

4. Thermal stress in a fully restrained bar does not depend on...

E

α

ΔT

its length

its length. $\sigma = -E\alpha\Delta T$.

Try this quiz again

How this helps save the planet

Taller wind turbines catch stronger wind but need taller towers. The U.S. Department of Energy reports that the hub height of utility-scale land-based turbines grew 83% since 1998–1999 to about 103.4 meters in 2023, taller than the Statue of Liberty, and that offshore hub heights are projected to reach about 150 meters by 2035. Winds are generally stronger higher up. DOE notes tower diameters are hard to truck under bridges, so it supports towers built on site, including spiral welding to minimize costly steel. Each tower is an axially loaded column that must carry the weight above it within its allowable stress.

On Saving Planets: Physics II

Sources: U.S. DOE: Wind turbines, the bigger the better (https://www.energy.gov/eere/articles/wind-turbines-bigger-better) · OpenStax UP2, 1.3 Thermal expansion (https://openstax.org/books/university-physics-volume-2/pages/1-3-thermal-expansion)

Free learning resources

- OpenStax UP2, 1.3 Thermal expansion (https://openstax.org/books/university-physics-volume-2/pages/1-3-thermal-expansion): Free textbook section.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Axial deformation.
- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures.

☐ **I finished this unit**

Torsion of Circular Shafts and Power Transmission

Shear strain and stress in twisted circular shafts, the polar moment of inertia for solid and hollow shafts, angle of twist, power and torque, shaft design, and statically indeterminate shafts.

Learning objectives

By the end of this unit you can:

- Compute shear stress with τ = Tρ/J.
- Compute the angle of twist φ = TL/(GJ).
- Convert power and speed to torque with P = Tω.
- Size solid and hollow shafts and explain why hollow shafts save material.

Key concepts

- In a twisted circular shaft, plane sections stay plane and shear strain grows linearly from the center: γ = ρφ/L.
- Shear stress is τ = Tρ/J, largest at the outer surface: τ_max = Tc/J.
- Polar moment of inertia: J = πd⁴/32 for a solid shaft and J = π(d_o⁴ − d_i⁴)/32 for a hollow one.
- Angle of twist φ = TL/(GJ) in radians; for several segments, add them.
- Power P = Tω. Slow shafts carry huge torques for the same power, which is why a gearbox speeds up a wind turbine’s rotation before the generator.

Key equations

$$\tau = \frac{T\rho}{J}, \quad \tau_{\max} = \frac{Tc}{J}$$

`τ=(Tρ)/(J), τ_=Tc/J`

Torsion formula

$$J = \frac{\pi d^4}{32}, \quad J = \frac{\pi (d_o^4 - d_i^4)}{32}$$

`J=(π d^4)/(32), J=(π(d_o^4-d_i^4))/(32)`

Polar moment of inertia

$$\phi = \frac{TL}{GJ}$$

`=TL/GJ`

Angle of twist

$$P = T\omega$$

Power

Worked examples, step by step

Example 4.1 A solid 50 mm steel shaft carries a torque of 1.5 kN·m. Find the maximum shear stress.

1. $J = \pi \times 0.05^4/32 = 6.136 \times 10^{-7}$ m⁴.
2. $\tau_{max} = Tc/J = 1500 \times 0.025/6.136 \times 10^{-7}$.

Answer: 61.1 MPa.

Example 4.2 The same shaft is 2 m long with G = 80 GPa. Find the angle of twist.

1. $\varphi = TL/(GJ) = 1500 \times 2/(80 \times 10^9 \times 6.136 \times 10^{-7})$.

Answer: 0.0611 rad = 3.50°.

Example 4.3 A motor delivers 50 kW at 1,500 rpm. Size a solid shaft for an allowable shear stress of 40 MPa.

1. $\omega = 1500 \times 2\pi/60 = 157.1$ rad/s; $T = P/\omega = 318.3$ N·m.
2. $\tau = 16T/(\pi d^3)$ gives $d = (16T/(\pi\tau))\text{^}(1/3)$.

Answer: d = 34.3 mm (use 35 mm).

Practice problems

Work each on paper first, then check.

1. A hollow shaft (d_o = 60 mm, d_i = 40 mm) carries 2 kN·m. Find τ_{max}.

   $J = 1.021 \times 10^{-6}$ m⁴; $\tau = 2000 \times 0.03/J = 58.8$ MPa.

2. Compare that hollow shaft with a solid 60 mm shaft: what fraction of the material and of the stiffness J does it keep?

   Area $1 - (40/60)^2 = 55.6\%$; $J\ 1 - (40/60)^4 = 80.2\%$.

3. A solid 30 mm shaft carries 500 N·m. Find τ_{max}.

$\tau = 16T/(\pi d^3) = 94.3$ MPa.

4. Sample values: a 3.4 MW wind turbine rotor turns at a sample 12 rpm. Find the torque on its main shaft.

   $\omega = 1.257$ rad/s; $T = P/\omega = 2.71$ MN·m (sample speed).

Common misconceptions

Myth: Shear stress is uniform across a shaft.

Actually: It is zero at the center and largest at the surface.

Myth: Torque and power are the same.

Actually: $P = T\omega$: the same power at low speed needs much more torque.

Myth: A hollow shaft is much weaker.

Actually: Material near the center carries little stress; removing it saves weight for a small loss in J.

Flashcards

Torsion formula

$\tau = T\rho/J$.

Solid J

$\pi d^4/32$.

Hollow J

$\pi(d_o^4 - d_i^4)/32$.

Angle of twist

$\varphi = TL/(GJ)$.

Power

$P = T\omega$.

Shaft sizing

d = (16T/(πτ))^(1/3).

Self-check quiz

1. In a solid shaft, maximum shear stress occurs...

at the center

at the outer surface

halfway out

everywhere equally

at the outer surface. $\tau \propto \rho$.

2. Doubling a solid shaft's diameter multiplies J by...

2

4

8

16

16. $J \propto d^4$.

3. Angle of twist is proportional to...

G

J

L

1/L

L. $\varphi = TL/(GJ)$.

4. A gearbox that speeds up a shaft 100 times (no losses) changes torque by a factor of...

100

1/100

1

10

1/100. $P = T\omega$ is constant.

Try this quiz again

How this helps save the planet

A wind turbine sends its power through a twisted shaft. The U.S. Department of Energy explains that the rotor connects to the generator either directly, in a direct-drive turbine, or through a shaft and a series of gears that speed up the rotation and allow a physically smaller generator. DOE also reports that the average capacity of newly installed U.S. wind turbines reached 3.4 MW in 2023. Because power is torque times speed, a slow, large rotor puts millions of newton-meters on its main shaft, and the torsion formulas of this unit size it.

On Saving Planets: Dynamics

Sources: U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work) · U.S. DOE: Wind turbines, the bigger the better (https://www.energy.gov/eere/articles/wind-turbines-bigger-better)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on torsion.
- OpenStax UP1, 10.6 Torque (https://openstax.org/books/university-physics-volume-1/pages/10-6-torque): Free textbook section on torque.
- PhET: Torque (https://phet.colorado.edu/en/simulations/torque): Interactive.

☐ **I finished this unit**

Shear and Moment Diagrams and Bending Stress

Internal shear and bending moment in beams, the relations dV/dx = −w and dM/dx = V, shear and moment diagrams, the flexure formula σ = My/I, the section modulus and efficient beam shapes.

Learning objectives

By the end of this unit you can:

- Draw shear and moment diagrams for common loads.
- Use dV/dx = −w and dM/dx = V to check diagrams.
- Compute bending stress with σ = My/I.
- Choose efficient cross-sections using the section modulus S = I/c.

Key concepts

- Cut a beam and the internal shear V and moment M hold the piece in equilibrium. Slopes follow dV/dx = −w and dM/dx = V, so M is largest where V crosses zero.
- Pure bending keeps plane sections plane; strain and stress grow linearly from the neutral axis, which passes through the centroid.
- The flexure formula is σ = −My/I; the largest stress is σ_max = Mc/I = M/S, where S = I/c is the section modulus.
- For a rectangle, I = bh³/12, so depth matters much more than width: a plank on edge is far stiffer and stronger than one lying flat.
- I-beams and hollow tubes put material far from the neutral axis, giving a large I for little mass.

Key equations

$$\frac{dV}{dx} = -w, \quad \frac{dM}{dx} = V$$

`dV/dx=-w, dM/dx=V`

Load, shear and moment

$$\sigma = -\frac{My}{I}, \quad \sigma_{\max} = \frac{Mc}{I} = \frac{M}{S}$$

`σ=-My/I, σ_=Mc/I=M/S`

Flexure formula

$$I = \frac{bh^3}{12}, \quad I = \frac{\pi d^4}{64}$$

`I=(bh^3)/(12), I=(π d^4)/(64)`

Rectangle and circle

$$M_{\max} = \frac{PL}{4}, \quad M_{\max} = \frac{wL^2}{8}$$

`M_=PL/4, M_=(wL^2)/(8)`

Simply supported: center load, uniform load

Worked examples, step by step

Example 5.1 A simply supported 4 m beam carries a 10 kN load at midspan. Find the reactions and the maximum moment.

1. By symmetry each reaction is 5 kN; V = ±5 kN.
2. M_max = PL/4 at midspan.

Answer: 10 kN·m.

Example 5.2 The beam is a 100 mm wide × 200 mm deep rectangle. Find the maximum bending stress.

1. $I = 0.1 \times 0.2^3/12 = 6.667 \times 10^{-5}$ m⁴; c = 0.1 m.
2. $\sigma = Mc/I = 10{,}000 \times 0.1/6.667 \times 10^{-5}$.

Answer: 15 MPa (tension at the bottom, compression at the top).

Example 5.3 A simply supported 6 m beam carries 5 kN/m along its length. Find V_max and M_max.

1. Reactions wL/2 = 15 kN each.
2. M_max = wL²/8 at midspan, where V = 0.

Answer: V_max = 15 kN; M_max = 22.5 kN·m.

Practice problems

Work each on paper first, then check.

1. A 2 m cantilever carries 3 kN at its tip. Find the largest moment and where it acts.

   M = 3 × 2 = 6 kN·m at the wall.

2. A 200 mm × 50 mm plank is used on edge instead of flat. By what factor does I grow?

   $(200/50)^2 = 16$ times.

3. A solid 40 mm circular rod carries M = 500 N·m. Find σ_{max}.

   $\sigma = 32M/(\pi d^3) = 79.6$ MPa.

4. Sample values: NREL's test center can apply 84 MN·m to a blade root. If a root section had a sample S = 0.4 m^3, what bending stress would that cause?

   $\sigma = M/S = 84/0.4 = 210$ MPa (sample section).

Common misconceptions

Myth: Maximum moment is always under the biggest load.

Actually: It is where V crosses zero, which may be elsewhere.

Myth: Bending stress is uniform across the section.

Actually: It is zero at the neutral axis and largest at the top and bottom.

Myth: Width and depth help equally.

Actually: I grows with h^3 but only with b.

Flashcards

Shear–moment

$dM/dx = V$; $dV/dx = -w$.

Flexure formula

$\sigma = My/I$.

Section modulus

$S = I/c$; $\sigma = M/S$.

Rectangle I

$bh^3/12$.

Center load

$M_{max} = PL/4$.

Uniform load

$M_{max} = wL^2/8$.

Self-check quiz

1. dM/dx equals...

w

V

σ

EI

V. Slope of the moment diagram is the shear.

2. The neutral axis passes through the...

top fiber

centroid

support

shear center always

centroid. For pure bending of a linear-elastic beam.

3. Doubling a rectangular beam's depth multiplies I by...

2

4

8

16

8. h^3.

4. M_max for a simply supported beam with uniform load w is...

wL/2

$wL^2/8$

$wL^2/2$

PL/4

$wL^2/8$. Standard result.

Try this quiz again

How this helps save the planet

Wind turbine blades are huge cantilever beams, and they are bent on purpose before they fly. NREL's fact sheet on the Wind Technology Testing Center in Boston, built with the U.S. Department of Energy and the Massachusetts Clean Energy Center, describes the only U.S. facility then able to test blades up to 90 meters long. Static tests pull blades horizontally and vertically to measure deflection and strains, applying up to an 84 meganewton-meter bending moment. The flexure formula of this unit turns that moment into the stress that decides whether a blade passes certification.

On Saving Planets: Statics

Sources: NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (https://docs.nlr.gov/docs/fy12osti/53581.pdf)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on beams.
- OpenStax UP1, 12.2 Examples of static equilibrium (https://openstax.org/books/university-physics-volume-1/pages/12-2-examples-of-static-equilibrium): Free textbook section on equilibrium.
- PhET: Balancing Act (https://phet.colorado.edu/en/simulations/balancing-act): Moments, interactive.

☐ **I finished this unit**

UNIT 6 · WEEK 9

Transverse Shear in Beams, Shear Flow and Composite Sections

Shear stress from transverse loads, the shear formula τ = VQ/(It), shear stress in rectangular and circular sections, shear flow in built-up and glued beams, fastener spacing, and beams of two materials.

Learning objectives

By the end of this unit you can:

- Compute Q and the shear stress τ = VQ/(It).
- Use τ_max = 1.5V/A for rectangles and 4V/(3A) for circles.
- Find shear flow q = VQ/I and the spacing of nails, bolts or glue strength in built-up beams.
- Explain how layered beams act together.

Key concepts

- A beam with changing moment also carries shear stress on horizontal planes; that is why a stack of loose boards slides and bends much more than one glued block.
- The shear formula τ = VQ/(It) uses Q = A'ȳ', the first moment about the neutral axis of the area beyond the cut.
- In a rectangle the shear stress is parabolic, zero at the top and bottom and 1.5V/A at the neutral axis.
- Shear flow q = VQ/I is the force per unit length that a glue line or a row of fasteners must carry; fastener spacing s = F_fastener/q.
- In most long beams bending stress governs; in short, deep beams and in wood, shear can govern.

Key equations

$$\tau = \frac{VQ}{It}, \quad Q = A'\bar{y}'$$

`τ=VQ/It, Q=A' y'`

Shear formula

$$\tau_{\max} = \frac{3V}{2A}$$

`τ_=3V/2A`

Rectangle

$$\tau_{\max} = \frac{4V}{3A}$$

τ_=4V/3A

Solid circle

$$q = \frac{VQ}{I}, \quad s = \frac{F}{q}$$

q=VQ/I, s=F/q

Shear flow and fastener spacing

Worked examples, step by step

Example 6.1 A 100 mm × 200 mm rectangular beam carries V = 15 kN. Find the maximum shear stress.

1. A = 0.02 m².
2. τ_max = 1.5V/A = 1.5 × 15,000/0.02.

Answer: 1.125 MPa at the neutral axis.

Example 6.2 Two 150 mm × 50 mm boards are glued flat on top of each other to make a 150 mm wide × 100 mm deep beam carrying V = 10 kN. Find the shear stress the glue must carry.

1. I = 0.15 × 0.1³/12 = 1.25 × 10⁻⁵ m⁴.
2. Q = 0.15 × 0.05 × 0.025 = 1.875 × 10⁻⁴ m³.
3. q = VQ/I = 150 N/mm; τ = q/b = 150/150.

Answer: τ = 1.0 MPa (the same as 1.5V/A).

Example 6.3 The same two boards are nailed instead, under V = 2 kN, and each nail can carry 900 N. Find the nail spacing.

1. q = VQ/I = 2000 × 1.875 × 10⁻⁴/1.25 × 10⁻⁵ = 30 N/mm.
2. s = F/q = 900/30.

Answer: 30 mm.

Practice problems

Work each on paper first, then check.

1. A solid 50 mm shaft carries V = 10 kN. Find τ_{max}.

   A = 1963 mm^2; $\tau = 4V/(3A) = 6.79$ MPa.

2. A 50 mm × 100 mm rectangle carries V = 5 kN. Find τ_{max}.

   1.5 × 5000/5000 = 1.5 MPa.

3. For a simply supported rectangular beam with a center load, show that $\sigma_{max}/\tau_{max} = 2L/h$, and evaluate it for L/h = 10.

   $\sigma = 1.5PL/(bh^2)$, $\tau = 0.75P/(bh)$, ratio 2L/h = 20: bending governs long beams.

4. Sample values: a mid-rise building's materials carry 1,000 t CO_2e of embodied emissions. Using the 13% to 26.5% range from the Forest Service paper, estimate the saving with mass timber.

   130 to 265 t CO_2e (sample building).

Common misconceptions

Myth: Shear stress is largest at the top and bottom, like bending stress.

Actually: It is zero there and largest at the neutral axis.

Myth: Q is the moment of inertia.

Actually: Q is a first moment of area (m^3); I is a second moment (m^4).

Myth: Glue only has to hold the boards together vertically.

Actually: It must carry the horizontal shear flow q.

Flashcards

Shear formula

$\tau = VQ/(It)$.

First moment Q

$A'\bar{y}'$ beyond the cut.

Rectangle

$\tau_{max} = 1.5V/A$.

Circle

$\tau_{max} = 4V/(3A)$.

Shear flow

Fastener spacing

$q = VQ/I$.

$s = F/q$.

Self-check quiz

1. For a rectangle, τ_max is...

V/A

1.5V/A

2V/A

4V/(3A)

1.5V/A. Parabolic distribution.

2. Shear flow has units of...

N/m^2

N/m

N·m

m^3

N/m. Force per length.

3. Q is...

$A'\bar{y}'$

$bh^3/12$

VQ/I

Mc/I

$A'\bar{y}'$. First moment of the area beyond the cut.

4. Halving the nail spacing lets the beam carry how much more shear?

the same

twice

four times

half

twice. q = F/s doubles.

Try this quiz again

How this helps save the planet

Glued wood layers that carry shear flow are building a lower-carbon skyline. A 2022 paper published through the USDA Forest Service's Forest Products Laboratory explains that cross-laminated timber is at the forefront of the mass timber movement, which lets designers build taller wood buildings, and that several studies show substituting mass timber for steel and concrete in mid-rise buildings can reduce the emissions from manufacturing, transporting and installing building materials by 13% to 26.5%. The authors also stress safeguards so that more demand for wood does not harm forest carbon and health. The glue lines between layers must carry the shear flow from this unit.

On Saving Planets: Forests

Sources: USDA Forest Service: What is the impact of mass timber utilization on climate and forests? (2022) (https://research.fs.usda.gov/treesearch/63853)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on beams.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Shear stress and shear modulus.

I finished this unit

UNIT 7 · WEEK 10

Stress Transformation, Mohr’s Circle, Failure Theories and Fatigue

Plane stress on rotated planes, principal stresses and maximum shear, Mohr’s circle, the Tresca and von Mises yield criteria, and an introduction to fatigue under repeated loading.

Learning objectives

By the end of this unit you can:

- Transform plane stress to any angle and find principal stresses and planes.
- Draw and use Mohr’s circle.
- Apply the Tresca and von Mises criteria to find a factor of safety.
- Explain fatigue and why parts can fail far below their static strength.

Key concepts

- The same stress state looks different on different planes. Rotating by θ changes σ and τ, but some planes have zero shear: the principal planes.
- Principal stresses are $\sigma_{1,2}$ = (σx + σy)/2 ± R, with R = √(((σx − σy)/2)² + τxy²). The maximum in-plane shear is R, on planes 45° from the principal planes.
- Mohr’s circle plots (σ, τ) for every plane: center at (σx + σy)/2, radius R. An angle θ on the element is 2θ on the circle.
- Ductile metals yield by shear. Tresca: the largest shear stress reaches σ_Y/2. Von Mises: σ_vm = √(σ_1^2 − $\sigma_1\sigma_2$ + σ_2^2) reaches σ_Y (it fits test data a little better).
- Repeated loading grows cracks from small flaws, so parts can break after millions of cycles at stresses well below yield. Designers use S–N curves and test full parts for their lifetime of cycles.

Key equations

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

```
σ_1,2=(σ_x+σ_y)/(2)((σ_x-σ_y)/(2))^2+τ_xy^2
```

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

```
2θ_p=2τ_xyσ_x-σ_y
```

Principal angle

$$\sigma_{vm} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

σ_vm=σ_1^2-σ_1σ_2+σ_2^2

Von Mises (plane stress)

$$\sigma_{vm} = \sqrt{\sigma^2 + 3\tau^2}$$

σ_vm=σ^2+3τ^2

Normal plus shear (shafts, beams)

Worked examples, step by step

Example 7.1 A point has σx = 80 MPa, σy = −40 MPa and τxy = 30 MPa. Find the principal stresses, the principal angle and the maximum in-plane shear.

1. Center = 20 MPa; R = √(60² + 30²) = 67.1 MPa.
2. σ_1 = 20 + 67.1; σ_2 = 20 − 67.1.
3. tan 2θ_p = 60/120 = 0.5.

Answer: σ_1 = 87.1 MPa, σ_2 = −47.1 MPa, θ_p = 13.3°, τ_max = 67.1 MPa.

Example 7.2 For that stress state, find the von Mises stress and the factor of safety for steel with σ_Y = 250 MPa.

1. σ_vm = √(87.1² − (87.1)(−47.1) + 47.1²).
2. FS = 250/σ_vm.

Answer: σ_vm = 117.9 MPa; FS = 2.12.

Example 7.3 Use Tresca for the same point.

1. σ_1 and σ_2 have opposite signs, so τ_abs max = (σ_1 − σ_2)/2 = 67.1 MPa.
2. FS = (σ_Y/2)/τ_max = 125/67.1.

Answer: FS = 1.86 (Tresca is a little more conservative).

Practice problems

Work each on paper first, then check.

1. A shaft surface is in pure shear, $\tau = 50$ MPa. Find the principal stresses and their planes.

   $\sigma_1 = 50$ MPa, $\sigma_2 = -50$ MPa, on planes at 45°.

2. A bar is in uniaxial tension at 100 MPa. Find the maximum shear stress and its plane.

   $\tau_{max} = 50$ MPa on planes at 45°.

3. A shaft carries $\sigma = 60$ MPa from bending and $\tau = 40$ MPa from torsion. Find the von Mises stress.

   $\sigma_{vm} = \sqrt{(60^2 + 3 \times 40^2)} = 91.7$ MPa.

4. Sample values: a part cycles once per second for 20 years. How many load cycles is that?

   $20 \times 365.25 \times 86{,}400 = 6.31 \times 10^8$ cycles.

Common misconceptions

Myth: The largest stress is always σx or σy.
Actually: Principal stresses on rotated planes can be larger.

Myth: An angle θ on the element is θ on Mohr's circle.
Actually: It is 2θ on the circle.

Myth: If a part survives one load, it survives that load forever.
Actually: Repeated loads can grow fatigue cracks far below the static strength.

Flashcards

Principal stresses

Center ± R.

Mohr's circle radius

$R = \sqrt{(((\sigma x - \sigma y)/2)^2 + \tau^2)}$.

Max in-plane shear

$\tau_{max} = R$, at 45°.

Tresca

$\tau_{max} = \sigma_Y/2$ at yield.

Von Mises

$\sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}$.

Fatigue

Failure after many cycles below yield.

Self-check quiz

1. On principal planes the shear stress is...

maximum

zero

equal to σ_1

R

zero. Definition.

2. Maximum in-plane shear planes are at what angle from principal planes?

30°

45°

60°

90°

45°. 90° on the circle.

3. Von Mises for uniaxial stress σ equals...

σ/2

σ

√3σ

2σ

σ. $\sigma_2 = 0$.

4. Fatigue failure is driven by...

one large load

repeated cycles of load

temperature alone

color

repeated cycles of load. Cracks grow cycle by cycle.

Try this quiz again

How this helps save the planet

A wind turbine blade must survive decades of bending back and forth. NREL's fact sheet on the Wind Technology Testing Center explains that fatigue tests cycle blades millions of times to simulate what a blade goes through in its lifetime on a turbine, using NREL's resonant excitation system with up to 21 meters of tip-to-tip motion to generate 20-plus years of cyclic field loads in a matter of months. Combined stress, failure theories and fatigue, all in this unit, decide whether a blade passes.

On Saving Planets: Statics

Sources: NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (https://docs.nlr.gov/docs/fy12osti/53581.pdf)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on stress transformation.
- MIT OpenCourseWare 2.002 Mechanics and Materials II (https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/): MIT lectures on yield, fracture and fatigue.

☐ **I finished this unit**

Thin-Walled Pressure Vessels and Combined Loading

Hoop and axial stress in thin cylinders, stress in thin spheres, required wall thickness, and combining axial, bending, torsion and pressure stresses at a point.

Learning objectives

By the end of this unit you can:

- Compute hoop and axial stress in thin-walled cylinders and spheres.
- Find the required wall thickness for a pressure and allowable stress.
- Superpose axial, bending, torsion and pressure stresses at a point.
- Find principal and von Mises stresses for combined loads.

Key concepts

- A vessel is thin-walled when r/t is about 10 or more; then the wall stress is nearly uniform through the thickness.
- Cylinder: hoop stress σ_h = pr/t, axial stress σ_a = pr/(2t). The hoop stress is twice the axial, so overpressured pipes split lengthwise.
- Sphere: σ = pr/(2t) in every direction, so a sphere needs half the wall of a cylinder of the same radius.
- Under combined loads, add the stresses from each load at a point (superposition), then use Mohr’s circle or a failure criterion.
- Units matter: 1 bar = 0.1 MPa and 1 psi = 6894.757 Pa (NIST).

Key equations

$$\sigma_h = \frac{pr}{t}, \quad \sigma_a = \frac{pr}{2t}$$

`σ_h=pr/t, σ_a=pr/2t`

Thin cylinder

$$\sigma = \frac{pr}{2t}$$

`σ=pr/2t`

Thin sphere

$$\sigma = \frac{P}{A} \pm \frac{Mc}{I}$$

`σ=P/AMc/I`

Axial plus bending

$$t = \frac{pr}{\sigma_{\text{allow}}}$$

`t=σ_allow`

Required cylinder wall

Worked examples, step by step

Example 8.1 Sample values: a hydrogen cylinder with inner radius 150 mm and wall 15 mm holds 700 bar (70 MPa). Find the hoop and axial stresses.

1. r/t = 10, thin-wall estimate.
2. σ_h = pr/t = 70 × 10; σ_a = σ_h/2.

Answer: σ_h = 700 MPa, σ_a = 350 MPa (sample tank, which is why such tanks use carbon-fiber composite).

Example 8.2 A sphere with the same radius, wall and pressure: find the stress.

1. σ = pr/(2t).

Answer: 350 MPa, half the cylinder's hoop stress.

Example 8.3 A solid 100 mm post carries a 10 kN compressive load and a 3 kN·m bending moment. Find the largest compressive and tensile stresses.

1. P/A = 10,000/7854 mm² = 1.27 MPa compression.
2. Mc/I = 32M/(πd³) = 30.6 MPa.

Answer: 31.8 MPa compression on one face; 29.3 MPa tension on the other.

Practice problems

Work each on paper first, then check.

1. A tank design is changed from 350 bar to 700 bar at the same radius and allowable stress. How must the wall thickness change?

   t = pr/σ: it doubles.

2. Sample values: a gas cylinder has r = 150 mm, t = 3 mm and p = 2 MPa. Find σ_h and σ_a.

   σ_h = 100 MPa; σ_a = 50 MPa.

3. Find the wall thickness of a cylinder with p = 35 MPa, r = 200 mm and σ_allow = 600 MPa.

   t = 35 × 200/600 = 11.7 mm.

4. Convert DOE's 10,000 psi to MPa with NIST's factor.

   10,000 × 6894.757 Pa = 68.9 MPa, about 700 bar.

Common misconceptions

Myth: Hoop and axial stresses are equal in a cylinder.
Actually: Hoop stress is twice the axial stress.

Myth: Thin-wall formulas work for any vessel.
Actually: Only when r/t is about 10 or more.

Myth: You can add stresses from different loads on different points.
Actually: Superpose at the same point and the same direction.

Flashcards

Hoop stress

σ_h = pr/t.

Axial stress

σ_a = pr/(2t).

Sphere

σ = pr/(2t).

Thin wall

r/t ≳ 10.

Superposition

Add stresses at a point.

bar and psi

1 bar = 0.1 MPa; 1 psi = 6894.757 Pa.

Self-check quiz

1. Hoop stress in a thin cylinder is...

pr/(2t)

pr/t

pt/r

2pr/t

pr/t. Equilibrium of a half-ring.

2. A sphere’s wall stress compared with a cylinder’s hoop stress (same p, r, t) is…

twice

equal

half

a quarter

half. pr/(2t).

3. An overpressured pipe usually splits…

around its circumference

lengthwise

at the ends only

never

lengthwise. Hoop stress is largest.

4. 700 bar is about…

7 MPa

70 MPa

700 MPa

0.7 MPa

70 MPa. 1 bar = 0.1 MPa.

Try this quiz again

How this helps save the planet

Hydrogen can power trucks, buses and industry without carbon emissions at the point of use, but it has to be stored. The U.S. Department of Energy explains that hydrogen has the highest energy per mass of any fuel but very low energy per volume, so storing it as a gas typically requires high-pressure tanks at 350–700 bar (5,000–10,000 psi), while storing it as a liquid requires cryogenic temperatures because it boils at −252.8 °C at ambient pressure. The hoop-stress formula of this unit shows why those tanks need very strong, light walls.

On Saving Planets: General Chemistry I

Sources: U.S. DOE: Hydrogen storage (https://www.energy.gov/eere/fuelcells/hydrogen-storage) · NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on pressure vessels.
- MIT OpenCourseWare 2.002 Mechanics and Materials II (https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/): MIT follow-on course.

☐ **I finished this unit**

Deflection of Beams

The elastic curve and EI v'' = M, integration with boundary conditions, standard deflection formulas, superposition, serviceability limits and statically indeterminate beams.

Learning objectives

By the end of this unit you can:

- Integrate EI v'' = M with boundary conditions to find slope and deflection.
- Use standard deflection formulas and superposition.
- Check deflection against a serviceability limit such as L/360.
- Solve propped cantilevers and other indeterminate beams.

Key concepts

- For small slopes, the elastic curve obeys EI d²v/dx² = M(x). Integrate twice and use the supports to fix the constants (v = 0 at pins; v = 0 and v' = 0 at a fixed end).
- EI is the flexural rigidity: stiffer material (E) or a deeper section (I) gives smaller deflections.
- Standard results: cantilever with tip load δ = PL³/(3EI); simply supported with center load PL³/(48EI); with uniform load 5wL⁴/(384EI).
- Deflections from separate loads add (superposition) while the beam stays linear-elastic.
- An extra support makes the beam indeterminate; remove it, compute the deflection there, and choose the reaction that cancels it.

Key equations

$$EI\frac{d^2v}{dx^2} = M(x)$$

`EI(d^2v)/(dx^2)=M(x)`

Elastic curve

$$\delta = \frac{PL^3}{3EI}, \quad \delta = \frac{wL^4}{8EI}$$

`=(PL^3)/(3EI), =(wL^4)/(8EI)`

Cantilever: tip load, uniform load

$$\delta = \frac{PL^3}{48EI}, \quad \delta = \frac{5wL^4}{384EI}$$

`=(PL^3)/(48EI), =(5wL^4)/(384EI)`

Simply supported: center load, uniform load

$$R_B = \frac{3wL}{8}$$

R_B=3wL/8

Propped cantilever, uniform load

Worked examples, step by step

Example 9.1 A 2 m steel cantilever (E = 200 GPa, I = 8 × 10^{-6} m^4) carries 1 kN at its tip. Find the tip deflection.

1. δ = $PL^3/(3EI)$ = 1000 × 8/(3 × 200 × 10^9 × 8 × 10^{-6}).

Answer: 1.67 mm.

Example 9.2 A simply supported 6 m steel beam (I = 1 × 10^{-4} m^4) carries 10 kN/m. Find the midspan deflection and compare with L/360.

1. δ = $5wL^4/(384EI)$ = 5 × 10,000 × 1296/(384 × 200 × 10^9 × 10^{-4}).
2. L/360 = 16.7 mm.

Answer: 8.44 mm, within the limit.

Example 9.3 The same beam is fixed at A and propped at B. Find the prop reaction.

1. Remove the prop: cantilever tip deflection $wL^4/(8EI)$.
2. The prop force R_B gives $R_BL^3/(3EI)$ upward; set them equal.

Answer: R_B = 3wL/8 = 22.5 kN.

Practice problems

Work each on paper first, then check.

1. A cantilever's length is doubled with the same tip load. By what factor does the tip deflection grow?

   2^3 = 8 times.

2. A simply supported wood beam ($E = 10$ GPa, $I = 5 \times 10^{-5}$ m^4, $L = 4$ m) carries 5 kN at midspan. Find δ.

   $PL^3/(48EI) = 13.3$ mm.

3. A 3 m steel cantilever ($I = 1 \times 10^{-5}$ m^4) carries 2 kN/m. Find the tip deflection.

   $wL^4/(8EI) = 10.1$ mm.

4. DOE says rotor swept areas have grown about 670% since 1998–1999. If swept area grows as diameter squared, by what factor did the average diameter grow?

   Swept area ×7.7, so diameter × $\sqrt{7.7} = 2.77$.

Common misconceptions

Myth: Deflection grows in step with length.
Actually: It grows with L^3 or L^4.

Myth: A stronger steel deflects less.
Actually: Deflection depends on E, which is about the same for all steels.

Myth: If a beam is strong enough, it is good enough.
Actually: It must also be stiff enough: floors that bounce or blades that hit the tower fail in service.

Flashcards

Elastic curve

$EIv'' = M$.

Cantilever tip load

$PL^3/(3EI)$.

Simply supported center load

$PL^3/(48EI)$.

Simply supported uniform load

$5wL^4/(384EI)$.

Superposition

Add deflections from each load.

Serviceability

Limit such as L/360.

Self-check quiz

1. Tip deflection of a cantilever with an end load:

$PL^3/(3EI)$

$PL^3/(48EI)$

$PL^2/(2EI)$

$5wL^4/(384EI)$

$PL^3/(3EI)$. Standard result.

2. EI is called the...

section modulus

flexural rigidity

polar moment

shear flow

flexural rigidity. Resistance to bending deformation.

3. At a fixed support...

v = 0 only

v = 0 and v' = 0

M = 0

V = 0

v = 0 and v' = 0. No deflection and no rotation.

4. Doubling the span of a simply supported beam with uniform load multiplies δ by...

4

8

16

2

16. L^4.

Try this quiz again

How this helps save the planet

Longer blades catch more wind, but they also deflect more. The U.S. Department of Energy reports that the average rotor diameter of newly installed U.S. turbines was over 133.8 meters in 2023, that 98% of new turbines that year had rotors of 115 meters or more, and that rotor swept areas have grown around 670% since 1998–1999, letting turbines produce more electricity even where winds are lighter. DOE notes blades cannot be folded or bent once built, which limits trucking, and it is researching more slender and flexible blades. Beam-deflection theory is how designers keep a flexible blade from striking the tower.

On Saving Planets: Physics II

Sources: U.S. DOE: Wind turbines, the bigger the better (https://www.energy.gov/eere/articles/wind-turbines-bigger-better)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on beam deflection.
- OpenStax Calculus Volume 1, 5.5 Substitution (https://openstax.org/books/calculus-volume-1/pages/5-5-substitution): Free calculus review for integrating the elastic curve.

☐ **I finished this unit**

Buckling of Columns

Stability and the critical load, Euler's formula for pinned columns, effective length for other end conditions, slenderness ratio and critical stress, inelastic buckling, imperfections, and thin shells with knockdown factors.

Learning objectives

By the end of this unit you can:

- Explain stable, neutral and unstable equilibrium in columns.
- Compute Euler critical loads with effective length factors.
- Use the slenderness ratio to tell buckling from yielding.
- Explain why imperfections and knockdown factors matter for thin shells such as rocket tanks.

Key concepts

- A slender column can fail suddenly by bending sideways at a load far below the crushing load. That load is the critical load P_cr.
- Euler: P_cr = π²EI/(KL)². K = 1 for pinned–pinned, 2 for fixed–free, 0.5 for fixed–fixed and about 0.7 for fixed–pinned.
- Buckling uses the smallest I of the section: a column buckles about its weak axis.
- Critical stress σ_cr = π²E/(KL/r)², with radius of gyration r = √(I/A). If σ_cr exceeds the yield strength, the short column yields first; between the two, inelastic buckling applies.
- Thin cylindrical shells, like rocket tanks, are very sensitive to tiny imperfections, so designers multiply the ideal buckling load by a knockdown factor taken from tests.

Key equations

$$P_{cr} = \frac{\pi^2 E I}{(KL)^2}$$

`P_cr=(π^2EI)/((KL)^2)`

Euler critical load

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}, \quad r = \sqrt{I/A}$$

`σ_cr=(π^2E)/((KL/r)^2), r=/A`

Critical stress

$$K = 1,\ 2,\ 0.5,\ 0.7$$

$K = 1,\ 2,\ 0.5,\ 0.7$

Pinned, fixed–free, fixed–fixed, fixed–pinned

Worked examples, step by step

Example 10.1 A pinned–pinned steel column (E = 200 GPa) is 3 m long with I = 2×10^{-6} m^4. Find P_{cr}.

1. $P_{cr} = \pi^2 \times 200 \times 10^9 \times 2 \times 10^{-6}/3^2$.

Answer: 438.6 kN.

Example 10.2 The same column is fixed at the base and free at the top. Find P_{cr}.

1. K = 2, so KL = 6 m and P_{cr} drops by 4.

Answer: 109.7 kN.

Example 10.3 A solid 50 mm steel rod, 2 m long and pinned at both ends, has σ_Y = 250 MPa. Does it buckle or yield first?

1. r = d/4 = 12.5 mm; KL/r = 2000/12.5 = 160.
2. $\sigma_{cr} = \pi^2 \times 200{,}000/160^2$ MPa.

Answer: σ_{cr} = 77.1 MPa < 250 MPa: it buckles first.

Practice problems

Work each on paper first, then check.

1. A column's length is doubled. What happens to P_{cr}?

   It drops to a quarter.

2. How does a fixed–fixed column compare with a pinned–pinned one of the same length?

   K = 0.5, so P_{cr} is 4 times larger.

3. NASA describes a test load of up to 750,000 pounds. Convert to meganewtons with NIST's factor.

   750,000 × 4.448222 N = 3.34 MN.

4. Sample values: NASA found removing weld lands can raise load capacity by up to 28%. What would a sample 10 MN cylinder carry?

   10 × 1.28 = 12.8 MN (sample cylinder).

Common misconceptions

Myth: Columns fail when the stress reaches yield.

Actually: Slender columns buckle at much lower stress.

Myth: A column buckles about its strong axis.

Actually: It buckles about the axis with the smallest I.

Myth: The Euler load is what a real column will carry.

Actually: Imperfections and eccentric loads lower it, especially for thin shells.

Flashcards

Euler load

$P_{cr} = \pi^2 EI/(KL)^2$.

Effective length

K = 1, 2, 0.5, 0.7.

Radius of gyration

$r = \sqrt{(I/A)}$.

Critical stress

$\pi^2 E/(KL/r)^2$.

Weak axis

Smallest I governs.

Knockdown factor

Test-based reduction for thin shells.

Self-check quiz

1. Euler's P_{cr} is proportional to...

L

1/L

$1/L^2$

L^2

1/L². π²EI/(KL)².

2. K for a fixed–free (flagpole) column:

0.5

0.7

1

2

2. Effective length 2L.

3. Slenderness ratio is…

KL/r

r/L

I/A

E/σ_Y

KL/r. Effective length over radius of gyration.

4. A knockdown factor accounts for…

temperature

imperfection sensitivity of thin shells

paint

gravity

imperfection sensitivity of thin shells. Real shells buckle below ideal theory.

Try this quiz again

How this helps save the planet

Lighter rockets launch the satellites that watch Earth's climate, and buckling sets how light their tanks can be. NASA's Engineering and Safety Center reports that its Shell Buckling Knockdown Factor team re-examined knockdown factors that had not been updated since the Apollo era of the 1960s. Eight subscale and two full-scale metallic cylinder tests led to new knockdown factors that are used on the Space Launch System core stage, giving both mass and cost savings. In one test, an 8-foot-diameter cylinder loaded up to 750,000 pounds buckled within 2.5% of the engineer's prediction, and early results showed that removing weld lands can raise load capacity by up to 28%.

On Saving Planets: Intro to Aerospace Engineering

Sources: NASA NESC: Shell Buckling Knockdown Factor project (https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lectures on buckling.
- NASA NESC: Shell Buckling Knockdown Factor project (https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/): How NASA tests buckling of rocket cylinders.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety
   4 problems
2. Strain and Material Behavior: Hooke's Law, Poisson's Ratio and the Stress–Strain Curve
   4 problems
3. Axially Loaded Members: Deformation, Indeterminate Bars and Thermal Stress
   4 problems
4. Torsion of Circular Shafts and Power Transmission
   4 problems
5. Shear and Moment Diagrams and Bending Stress
   4 problems
6. Transverse Shear in Beams, Shear Flow and Composite Sections
   4 problems
7. Stress Transformation, Mohr's Circle, Failure Theories and Fatigue
   4 problems
8. Thin-Walled Pressure Vessels and Combined Loading
   4 problems
9. Deflection of Beams
   4 problems
10. Buckling of Columns

4 problems

The lab (Materials lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation or with household items. Stretched rubber bands and snapping spaghetti can fly: wear glasses, keep loads small, and keep faces away from parts under load.

Safety reference: OSHA: Hand and power tools (https://www.osha.gov/hand-power-tools)

Lab A: Hooke’s law and stiffness

WEEK 2

Goal: Load a spring (or rubber band) in steps, show that stretch grows in proportion to force, and find the stiffness k.

$$F = kx, \quad U = \tfrac{1}{2}kx^2$$

```
F=kx, U=12kx^2
```

Linear spring and stored energy

Procedure

1. Open PhET Hooke’s Law, choose one spring and set its constant; apply forces in equal steps and record the displacement.
2. Home: hang a bag from a rubber band and add coins of known mass (F = mg), measuring the stretch with a ruler each time.
3. Plot F against x and fit a line through the origin; the slope is k.
4. Unload step by step and note whether the rubber band returns to its starting length.

Data table

F (N)	x (mm) for k = 50 N/m
0.5	10
1.0	20
1.5	30
2.0	40

Sample values for a spring with k = 50 N/m; your own spring sets your slope.

Sample calculation

1. k = F/x = 0.5 N/0.010 m = 50 N/m in every row.
2. Stored energy at 2 N: U = ½ × 50 × 0.04² = 0.04 J.

> **Safety**
>
> - Wear glasses; a stretched rubber band can snap back.

Home or virtual alternative

- The rubber-band step is the home version.

PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law)

Planet link: Stiffness tests like this one, scaled up, are how engineers qualify every bolt, cable and bridge member. ASCE Report Card for America's Infrastructure: Bridges (https://infrastructurereportcard.org/cat-item/bridges-infrastructure/)

☐ **I completed this lab (wet or virtual)**

Lab B: Stress, strain and the modulus of a rubber band

WEEK 3

Goal: Turn force and stretch into stress and strain, estimate a modulus, and see where the response stops being linear.

$$\sigma = \frac{F}{A}, \quad \varepsilon = \frac{\delta}{L_0}, \quad E = \frac{\sigma}{\varepsilon}$$

σ=F/A, =()/(L_0), E=(σ)/()

Engineering stress and strain

Procedure

1. Measure a flat rubber band's width and thickness with a ruler (or count layers of a known thickness) and its unstretched length L_0.
2. Hang known masses and record the stretch δ for each.
3. Compute σ and ε and plot σ against ε. Fit the straight early part to estimate E.
4. Keep adding small masses and note where the curve bends: rubber is not linear at large strain.

Data table

F (N)	δ (mm)	σ (MPa)	ε
1	8	0.667	0.10
2	16	1.333	0.20
3	24	2.000	0.30

Sample values for a band with A = 1.5 mm² and L_0 = 80 mm in its near-linear range.

Sample calculation

1. σ = 1 N/1.5 mm² = 0.667 MPa; ε = 8/80 = 0.10.
2. E = 0.667/0.10 = 6.67 MPa, about 30,000 times less stiff than steel (200 GPa).

> **Safety**
>
> - Wear glasses and keep the band pointed away from faces; stop before it breaks.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs)

Planet link: Knowing stiffness and strength per kilogram is how lighter car bodies are chosen, and DOE links a 10% weight cut to 6% to 8% better fuel economy. U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks)

- [ ] **I completed this lab (wet or virtual)**

Lab C: Twisting a foam rod

WEEK 6

Goal: Twist a foam rod with a known torque, measure the angle of twist, and check that φ grows in proportion to T as in φ = TL/(GJ).

$$\phi = \frac{TL}{GJ}, \quad J = \frac{\pi d^4}{32}$$

`=TL/GJ, J=(π d^4)/(32)`

Angle of twist of a solid rod

Procedure

1. Draw a straight line along a solid foam rod and mark two rings 0.5 m apart.
2. Hold one end fixed and apply a torque with a lever arm and a known weight (T = F × arm).
3. Photograph the line and read the twist angle between the rings; repeat for three torques.
4. Plot φ against T; the slope is L/(GJ). Note the spiral the straight line becomes: that is shear strain.

Data table

T (N·m)	φ (°)
0.1	4.67
0.2	9.34
0.3	14.0

Values computed for a sample rod with d = 50 mm, L = 0.5 m and G = 1 MPa.

Sample calculation

1. J = π × 0.05⁴/32 = 6.136 × 10⁻⁷ m⁴.
2. φ = 0.2 × 0.5/(10⁶ × 6.136 × 10⁻⁷) = 0.163 rad = 9.34°.
3. Surface shear strain γ = cφ/L = 0.025 × 0.163/0.5 = 0.00815.

> **Safety**
>
> - Keep weights small and low over a soft surface.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Torque (https://phet.colorado.edu/en/simulations/torque)

Planet link: Wind turbine shafts carry the same kind of twisting load, millions of times larger. U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work)

- [ ] **I completed this lab (wet or virtual)**

Lab D: Shape matters, a ruler flat and on edge

WEEK 8

Goal: Load a ruler as a cantilever lying flat and then on edge and compare deflection and stress with I = bh³/12.

$$I = \frac{bh^3}{12}, \quad \delta = \frac{PL^3}{3EI}, \quad \sigma = \frac{Mc}{I}$$

`I=(bh^3)/(12), =(PL^3)/(3EI), σ=Mc/I`

Section shape and cantilever bending

Procedure

1. Clamp a 30 mm × 3 mm plastic ruler flat to a table with 250 mm sticking out.
2. Hang a 0.5 N load (about 51 g) from the tip and measure the deflection against a second ruler.
3. Turn the ruler on edge (clamp it between two books) and repeat with the same load.
4. Compare the ratio of deflections with the ratio of I values.

Data table

Orientation	I (m⁴)	δ at 0.5 N (mm)	σ_max (MPa)
Flat (h = 3 mm)	6.75×10^{-11}	12.9	2.78
On edge (h = 30 mm)	6.75×10^{-9}	0.129	0.278

Values computed for a sample plastic ruler with E = 3 GPa.

Sample calculation

1. I ratio = $(30/3)^2 = 100$, so the edge deflection is 100 times smaller.
2. M = 0.5 × 0.25 = 0.125 N·m; flat $\sigma = 0.125 \times 0.0015/6.75 \times 10^{-11} = 2.78$ MPa.

> **Safety**
>
> - Do not overload the ruler; plastic can snap.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Balancing Act (https://phet.colorado.edu/en/simulations/balancing-act)

Planet link: Putting material far from the neutral axis is how blades, beams and towers save material. NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (https://docs.nlr.

gov/docs/fy12osti/53581.pdf)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Cantilever deflection and Young’s modulus

WEEK 12

Goal: Measure tip deflection against load for a flat ruler cantilever and back out Young’s modulus from $\delta = PL^3/(3EI)$.

$$E = \frac{L^3}{3I\,(\delta/P)}$$

```
E=(L^3)/(3I (/P))
```

From the slope of δ against P

Procedure

1. Use the flat ruler from Lab D with 250 mm overhang.
2. Hang 0.25, 0.5 and 1.0 N at the tip and record the deflection each time.
3. Plot δ against P and fit a line through the origin.
4. Compute E from the slope and compare with a typical value for the plastic.

Data table

P (N)	δ (mm)
0.25	6.43
0.50	12.9
1.00	25.7

Values computed for a sample plastic ruler with E = 3 GPa, b = 30 mm, h = 3 mm.

Sample calculation

1. Slope $\delta/P = 25.7$ mm/N.
2. $E = 0.25^3/(3 \times 6.75 \times 10^{-11} \times 0.0257) = 3.0$ GPa.

> **Safety**
>
> - Keep loads small and catch them if they slip.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Hooke’s Law (https://phet.colorado.edu/en/simulations/hookes-law)

Planet link: Longer, more flexible wind blades are designed with the same deflection formula. U.S. DOE: Wind turbines, the bigger the better (https://www.energy.gov/eere/articles/wind-turbines-bigger-better)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Buckling spaghetti

WEEK 14

Goal: Find the load that buckles strands of dry spaghetti of different lengths and check that P_cr falls as 1/L².

$$P_{cr} = \frac{\pi^2 E I}{L^2}, \quad I = \frac{\pi d^4}{64}$$

`P_cr=(π^2EI)/(L^2), I=(π d^4)/(64)`

Pinned–pinned column

Procedure

1. Cut strands to 100, 150 and 200 mm. Stand each upright on a kitchen scale and press down slowly on top with a flat object.
2. Record the highest scale reading just as the strand bows sideways (convert grams to newtons).
3. Repeat three times per length and average.
4. Plot P_cr against 1/L²; the line should pass near the origin.

Data table

L (mm)	P_cr (N)
100	1.526
150	0.678
200	0.381

Values computed for a sample strand with d = 1.8 mm and E = 3 GPa; real spaghetti varies.

Sample calculation

1. $I = \pi \times 1.8^4/64 = 0.515$ mm⁴ $= 5.153 \times 10^{-13}$ m⁴.
2. P_{cr} at 0.1 m $= \pi^2 \times 3 \times 10^9 \times 5.153 \times 10^{-13}/0.01 = 1.526$ N (about 156 g on the scale).
3. Doubling L from 100 to 200 mm cuts P_{cr} by 4.

> **Safety**
>
> - Wear glasses; spaghetti snaps into sharp pieces.

Home or virtual alternative

- The whole lab is a home lab.

OpenStax UP1, 12.4 Elasticity and plasticity (https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity)

Planet link: NASA tests full-size rocket cylinders to buckling to set knockdown factors that save mass. NASA NESC: Shell Buckling Knockdown Factor project (https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/)

☐ **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. A 16 mm steel rod carries 30 kN in tension. Find the stress.

    1. $A = \pi \times 16^2/4 = 201.1\ \text{mm}^2$.

    Answer: 149.2 MPa.

2. An aluminum bar (E = 70 GPa) 1.5 m long with A = 300 mm² carries 12 kN. Find its elongation.

    1. $\delta = PL/(AE)$.

    Answer: 0.857 mm.

3. A steel bar (E = 200 GPa, $\alpha = 12 \times 10^{-6}/°C$) is held between rigid walls and heated 25 °C. Find the stress.

    1. $\sigma = E\alpha\Delta T$.

    Answer: 60 MPa compression.

4. A solid 40 mm shaft carries 800 N·m. Find τ_max.

    1. $\tau = 16T/(\pi d^3)$.

    Answer: 63.7 MPa.

5. A simply supported 5 m beam, 120 mm wide × 240 mm deep, carries 4 kN/m. Find σ_max.

    1. $M = wL^2/8 = 12.5\ \text{kN·m}$; $I = 1.382 \times 10^{-4}\ \text{m}^4$.

    Answer: 10.9 MPa.

Final practice set (all units)

1. An 80 mm × 160 mm rectangular beam carries V = 12 kN. Find τ_max.

    1. τ = 1.5V/A.

    Answer: 1.41 MPa.

2. Find the principal stresses for σx = 50 MPa, σy = 10 MPa, τxy = 15 MPa.

    1. Center 30 MPa; R = √(20² + 15²) = 25 MPa.

    Answer: σ_1 = 55 MPa, σ_2 = 5 MPa.

3. A thin cylinder has r = 0.5 m, t = 10 mm and p = 2 MPa. Find σ_h and σ_a.

    1. pr/t and pr/(2t).

    Answer: 100 MPa and 50 MPa.

4. A 1.5 m steel cantilever (I = 4 × 10⁻⁶ m⁴) carries 2 kN at its tip. Find the tip deflection.

    1. δ = PL³/(3EI).

    Answer: 2.81 mm.

5. A pinned steel column 4 m long has I = 5 × 10⁻⁶ m⁴. Find P_cr.

    1. π²EI/L².

    Answer: 616.9 kN.

6. A shaft point has σ = 80 MPa and τ = 30 MPa. Find the von Mises stress.

    1. √(σ² + 3τ²).

    Answer: 95.4 MPa.

Formula sheet

$$\sigma = P/A,\ \tau = V/A,\ \sigma_b = P/(td)$$

$$\varepsilon = \delta/L,\ \sigma = E\varepsilon,\ G = E/(2(1+\nu))$$

$$\delta = PL/(AE),\ \delta_T = \alpha \Delta T L$$

$$\tau = T\rho/J,\ \phi = TL/(GJ),\ J = \pi d^4/32$$

$$P = T\omega$$

$$dM/dx = V,\ dV/dx = -w$$

$$\sigma = My/I,\ I = bh^3/12$$

$$\tau = VQ/(It),\ q = VQ/I$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{vm} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$\sigma_h = pr/t,\ \sigma_a = pr/(2t)$$

$$EIv'' = M$$

$$\delta = PL^3/(3EI),\ PL^3/(48EI),\ 5wL^4/(384EI)$$

$$P_{cr} = \pi^2 EI/(KL)^2$$

Constants

Constant	Value
1 megapascal (MPa)	10^6 Pa = 1 N/mm²
1 bar	10^5 Pa = 0.1 MPa
1 psi (lbf/in²)	6894.757 Pa
1 pound-force (lbf)	4.448222 N
1 revolution per minute	0.1047198 rad/s
Typical steel (textbook values)	E = 200 GPa, G = 80 GPa, ν = 0.3, α = 12 × 10^{-6}/°C, yield about 250 MPa
Typical aluminum alloy (textbook values)	E = 70 GPa, G = 26 GPa, ν = 0.33, α = 23 × 10^{-6}/°C

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free textbooks, lectures and simulations alongside the course.

- MIT OpenCourseWare 2.001 Mechanics and Materials I (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): Free MIT lecture notes, problem sets and exams.
- MIT OpenCourseWare 2.002 Mechanics and Materials II (https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/): Free MIT follow-on course: plasticity, fracture and fatigue.
- OpenStax University Physics Volume 1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook introduction to stress and strain.
- OpenStax University Physics Volume 1, 12.4 Elasticity and plasticity (https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity): Yield, plastic flow and fracture.
- PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law): Springs, stiffness and energy.
- PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs): Stiffness and stretch.

Glossary

Bearing stress
Contact pressure between a fastener and a plate, P/(td).

Elastic curve
Deflected shape of a beam's neutral axis.

Factor of safety
Failure load divided by allowable load.

Fatigue
Failure from repeated loading, often below yield.

Flexural rigidity
EI, a beam's resistance to bending deformation.

Hoop stress
Circumferential stress pr/t in a pressurized cylinder.

Knockdown factor
Test-based reduction of the ideal buckling load of thin shells.

Neutral axis
Line of zero bending stress through the centroid.

Poisson's ratio
Ratio of lateral to axial strain, with a minus sign.

Polar moment of inertia
J, a shaft's resistance to twisting.

Principal stresses
Normal stresses on planes with zero shear.

Section modulus
S = I/c.

Shear flow
Shear force per unit length, VQ/I.

Slenderness ratio
KL/r of a column.

Strain
Deformation per unit length.

Stress
Internal force per unit area.

Von Mises stress
Equivalent stress compared with the yield strength.

Yield strength
Stress at which permanent deformation begins.

Young’s modulus
E, slope of the elastic stress–strain line.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Finds internal loads with a free-body diagram, picks the right deformation model, checks assumptions (elastic, thin-walled, slender), and reports stresses with a factor of safety.
3 · Proficient	Solves the practice problems with occasional unit or sign slips.
2 · Developing	Follows worked examples but mixes up I, J and Q or forgets compatibility.
1 · Beginning	Plugs numbers into formulas without diagrams; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	25%
Labs	20%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Dynamics ENGR 212
- Linear Algebra MATH 230
- Calculus III MATH 210
- All courses