

About this course

This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.

Its scope follows a one-semester sophomore mechanics of materials (strength of materials) course. It is modeled on the topics and depth of MIT 2.001 Mechanics and Materials I (on MIT OpenCourseWare), Purdue ME 323 Mechanics of Materials and Georgia Tech COE 3001 Mechanics of Deformable Bodies, and on the free OpenStax University Physics, which we link where it helps. None of those authors or schools endorses this page.

What we add: every unit ties to real clean-energy, infrastructure or space work with a cited public source (bridges, lighter cars, wind turbine towers, shafts and blades, mass timber, hydrogen tanks, NASA rocket structures), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a cheap home version.

Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Mechanics of materials decides how much material a structure needs. Using just enough steel, aluminum, composite or wood, and no more, makes cars lighter and more efficient, wind turbines taller, buildings lower in carbon, and rockets able to carry more to orbit.

It is also how we keep what we have built safe: bridges, towers and tanks must carry their loads for decades without yielding, cracking from fatigue or buckling. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values, and material properties are typical textbook values.

Before you start

Prerequisites

- Statics: free-body diagrams, equilibrium, centroids and area moments of inertia.
- Calculus I and II: derivatives and integrals; Unit 9 integrates the beam equation twice.
- Physics I helps with springs, energy and units.
- Dynamics is not required.

How to study

- Start every problem with statics: a free-body diagram and the internal force, torque or moment at the cut.
- Then ask how the part deforms: stretch, twist or bend, and which formula fits that deformation.
- Check that the formula applies: linear elastic, small deformations, thin walls or long slender members where required.
- Solve with symbols first, then put in SI numbers (N, m, Pa), and check units: $1 \text{ MPa} = 1 \text{ N/mm}^2$.
- Compare the stress with the material's allowable stress and state a factor of safety.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

Lab equipment (for the wet labs)

- A computer or tablet for the free PhET simulations (they run in a browser).
- Rubber bands, a bag for coins or washers, and a kitchen scale (to know each mass).
- A 30 cm plastic ruler, a second ruler or tape measure, and tape or a clamp to hold it to a table edge.
- A foam rod or pool-noodle offcut and a marker for torsion lines.
- Dry spaghetti and small weights for the buckling lab.
- Safety glasses, a smartphone camera and spreadsheet software for graphs and fits.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety	Internal forces, normal and shear stress	MIT 2.001; OpenStax UP1 12.3
2	Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety	Bearing stress, allowable stress, factor of safety; Lab A	MIT 2.001
3	Strain and Material Behavior: Hooke's Law, Poisson's Ratio and the Stress–Strain Curve	Strain, stress–strain curve, Hooke's law, Poisson's ratio; Lab B	UP1 12.3–12.4
4	Axially Loaded Members: Deformation, Indeterminate Bars and Thermal Stress	Axial deformation, indeterminate bars, thermal stress	UP2 1.3
5	Torsion of Circular Shafts and Power Transmission	Torsion formula, polar moment, angle of twist	MIT 2.001

Week	Unit	Topics	Reading
6	Torsion of Circular Shafts and Power Transmission	Power transmission, shaft design; Lab C	MIT 2.001
7	Shear and Moment Diagrams and Bending Stress	Shear and moment diagrams	MIT 2.001; UP1 12.2
8	Shear and Moment Diagrams and Bending Stress	Flexure formula, section modulus; Lab D; midterm practice	MIT 2.001
9	Transverse Shear in Beams, Shear Flow and Composite Sections	Transverse shear, shear flow, built-up beams	MIT 2.001
10	Stress Transformation, Mohr's Circle, Failure Theories and Fatigue	Stress transformation, Mohr's circle, failure theories, fatigue	MIT 2.001, 2.002
11	Thin-Walled Pressure Vessels and Combined Loading	Pressure vessels and combined loading	MIT 2.001
12	Deflection of Beams	Elastic curve, integration, standard formulas; Lab E	MIT 2.001
13	Deflection of Beams	Superposition and indeterminate beams	MIT 2.001
14	Buckling of Columns	Column buckling and knockdown factors; Lab F	MIT 2.001
15	Review	Review and final practice set	All units

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Stress: Normal, Shear and Bearing; Allowable Stress and Factor of Safety

Internal forces found by cutting a member, average normal stress in rods, average shear stress in pins and bolts (single and double shear), bearing stress, allowable stress design and the factor of safety.

Learning objectives

By the end of this unit you can:

- Find internal forces with a free-body diagram of a cut section.
- Compute average normal, shear and bearing stresses.
- Tell single shear from double shear in connections.
- Size a member from an allowable stress and a factor of safety.

Key concepts

- Stress is internal force per unit area. Its SI unit is the pascal (1 Pa = 1 N/m²); engineers use MPa, and 1 MPa = 1 N/mm².
- Normal stress $\sigma = P/A$ acts perpendicular to a cut; tension is positive and compression negative.
- Average shear stress $\tau = V/A$ acts in the plane of the cut. A pin in double shear has two cut planes, so each carries $V = P/2$.
- Bearing stress $\sigma_b = P/(td)$ is the contact pressure between a bolt and the hole in a plate of thickness t .
- Factor of safety FS = failure load/allowable load (or failure stress/allowable stress). Design keeps working stress at or below $\sigma_{\text{allow}} = \sigma_{\text{fail}}/\text{FS}$.

Key equations

$$\sigma = \frac{P}{A}$$

Average normal stress

$$\tau_{\text{avg}} = \frac{V}{A}$$

Average shear stress

$$\sigma_b = \frac{P}{td}$$

Bearing stress

$$FS = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}$$

Factor of safety

Worked examples, step by step

Example 1.1 A steel rod of 20 mm diameter carries a tensile load of 50 kN. Find the normal stress.

1. $A = \pi d^2/4 = \pi \times 20^2/4 = 314.2 \text{ mm}^2$.
2. $\sigma = P/A = 50,000 \text{ N}/314.2 \text{ mm}^2$.

Answer: 159.2 MPa (tension).

Example 1.2 A 12 mm pin in double shear carries a 20 kN load. Find the average shear stress in the pin.

1. Each of the two shear planes carries $V = 10 \text{ kN}$.
2. $A = \pi \times 12^2/4 = 113.1 \text{ mm}^2$; $\tau = 10,000/113.1$.

Answer: 88.4 MPa.

Example 1.3 A round steel hanger must carry 100 kN. The steel yields at 250 MPa and the factor of safety is 2. Find the smallest diameter.

1. $\sigma_{\text{allow}} = 250/2 = 125 \text{ MPa}$.
2. $A = P/\sigma_{\text{allow}} = 100,000/125 = 800 \text{ mm}^2$.
3. $d = \sqrt{(4A/\pi)}$.

Answer: $d = 31.9 \text{ mm}$ (use 32 mm).

Practice problems

Work each on paper first, then check.

1. A 100 mm × 100 mm square post carries 200 kN in compression. Find the stress.

$$\sigma = 200,000/10,000 = 20 \text{ MPa (compression)}.$$

2. A 10 mm bolt in single shear carries 8 kN. Find the average shear stress.

$$A = 78.54 \text{ mm}^2; \tau = 101.9 \text{ MPa}.$$

3. The same 10 mm bolt passes through a 6 mm thick plate. Find the bearing stress.

$$\sigma_b = 8000 / (6 \times 10) = 133 \text{ MPa}.$$

4. A link fails at 90 kN and works at 30 kN. Find the factor of safety.

$$FS = 90/30 = 3.$$

Common misconceptions

Myth: Stress is the same as force.

Actually: Stress is force per area; a thin wire and a thick bar with the same force have very different stresses.

Myth: A pin in double shear carries the full load on each plane.

Actually: Each of the two planes carries half.

Myth: A higher factor of safety is always better.

Actually: It also means heavier, costlier parts; designers choose FS from codes and the uncertainty in loads and materials.

Flashcards

Normal stress

$$\sigma = P/A.$$

Shear stress

$$\tau = V/A.$$

Double shear

Each plane carries $P/2$.

Bearing stress

$$\sigma_b = P/(td).$$

Factor of safety

$$FS = \text{failure/allowable}.$$

MPa

$$1 \text{ MPa} = 1 \text{ N/mm}^2.$$

Self-check quiz

1. 1 MPa equals...

$$1 \text{ N/m}^2$$

1 N/mm²

1 kN/m²

1 N/cm²

1 N/mm². $10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$.

2. A pin in double shear has how many shear planes? _____

1

2

3

4

2. Two cut planes share the load.

3. Doubling a rod's diameter changes its axial stress (same load) by a factor of... _____

$\frac{1}{2}$

$\frac{1}{4}$

2

4

$\frac{1}{4}$. A grows with d^2 .

4. Bearing stress is... _____

$P/(td)$

P/A of the bolt

V/A

Mc/I

P/(td). Contact pressure on the projected area td.

Try this quiz again

How this helps save the planet

Stress limits decide which bridges trucks may cross. The ASCE Report Card for America's Infrastructure counts 623,218 bridges in the United States, and 63,085 of them were posted for load in 2024, meaning they restrict the weight of vehicles that can safely cross, which is about 10.1% of all bridges. ASCE warns that trucks heavier than bridges were designed for threaten to overstress bridge elements and cause metal fatigue and cracking. Keeping working stress below the allowable stress, as in this unit, is how engineers rate and repair them, and preserving a bridge costs less than replacing it.

On Saving Planets: [Statics](#)

Sources: [ASCE Report Card for America's Infrastructure: Bridges](https://infrastructurereportcard.org/cat-item/bridges-infrastructure/) (https://infrastructurereportcard.org/cat-item/bridges-infrastructure/) · [OpenStax UP1, 12.3 Stress, strain and elastic modulus](https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus) (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus)

Free learning resources

- [OpenStax UP1, 12.3 Stress, strain and elastic modulus](https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus) (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): MIT lecture notes on stress.
- [PhET: Hooke's Law](https://phet.colorado.edu/en/simulations/hooks-law) (https://phet.colorado.edu/en/simulations/hooks-law): Interactive.



I finished this unit

Strain and Material Behavior: Hooke's Law, Poisson's Ratio and the Stress–Strain Curve

Normal and shear strain, the tensile test and the stress–strain diagram, Young's modulus, yield and ultimate strength, ductility, Poisson's ratio, the shear modulus, strain energy and specific stiffness.

Learning objectives

By the end of this unit you can:

- Compute normal and shear strain from deformations.
- Read E , yield strength, ultimate strength and ductility from a stress–strain curve.
- Use Hooke's law, Poisson's ratio and $G = E/(2(1 + \nu))$.
- Compare materials by stiffness and strength per unit mass.

Key concepts

- Normal strain $\epsilon = \delta/L$ is a stretch per unit length (dimensionless, often in $\mu\text{m}/\text{m}$). Shear strain γ is the change in a right angle, in radians.
- In the linear-elastic range, Hooke's law holds: $\sigma = E\epsilon$ and $\tau = G\gamma$. E is Young's modulus, the slope of the stress–strain curve.
- Beyond the yield strength, the material deforms plastically and does not spring back; the ultimate strength is the peak engineering stress, and ductile metals neck before they break.
- Stretching a bar makes it thinner: lateral strain = $-\nu \times$ axial strain, with Poisson's ratio ν about 0.3 for metals. For isotropic materials $G = E/(2(1 + \nu))$.
- Specific stiffness E/ρ and specific strength σ/ρ compare materials per kilogram; that is why aluminum, magnesium and carbon fiber can replace steel in light structures.

Key equations

$$\epsilon = \frac{\delta}{L}, \quad \sigma = E\epsilon$$

Strain and Hooke's law

$$\epsilon_{\text{lat}} = -\nu \epsilon_{\text{axial}}$$

Poisson's ratio

$$G = \frac{E}{2(1 + \nu)}$$

$$u_r = \frac{\sigma_Y^2}{2E}$$

Worked examples, step by step

Example 2.1 A 2 m steel bar stretches 1.2 mm when the stress is 120 MPa. Find the strain and Young's modulus.

1. $\varepsilon = 1.2/2000 = 6 \times 10^{-4}$.
2. $E = \sigma/\varepsilon = 120 \text{ MPa}/0.0006$.

Answer: $\varepsilon = 600 \text{ } \mu\text{m/m}$; $E = 200 \text{ GPa}$.

Example 2.2 An aluminum alloy has $E = 70 \text{ GPa}$ and $\nu = 0.33$. Find its shear modulus.

1. $G = E/(2(1 + \nu)) = 70/2.66$.

Answer: $G = 26.3 \text{ GPa}$.

Example 2.3 A 20 mm steel rod ($\nu = 0.3$) has an axial strain of 0.001. How much does its diameter change?

1. Lateral strain $= -0.3 \times 0.001 = -0.0003$.
2. $\Delta d = -0.0003 \times 20 \text{ mm}$.

Answer: It shrinks by 0.006 mm.

Practice problems

Work each on paper first, then check.

1. Typical values: steel has $E = 200 \text{ GPa}$ and $\rho = 7850 \text{ kg/m}^3$; aluminum has $E = 70 \text{ GPa}$ and $\rho = 2700 \text{ kg/m}^3$. Compare E/ρ .

Steel $25.5 \times 10^6 \text{ m}^2/\text{s}^2$, aluminum $25.9 \times 10^6 \text{ m}^2/\text{s}^2$: almost the same stiffness per kilogram.

2. A tensile specimen with a 50 mm gauge length measures 62 mm at fracture. Find the elongation at break.

$$(62 - 50)/50 = 24\%: \text{a ductile metal.}$$

3. Find the modulus of resilience of steel with $\sigma_Y = 250 \text{ MPa}$ and $E = 200 \text{ GPa}$.

$$u_r = 250^2 / (2 \times 200,000) = 0.156 \text{ MPa} = 156 \text{ kJ/m}^3.$$

4. Sample values: a car that gets 30 mpg is made 10% lighter. Using DOE's 6% to 8% figure, estimate its new fuel economy.

$$30 \times 1.06 = 31.8 \text{ to } 30 \times 1.08 = 32.4 \text{ mpg (sample car).}$$

Common misconceptions

Myth: Stronger means stiffer.

Actually: Strength (yield) and stiffness (E) are different; most steels share $E = 200 \text{ GPa}$ whatever their strength.

Myth: Strain has units of length.

Actually: It is length per length, dimensionless.

Myth: A material breaks at its yield strength.

Actually: Yield starts permanent deformation; ductile metals carry more load up to the ultimate strength.

Flashcards

Normal strain

$$\varepsilon = \delta/L.$$

Hooke's law

$$\sigma = E\varepsilon, \tau = G\gamma.$$

Poisson's ratio

$$\varepsilon_{\text{lat}} = -\nu\varepsilon.$$

Shear modulus

$$G = E/(2(1 + \nu)).$$

Yield vs ultimate

Start of plastic flow vs peak stress.

Specific stiffness

$$E/\rho.$$

Self-check quiz

1. Young's modulus is the slope of...

load vs time

stress vs strain (elastic part)

strain vs temperature

force vs area

stress vs strain (elastic part). $\sigma = E\epsilon$.

2. Poisson's ratio for most metals is about...

0.03

0.3

3

30

0.3. About 0.3.

3. G for E = 200 GPa and $\nu = 0.25$ is...

50 GPa

80 GPa

100 GPa

160 GPa

80 GPa. $200/2.5 = 80$ GPa.

4. Permanent deformation starts at the...

ultimate strength

yield strength

fracture point

proportional limit only for wood

yield strength. Yield.

Try this quiz again

How this helps save the planet

Choosing materials by stiffness and strength per kilogram saves fuel. The U.S. Department of Energy says a 10% reduction in vehicle weight can result in a 6% to 8% fuel economy improvement, and that replacing cast iron and traditional steel with high-strength steel, magnesium and aluminum alloys, carbon fiber and polymer composites can cut the weight of a vehicle's body and chassis by up to 50 percent. DOE adds that lightweight materials are especially important for electric vehicles, where they offset the weight of batteries and motors and can increase all-electric range.

On Saving Planets: [General Chemistry I](#)

Sources: U.S. DOE: [Lightweight materials for cars and trucks](https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks) (<https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks>) · [OpenStax UP1, 12.4 Elasticity and plasticity](https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity) (<https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity>)

Free learning resources

- [OpenStax UP1, 12.3 Stress, strain and elastic modulus](https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus) (<https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus>): Free textbook section.
- [OpenStax UP1, 12.4 Elasticity and plasticity](https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity) (<https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity>): Stress–strain curves.
- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures.



I finished this unit

Axially Loaded Members: Deformation, Indeterminate Bars and Thermal Stress

Elongation of bars $\delta = PL/(AE)$, bars with several segments, statically indeterminate bars solved with compatibility, thermal strain and thermal stress, and stress concentrations.

Learning objectives

By the end of this unit you can:

- Compute the elongation of uniform and stepped bars.
- Solve statically indeterminate axial problems with equilibrium plus compatibility.
- Compute thermal expansion and the stress in a restrained bar.
- Explain stress concentrations at holes and fillets.

Key concepts

- A uniform bar under axial load stretches $\delta = PL/(AE)$. For several segments, add the δ of each: $\delta = \sum P_i L_i / (A_i E_i)$.
- If equilibrium alone cannot give the forces (statically indeterminate), add a compatibility equation: how the deformations must fit together.
- A temperature change makes a free bar grow $\delta_T = \alpha \Delta T L$ with no stress. If the bar is held, the growth is pushed back and $\sigma = -E\alpha \Delta T$ appears.
- Rails, pipelines, bridges and pavements need expansion gaps or joints for this reason.
- Holes, notches and sharp corners raise local stress: $\sigma_{\max} = K \sigma_{\text{avg}}$, where K is the stress concentration factor (often 2 to 3 for a small hole).

Key equations

$$\delta = \frac{PL}{AE}$$

Axial deformation

$$\delta = \sum \frac{P_i L_i}{A_i E_i}$$

Stepped bar

$$\delta_T = \alpha \Delta T L$$

Thermal expansion

$$\sigma = -E\alpha \Delta T$$

Fully restrained bar

Worked examples, step by step

Example 3.1 A 3 m steel rod ($E = 200 \text{ GPa}$) with $A = 500 \text{ mm}^2$ carries 60 kN. Find its elongation.

$$1. \delta = PL/(AE) = 60,000 \times 3000/(500 \times 200,000) \text{ in N and mm.}$$

Answer: 1.8 mm.

Example 3.2 A steel bar ($E = 200 \text{ GPa}$, $\alpha = 12 \times 10^{-6}/^\circ\text{C}$) is fixed between two rigid walls and heated by 40°C . Find the stress.

1. Free growth would be $\alpha\Delta T L$; the walls push it back.

$$2. \sigma = -E\alpha\Delta T = -200,000 \times 12 \times 10^{-6} \times 40 \text{ MPa.}$$

Answer: 96 MPa compression.

Example 3.3 A uniform bar fixed at both ends carries an axial load $P = 30 \text{ kN}$ at a point 1 m from end A and 2 m from end B. Find both wall reactions.

1. Equilibrium: $R_A + R_B = 30 \text{ kN}$.

2. Compatibility: the stretch of segment A equals the shortening of segment B, so $R_A \times 1 = R_B \times 2$.

Answer: $R_A = 20 \text{ kN}$, $R_B = 10 \text{ kN}$.

Practice problems

Work each on paper first, then check.

1. A 25 m steel rail warms by 30°C with nothing holding it. How much does it grow?

$$\delta = 12 \times 10^{-6} \times 30 \times 25,000 = 9 \text{ mm.}$$

2. A bar has an aluminum segment ($E = 70 \text{ GPa}$, $A = 400 \text{ mm}^2$, $L = 1 \text{ m}$) and a steel segment ($A = 200 \text{ mm}^2$, $L = 0.5 \text{ m}$) in series, carrying 20 kN. Find the total stretch.

$$0.714 + 0.250 = 0.964 \text{ mm.}$$

3. Find the strain in aluminum at 100 MPa ($E = 70 \text{ GPa}$).

$$\epsilon = 100/70,000 = 1.43 \times 10^{-3}.$$

4. DOE says land-based hub heights reached about 103.4 m in 2023 after growing 83% since 1998–1999. What was the earlier average?

$$103.4/1.83 = 56.5 \text{ m}.$$

Common misconceptions

Myth: Thermal expansion always causes stress.

Actually: Only when the expansion is restrained.

Myth: An indeterminate bar can be solved with equilibrium alone.

Actually: You need compatibility (deformation) equations too.

Myth: A small hole barely matters.

Actually: It can triple the local stress, which is where fatigue cracks start.

Flashcards

Axial deformation

$$\delta = PL/(AE).$$

Stepped bar

Add $PL/(AE)$ for each segment.

Compatibility

Deformations must fit together.

Thermal strain

$$\alpha \Delta T.$$

Thermal stress

$$\sigma = -E\alpha \Delta T \text{ if fully held.}$$

Stress concentration

$$\sigma_{\text{max}} = K\sigma_{\text{avg}}.$$

Self-check quiz

1. Doubling the length of a bar (same load and area) makes δ ...

half

the same

double

four times

double. $\delta \propto L$.

2. A restrained bar that is heated is in...

tension

compression

no stress

shear only

compression. The supports push back.

3. Statically indeterminate problems need...

only equilibrium

compatibility of deformations

dynamics

no free-body diagram

compatibility of deformations. More unknowns than equilibrium equations.

4. Thermal stress in a fully restrained bar does not depend on...

E

α

ΔT

its length

its length. $\sigma = -E\alpha\Delta T$.

Try this quiz again

How this helps save the planet

Taller wind turbines catch stronger wind but need taller towers. The U.S. Department of Energy reports that the hub height of utility-scale land-based turbines grew 83% since 1998–1999 to about 103.4 meters in 2023, taller than the Statue of Liberty, and that offshore hub heights are projected to reach about 150 meters by 2035. Winds are generally stronger higher up. DOE notes tower diameters are hard to truck under bridges, so it supports towers built on site, including spiral welding to minimize costly steel. Each tower is an axially loaded column that must carry the weight above it within its allowable stress.

On Saving Planets: [Physics II](#)

Sources: U.S. DOE: Wind turbines, the bigger the better (<https://www.energy.gov/eere/articles/wind-turbines-bigger-better>) · OpenStax UP2, 1.3 Thermal expansion (<https://openstax.org/books/university-physics-volume-2/pages/1-3-thermal-expansion>)

Free learning resources

- [OpenStax UP2, 1.3 Thermal expansion](https://openstax.org/books/university-physics-volume-2/pages/1-3-thermal-expansion) (<https://openstax.org/books/university-physics-volume-2/pages/1-3-thermal-expansion>): Free textbook section.
- [OpenStax UP1, 12.3 Stress, strain and elastic modulus](https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus) (<https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus>): Axial deformation.
- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures.



I finished this unit

Torsion of Circular Shafts and Power Transmission

Shear strain and stress in twisted circular shafts, the polar moment of inertia for solid and hollow shafts, angle of twist, power and torque, shaft design, and statically indeterminate shafts.

Learning objectives

By the end of this unit you can:

- Compute shear stress with $\tau = T\rho/J$.
- Compute the angle of twist $\phi = TL/(GJ)$.
- Convert power and speed to torque with $P = T\omega$.
- Size solid and hollow shafts and explain why hollow shafts save material.

Key concepts

- In a twisted circular shaft, plane sections stay plane and shear strain grows linearly from the center: $\gamma = \rho\phi/L$.
- Shear stress is $\tau = T\rho/J$, largest at the outer surface: $\tau_{\max} = Tc/J$.
- Polar moment of inertia: $J = \pi d^4/32$ for a solid shaft and $J = \pi(d_o^4 - d_i^4)/32$ for a hollow one.
- Angle of twist $\phi = TL/(GJ)$ in radians; for several segments, add them.
- Power $P = T\omega$. Slow shafts carry huge torques for the same power, which is why a gearbox speeds up a wind turbine's rotation before the generator.

Key equations

$$\tau = \frac{T\rho}{J}, \quad \tau_{\max} = \frac{Tc}{J}$$

Torsion formula

$$J = \frac{\pi d^4}{32}, \quad J = \frac{\pi(d_o^4 - d_i^4)}{32}$$

Polar moment of inertia

$$\phi = \frac{TL}{GJ}$$

Angle of twist

$$P = T\omega$$

Power

Worked examples, step by step

Example 4.1 A solid 50 mm steel shaft carries a torque of 1.5 kN·m. Find the maximum shear stress.

1. $J = \pi \times 0.05^4/32 = 6.136 \times 10^{-7} \text{ m}^4$.
2. $\tau_{\text{max}} = Tc/J = 1500 \times 0.025/6.136 \times 10^{-7}$.

Answer: 61.1 MPa.

Example 4.2 The same shaft is 2 m long with $G = 80 \text{ GPa}$. Find the angle of twist.

1. $\phi = TL/(GJ) = 1500 \times 2/(80 \times 10^9 \times 6.136 \times 10^{-7})$.

Answer: 0.0611 rad = 3.50°.

Example 4.3 A motor delivers 50 kW at 1,500 rpm. Size a solid shaft for an allowable shear stress of 40 MPa.

1. $\omega = 1500 \times 2\pi/60 = 157.1 \text{ rad/s}$; $T = P/\omega = 318.3 \text{ N}\cdot\text{m}$.
2. $\tau = 16T/(\pi d^3)$ gives $d = (16T/(\pi\tau))^{1/3}$.

Answer: $d = 34.3 \text{ mm}$ (use 35 mm).

Practice problems

Work each on paper first, then check.

1. A hollow shaft ($d_o = 60 \text{ mm}$, $d_i = 40 \text{ mm}$) carries 2 kN·m. Find τ_{max} .

$$J = 1.021 \times 10^{-6} \text{ m}^4; \tau = 2000 \times 0.03/J = 58.8 \text{ MPa}.$$

2. Compare that hollow shaft with a solid 60 mm shaft: what fraction of the material and of the stiffness J does it keep?

$$\text{Area } 1 - (40/60)^2 = 55.6\%; J \text{ } 1 - (40/60)^4 = 80.2\%.$$

3. A solid 30 mm shaft carries 500 N·m. Find τ_{max} .

$$\tau = 16T/(\pi d^3) = 94.3 \text{ MPa}.$$

4. Sample values: a 3.4 MW wind turbine rotor turns at a sample 12 rpm. Find the torque on its main shaft.

$\omega = 1.257 \text{ rad/s}$; $T = P/\omega = 2.71 \text{ MN}\cdot\text{m}$ (sample speed).

Common misconceptions

Myth: Shear stress is uniform across a shaft.

Actually: It is zero at the center and largest at the surface.

Myth: Torque and power are the same.

Actually: $P = T\omega$: the same power at low speed needs much more torque.

Myth: A hollow shaft is much weaker.

Actually: Material near the center carries little stress; removing it saves weight for a small loss in J .

Flashcards

Torsion formula

$$\tau = T\rho/J.$$

Solid J

$$\pi d^4/32.$$

Hollow J

$$\pi(d_o^4 - d_i^4)/32.$$

Angle of twist

$$\phi = TL/(GJ).$$

Power

$$P = T\omega.$$

Shaft sizing

$$d = (16T/(\pi\tau))^{1/3}.$$

Self-check quiz

1. In a solid shaft, maximum shear stress occurs...

at the center

at the outer surface

halfway out

everywhere equally

at the outer surface. $\tau \propto \rho$.

2. Doubling a solid shaft's diameter multiplies J by...

2

4

8

16

16. $J \propto d^4$.

3. Angle of twist is proportional to...

G

J

L

1/L

L. $\phi = TL/(GJ)$.

4. A gearbox that speeds up a shaft 100 times (no losses) changes torque by a factor of...

100

1/100

1

10

1/100. $P = T\omega$ is constant.

Try this quiz again

How this helps save the planet

A wind turbine sends its power through a twisted shaft. The U.S. Department of Energy explains that the rotor connects to the generator either directly, in a direct-drive turbine, or through a shaft

and a series of gears that speed up the rotation and allow a physically smaller generator. DOE also reports that the average capacity of newly installed U.S. wind turbines reached 3.4 MW in 2023. Because power is torque times speed, a slow, large rotor puts millions of newton-meters on its main shaft, and the torsion formulas of this unit size it.

On Saving Planets: [Dynamics](#)

Sources: [U.S. DOE: How do wind turbines work?](https://www.energy.gov/eere/wind/how-do-wind-turbines-work) (<https://www.energy.gov/eere/wind/how-do-wind-turbines-work>) · [U.S. DOE: Wind turbines, the bigger the better](https://www.energy.gov/eere/articles/wind-turbines-bigger-better) (<https://www.energy.gov/eere/articles/wind-turbines-bigger-better>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on torsion.
- [OpenStax UP1, 10.6 Torque](https://openstax.org/books/university-physics-volume-1/pages/10-6-torque) (<https://openstax.org/books/university-physics-volume-1/pages/10-6-torque>): Free textbook section on torque.
- [PhET: Torque](https://phet.colorado.edu/en/simulations/torque) (<https://phet.colorado.edu/en/simulations/torque>): Interactive.



I finished this unit

Shear and Moment Diagrams and Bending Stress

Internal shear and bending moment in beams, the relations $dV/dx = -w$ and $dM/dx = V$, shear and moment diagrams, the flexure formula $\sigma = My/I$, the section modulus and efficient beam shapes.

Learning objectives

By the end of this unit you can:

- Draw shear and moment diagrams for common loads.
- Use $dV/dx = -w$ and $dM/dx = V$ to check diagrams.
- Compute bending stress with $\sigma = My/I$.
- Choose efficient cross-sections using the section modulus $S = I/c$.

Key concepts

- Cut a beam and the internal shear V and moment M hold the piece in equilibrium. Slopes follow $dV/dx = -w$ and $dM/dx = V$, so M is largest where V crosses zero.
- Pure bending keeps plane sections plane; strain and stress grow linearly from the neutral axis, which passes through the centroid.
- The flexure formula is $\sigma = -My/I$; the largest stress is $\sigma_{\max} = Mc/I = M/S$, where $S = I/c$ is the section modulus.
- For a rectangle, $I = bh^3/12$, so depth matters much more than width: a plank on edge is far stiffer and stronger than one lying flat.
- I-beams and hollow tubes put material far from the neutral axis, giving a large I for little mass.

Key equations

$$\frac{dV}{dx} = -w, \quad \frac{dM}{dx} = V$$

Load, shear and moment

$$\sigma = -\frac{My}{I}, \quad \sigma_{\max} = \frac{Mc}{I} = \frac{M}{S}$$

Flexure formula

$$I = \frac{bh^3}{12}, \quad I = \frac{\pi d^4}{64}$$

Rectangle and circle

$$M_{\max} = \frac{PL}{4}, \quad M_{\max} = \frac{wL^2}{8}$$

Simply supported: center load, uniform load

Worked examples, step by step

Example 5.1 A simply supported 4 m beam carries a 10 kN load at midspan. Find the reactions and the maximum moment.

1. By symmetry each reaction is 5 kN; $V = \pm 5$ kN.
2. $M_{\max} = PL/4$ at midspan.

Answer: 10 kN·m.

Example 5.2 The beam is a 100 mm wide × 200 mm deep rectangle. Find the maximum bending stress.

1. $I = 0.1 \times 0.2^3/12 = 6.667 \times 10^{-5} \text{ m}^4$; $c = 0.1$ m.
2. $\sigma = Mc/I = 10,000 \times 0.1/6.667 \times 10^{-5}$.

Answer: 15 MPa (tension at the bottom, compression at the top).

Example 5.3 A simply supported 6 m beam carries 5 kN/m along its length. Find $V_{\max}$ and $M_{\max}$.

1. Reactions $wL/2 = 15$ kN each.
2. $M_{\max} = wL^2/8$ at midspan, where $V = 0$.

Answer: $V_{\max} = 15$ kN; $M_{\max} = 22.5$ kN·m.

Practice problems

Work each on paper first, then check.

1. A 2 m cantilever carries 3 kN at its tip. Find the largest moment and where it acts.

$$M = 3 \times 2 = 6 \text{ kN}\cdot\text{m at the wall.}$$

2. A 200 mm × 50 mm plank is used on edge instead of flat. By what factor does I grow?

$$(200/50)^2 = 16 \text{ times.}$$

3. A solid 40 mm circular rod carries $M = 500 \text{ N}\cdot\text{m}$. Find σ_{max} .

$$\sigma = 32M/(\pi d^3) = 79.6 \text{ MPa}.$$

4. Sample values: NREL's test center can apply $84 \text{ MN}\cdot\text{m}$ to a blade root. If a root section had a sample $S = 0.4 \text{ m}^3$, what bending stress would that cause?

$$\sigma = M/S = 84/0.4 = 210 \text{ MPa (sample section)}.$$

Common misconceptions

Myth: Maximum moment is always under the biggest load.

Actually: It is where V crosses zero, which may be elsewhere.

Myth: Bending stress is uniform across the section.

Actually: It is zero at the neutral axis and largest at the top and bottom.

Myth: Width and depth help equally.

Actually: I grows with h^3 but only with b .

Flashcards

Shear-moment

$$dM/dx = V; dV/dx = -w.$$

Flexure formula

$$\sigma = My/I.$$

Section modulus

$$S = I/c; \sigma = M/S.$$

Rectangle I

$$bh^3/12.$$

Center load

$$M_{\text{max}} = PL/4.$$

Uniform load

$$M_{\text{max}} = wL^2/8.$$

Self-check quiz

1. dM/dx equals...

w

V

σ

EI

V. Slope of the moment diagram is the shear.

2. The neutral axis passes through the...

top fiber

centroid

support

shear center always

centroid. For pure bending of a linear-elastic beam.

3. Doubling a rectangular beam's depth multiplies I by...

2

4

8

16

8. h^3 .

4. $M_{\max}$ for a simply supported beam with uniform load w is...

$wL/2$

$wL^2/8$

$wL^2/2$

$PL/4$

$wL^2/8$. Standard result.

Try this quiz again

How this helps save the planet

Wind turbine blades are huge cantilever beams, and they are bent on purpose before they fly. NREL's fact sheet on the Wind Technology Testing Center in Boston, built with the U.S. Department of Energy and the Massachusetts Clean Energy Center, describes the only U.S. facility then able to test blades up to 90 meters long. Static tests pull blades horizontally and vertically to measure deflection and strains, applying up to an 84 meganewton-meter bending moment. The flexure formula of this unit turns that moment into the stress that decides whether a blade passes certification.

On Saving Planets: [Statics](#)

Sources: NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (<https://docs.nrl.gov/docs/fy12osti/53581.pdf>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on beams.
- [OpenStax UP1, 12.2 Examples of static equilibrium](https://openstax.org/books/university-physics-volume-1/pages/12-2-examples-of-static-equilibrium) (<https://openstax.org/books/university-physics-volume-1/pages/12-2-examples-of-static-equilibrium>): Free textbook section on equilibrium.
- [PhET: Balancing Act](https://phet.colorado.edu/en/simulations/balancing-act) (<https://phet.colorado.edu/en/simulations/balancing-act>): Moments, interactive.



I finished this unit

Transverse Shear in Beams, Shear Flow and Composite Sections

Shear stress from transverse loads, the shear formula $\tau = VQ/(It)$, shear stress in rectangular and circular sections, shear flow in built-up and glued beams, fastener spacing, and beams of two materials.

Learning objectives

By the end of this unit you can:

- Compute Q and the shear stress $\tau = VQ/(It)$.
- Use $\tau_{\max} = 1.5V/A$ for rectangles and $4V/(3A)$ for circles.
- Find shear flow $q = VQ/I$ and the spacing of nails, bolts or glue strength in built-up beams.
- Explain how layered beams act together.

Key concepts

- A beam with changing moment also carries shear stress on horizontal planes; that is why a stack of loose boards slides and bends much more than one glued block.
- The shear formula $\tau = VQ/(It)$ uses $Q = A'\bar{y}'$, the first moment about the neutral axis of the area beyond the cut.
- In a rectangle the shear stress is parabolic, zero at the top and bottom and $1.5V/A$ at the neutral axis.
- Shear flow $q = VQ/I$ is the force per unit length that a glue line or a row of fasteners must carry; fastener spacing $s = F_{\text{fastener}}/q$.
- In most long beams bending stress governs; in short, deep beams and in wood, shear can govern.

Key equations

$$\tau = \frac{VQ}{It}, \quad Q = A'\bar{y}'$$

Shear formula

$$\tau_{\max} = \frac{3V}{2A}$$

Rectangle

$$\tau_{\max} = \frac{4V}{3A}$$

Solid circle

$$q = \frac{VQ}{I}, \quad s = \frac{F}{q}$$

Shear flow and fastener spacing

Worked examples, step by step

Example 6.1 A 100 mm × 200 mm rectangular beam carries $V = 15$ kN. Find the maximum shear stress.

1. $A = 0.02 \text{ m}^2$.
2. $\tau_{\text{max}} = 1.5V/A = 1.5 \times 15,000/0.02$.

Answer: 1.125 MPa at the neutral axis.

Example 6.2 Two 150 mm × 50 mm boards are glued flat on top of each other to make a 150 mm wide × 100 mm deep beam carrying $V = 10$ kN. Find the shear stress the glue must carry.

1. $I = 0.15 \times 0.1^3/12 = 1.25 \times 10^{-5} \text{ m}^4$.
2. $Q = 0.15 \times 0.05 \times 0.025 = 1.875 \times 10^{-4} \text{ m}^3$.
3. $q = VQ/I = 150 \text{ N/mm}$; $\tau = q/b = 150/150$.

Answer: $\tau = 1.0$ MPa (the same as $1.5V/A$).

Example 6.3 The same two boards are nailed instead, under $V = 2$ kN, and each nail can carry 900 N. Find the nail spacing.

1. $q = VQ/I = 2000 \times 1.875 \times 10^{-4}/1.25 \times 10^{-5} = 30 \text{ N/mm}$.
2. $s = F/q = 900/30$.

Answer: 30 mm.

Practice problems

Work each on paper first, then check.

1. A solid 50 mm shaft carries $V = 10$ kN. Find τ_{max} .

$$A = 1963 \text{ mm}^2; \tau = 4V/(3A) = 6.79 \text{ MPa}.$$

2. A 50 mm × 100 mm rectangle carries $V = 5$ kN. Find τ_{max} .

$$1.5 \times 5000/5000 = 1.5 \text{ MPa.}$$

3. For a simply supported rectangular beam with a center load, show that $\sigma_{\text{max}}/\tau_{\text{max}} = 2L/h$, and evaluate it for $L/h = 10$.

$$\sigma = 1.5PL/(bh^2), \tau = 0.75P/(bh), \text{ ratio } 2L/h = 20: \text{ bending governs long beams.}$$

4. Sample values: a mid-rise building's materials carry 1,000 t CO₂e of embodied emissions. Using the 13% to 26.5% range from the Forest Service paper, estimate the saving with mass timber.

$$130 \text{ to } 265 \text{ t CO}_2\text{e (sample building).}$$

Common misconceptions

Myth: Shear stress is largest at the top and bottom, like bending stress.

Actually: It is zero there and largest at the neutral axis.

Myth: Q is the moment of inertia.

Actually: Q is a first moment of area (m³); I is a second moment (m⁴).

Myth: Glue only has to hold the boards together vertically.

Actually: It must carry the horizontal shear flow q.

Flashcards

Shear formula

$$\tau = VQ/(It).$$

First moment Q

A' $\bar{y}$ ' beyond the cut.

Rectangle

$$\tau_{\text{max}} = 1.5V/A.$$

Circle

$$\tau_{\text{max}} = 4V/(3A).$$

Shear flow

$$q = VQ/I.$$

Fastener spacing

$$s = F/q.$$

Self-check quiz

1. For a rectangle, τ_{max} is...

$$V/A$$

1.5V/A

2V/A

4V/(3A)

1.5V/A. Parabolic distribution.

2. Shear flow has units of...

N/m²

N/m

N·m

m³

N/m. Force per length.

3. Q is...

A'y'

bh³/12

VQ/I

Mc/I

A'y'. First moment of the area beyond the cut.

4. Halving the nail spacing lets the beam carry how much more shear?

the same

twice

four times

half

twice. $q = F/s$ doubles.

Try this quiz again

How this helps save the planet

Glued wood layers that carry shear flow are building a lower-carbon skyline. A 2022 paper published through the USDA Forest Service's Forest Products Laboratory explains that cross-laminated timber is at the forefront of the mass timber movement, which lets designers build taller wood buildings, and that several studies show substituting mass timber for steel and concrete in mid-rise buildings can reduce the emissions from manufacturing, transporting and installing building materials by 13% to 26.5%. The authors also stress safeguards so that more demand for wood does not harm forest carbon and health. The glue lines between layers must carry the shear flow from this unit.

On Saving Planets: [Forests](#)

Sources: [USDA Forest Service: What is the impact of mass timber utilization on climate and forests? \(2022\)](#) (<https://research.fs.usda.gov/treesearch/63853>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on beams.
- [OpenStax UP1, 12.3 Stress, strain and elastic modulus](https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus) (<https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus>): Shear stress and shear modulus.



I finished this unit

Stress Transformation, Mohr's Circle, Failure Theories and Fatigue

Plane stress on rotated planes, principal stresses and maximum shear, Mohr's circle, the Tresca and von Mises yield criteria, and an introduction to fatigue under repeated loading.

Learning objectives

By the end of this unit you can:

- Transform plane stress to any angle and find principal stresses and planes.
- Draw and use Mohr's circle.
- Apply the Tresca and von Mises criteria to find a factor of safety.
- Explain fatigue and why parts can fail far below their static strength.

Key concepts

- The same stress state looks different on different planes. Rotating by θ changes σ and τ , but some planes have zero shear: the principal planes.
- Principal stresses are $\sigma_{1,2} = (\sigma_x + \sigma_y)/2 \pm R$, with $R = \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$. The maximum in-plane shear is R , on planes 45° from the principal planes.
- Mohr's circle plots (σ, τ) for every plane: center at $(\sigma_x + \sigma_y)/2$, radius R . An angle θ on the element is 2θ on the circle.
- Ductile metals yield by shear. Tresca: the largest shear stress reaches $\sigma_Y/2$. Von Mises: $\sigma_{vm} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}$ reaches σ_Y (it fits test data a little better).
- Repeated loading grows cracks from small flaws, so parts can break after millions of cycles at stresses well below yield. Designers use S-N curves and test full parts for their lifetime of cycles.

Key equations

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal stresses

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Principal angle

$$\sigma_{vm} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$\sigma_{vm} = \sqrt{\sigma^2 + 3\tau^2}$$

Normal plus shear (shafts, beams)

Worked examples, step by step

Example 7.1 A point has $\sigma_x = 80$ MPa, $\sigma_y = -40$ MPa and $\tau_{xy} = 30$ MPa. Find the principal stresses, the principal angle and the maximum in-plane shear.

1. Center = 20 MPa; $R = \sqrt{(60^2 + 30^2)} = 67.1$ MPa.
2. $\sigma_1 = 20 + 67.1$; $\sigma_2 = 20 - 67.1$.
3. $\tan 2\theta_p = 60/120 = 0.5$.

Answer: $\sigma_1 = 87.1$ MPa, $\sigma_2 = -47.1$ MPa, $\theta_p = 13.3^\circ$, $\tau_{\max} = 67.1$ MPa.

Example 7.2 For that stress state, find the von Mises stress and the factor of safety for steel with $\sigma_Y = 250$ MPa.

1. $\sigma_{vm} = \sqrt{(87.1)^2 - (87.1)(-47.1) + 47.1^2}$.
2. $FS = 250/\sigma_{vm}$.

Answer: $\sigma_{vm} = 117.9$ MPa; $FS = 2.12$.

Example 7.3 Use Tresca for the same point.

1. σ_1 and σ_2 have opposite signs, so $\tau_{\text{abs max}} = (\sigma_1 - \sigma_2)/2 = 67.1$ MPa.
2. $FS = (\sigma_Y/2)/\tau_{\max} = 125/67.1$.

Answer: $FS = 1.86$ (Tresca is a little more conservative).

Practice problems

Work each on paper first, then check.

1. A shaft surface is in pure shear, $\tau = 50$ MPa. Find the principal stresses and their planes.

$\sigma_1 = 50$ MPa, $\sigma_2 = -50$ MPa, on planes at 45° .

2. A bar is in uniaxial tension at 100 MPa. Find the maximum shear stress and its plane.

$\tau_{\max} = 50 \text{ MPa}$ on planes at 45° .

3. A shaft carries $\sigma = 60 \text{ MPa}$ from bending and $\tau = 40 \text{ MPa}$ from torsion. Find the von Mises stress.

$$\sigma_{\text{vm}} = \sqrt{(60^2 + 3 \times 40^2)} = 91.7 \text{ MPa}.$$

4. Sample values: a part cycles once per second for 20 years. How many load cycles is that?

$$20 \times 365.25 \times 86,400 = 6.31 \times 10^8 \text{ cycles}.$$

Common misconceptions

Myth: The largest stress is always σ_x or σ_y .

Actually: Principal stresses on rotated planes can be larger.

Myth: An angle θ on the element is θ on Mohr's circle.

Actually: It is 2θ on the circle.

Myth: If a part survives one load, it survives that load forever.

Actually: Repeated loads can grow fatigue cracks far below the static strength.

Flashcards

Principal stresses

Center $\pm R$.

Mohr's circle radius

$$R = \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau^2}.$$

Max in-plane shear

$$\tau_{\max} = R, \text{ at } 45^\circ.$$

Tresca

$$\tau_{\max} = \sigma_Y/2 \text{ at yield}.$$

Von Mises

$$\sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}.$$

Fatigue

Failure after many cycles below yield.

Self-check quiz

1. On principal planes the shear stress is...

maximum

zero

equal to σ_1

R

zero. Definition.

2. Maximum in-plane shear planes are at what angle from principal planes?

30°

45°

60°

90°

45°. 90° on the circle.

3. Von Mises for uniaxial stress σ equals...

$\sigma/2$

σ

$\sqrt{3}\sigma$

2σ

σ . $\sigma_2 = 0$.

4. Fatigue failure is driven by...

one large load

repeated cycles of load

temperature alone

color

repeated cycles of load. Cracks grow cycle by cycle.

Try this quiz again

How this helps save the planet

A wind turbine blade must survive decades of bending back and forth. NREL's fact sheet on the Wind Technology Testing Center explains that fatigue tests cycle blades millions of times to simulate what a blade goes through in its lifetime on a turbine, using NREL's resonant excitation system with up to 21 meters of tip-to-tip motion to generate 20-plus years of cyclic field loads in a matter of months. Combined stress, failure theories and fatigue, all in this unit, decide whether a blade passes.

On Saving Planets: [Statics](#)

Sources: NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (<https://docs.nrlr.gov/docs/fy12osti/53581.pdf>)

Free learning resources

- MIT OpenCourseWare 2.001 Mechanics and Materials I (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on stress transformation.
- MIT OpenCourseWare 2.002 Mechanics and Materials II (<https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/>): MIT lectures on yield, fracture and fatigue.



I finished this unit

Thin-Walled Pressure Vessels and Combined Loading

Hoop and axial stress in thin cylinders, stress in thin spheres, required wall thickness, and combining axial, bending, torsion and pressure stresses at a point.

Learning objectives

By the end of this unit you can:

- Compute hoop and axial stress in thin-walled cylinders and spheres.
- Find the required wall thickness for a pressure and allowable stress.
- Superpose axial, bending, torsion and pressure stresses at a point.
- Find principal and von Mises stresses for combined loads.

Key concepts

- A vessel is thin-walled when r/t is about 10 or more; then the wall stress is nearly uniform through the thickness.
- Cylinder: hoop stress $\sigma_h = pr/t$, axial stress $\sigma_a = pr/(2t)$. The hoop stress is twice the axial, so overpressured pipes split lengthwise.
- Sphere: $\sigma = pr/(2t)$ in every direction, so a sphere needs half the wall of a cylinder of the same radius.
- Under combined loads, add the stresses from each load at a point (superposition), then use Mohr's circle or a failure criterion.
- Units matter: 1 bar = 0.1 MPa and 1 psi = 6894.757 Pa (NIST).

Key equations

$$\sigma_h = \frac{pr}{t}, \quad \sigma_a = \frac{pr}{2t}$$

Thin cylinder

$$\sigma = \frac{pr}{2t}$$

Thin sphere

$$\sigma = \frac{P}{A} \pm \frac{Mc}{I}$$

Axial plus bending

$$t = \frac{pr}{\sigma_{\text{allow}}}$$

Worked examples, step by step

Example 8.1 Sample values: a hydrogen cylinder with inner radius 150 mm and wall 15 mm holds 700 bar (70 MPa). Find the hoop and axial stresses.

1. $r/t = 10$, thin-wall estimate.
2. $\sigma_h = pr/t = 70 \times 10$; $\sigma_a = \sigma_h/2$.

Answer: $\sigma_h = 700$ MPa, $\sigma_a = 350$ MPa (sample tank, which is why such tanks use carbon-fiber composite).

Example 8.2 A sphere with the same radius, wall and pressure: find the stress.

1. $\sigma = pr/(2t)$.

Answer: 350 MPa, half the cylinder's hoop stress.

Example 8.3 A solid 100 mm post carries a 10 kN compressive load and a 3 kN·m bending moment. Find the largest compressive and tensile stresses.

1. $P/A = 10,000/7854 \text{ mm}^2 = 1.27$ MPa compression.
2. $Mc/I = 32M/(\pi d^3) = 30.6$ MPa.

Answer: 31.8 MPa compression on one face; 29.3 MPa tension on the other.

Practice problems

Work each on paper first, then check.

1. A tank design is changed from 350 bar to 700 bar at the same radius and allowable stress. How must the wall thickness change?

$t = pr/\sigma$: it doubles.

2. Sample values: a gas cylinder has $r = 150$ mm, $t = 3$ mm and $p = 2$ MPa. Find σ_h and σ_a .

$\sigma_h = 100$ MPa; $\sigma_a = 50$ MPa.

3. Find the wall thickness of a cylinder with $p = 35$ MPa, $r = 200$ mm and $\sigma_{\text{allow}} = 600$ MPa.

$$t = 35 \times 200/600 = 11.7 \text{ mm.}$$

4. Convert DOE's 10,000 psi to MPa with NIST's factor.

$$10,000 \times 6894.757 \text{ Pa} = 68.9 \text{ MPa, about 700 bar.}$$

Common misconceptions

Myth: Hoop and axial stresses are equal in a cylinder.

Actually: Hoop stress is twice the axial stress.

Myth: Thin-wall formulas work for any vessel.

Actually: Only when r/t is about 10 or more.

Myth: You can add stresses from different loads on different points.

Actually: Superpose at the same point and the same direction.

Flashcards

Hoop stress

$$\sigma_h = pr/t.$$

Axial stress

$$\sigma_a = pr/(2t).$$

Sphere

$$\sigma = pr/(2t).$$

Thin wall

$$r/t \geq 10.$$

Superposition

Add stresses at a point.

bar and psi

$$1 \text{ bar} = 0.1 \text{ MPa}; 1 \text{ psi} = 6894.757 \text{ Pa.}$$

Self-check quiz

1. Hoop stress in a thin cylinder is...

$$pr/(2t)$$

$$pr/t$$

$$pt/r$$

$2pr/t$

pr/t . Equilibrium of a half-ring.

2. A sphere's wall stress compared with a cylinder's hoop stress (same p , r , t) is...

twice

equal

half

a quarter

half. $pr/(2t)$.

3. An overpressured pipe usually splits...

around its circumference

lengthwise

at the ends only

never

lengthwise. Hoop stress is largest.

4. 700 bar is about...

7 MPa

70 MPa

700 MPa

0.7 MPa

70 MPa. 1 bar = 0.1 MPa.

Try this quiz again

How this helps save the planet

Hydrogen can power trucks, buses and industry without carbon emissions at the point of use, but it has to be stored. The U.S. Department of Energy explains that hydrogen has the highest energy per mass of any fuel but very low energy per volume, so storing it as a gas typically requires high-pressure tanks at 350–700 bar (5,000–10,000 psi), while storing it as a liquid requires cryogenic temperatures because it boils at $-252.8\text{ }^{\circ}\text{C}$ at ambient pressure. The hoop-stress formula of this unit shows why those tanks need very strong, light walls.

On Saving Planets: [General Chemistry I](#)

Sources: U.S. DOE: [Hydrogen storage](https://www.energy.gov/eere/fuelcells/hydrogen-storage) (<https://www.energy.gov/eere/fuelcells/hydrogen-storage>) · NIST Guide to the SI, [Appendix B.9: conversion factors](https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9) (<https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on pressure vessels.
- [MIT OpenCourseWare 2.002 Mechanics and Materials II](https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/) (<https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/>): MIT follow-on course.



I finished this unit

Deflection of Beams

The elastic curve and $EI v'' = M$, integration with boundary conditions, standard deflection formulas, superposition, serviceability limits and statically indeterminate beams.

Learning objectives

By the end of this unit you can:

- Integrate $EI v'' = M$ with boundary conditions to find slope and deflection.
- Use standard deflection formulas and superposition.
- Check deflection against a serviceability limit such as $L/360$.
- Solve propped cantilevers and other indeterminate beams.

Key concepts

- For small slopes, the elastic curve obeys $EI \frac{d^2v}{dx^2} = M(x)$. Integrate twice and use the supports to fix the constants ($v = 0$ at pins; $v = 0$ and $v' = 0$ at a fixed end).
- EI is the flexural rigidity: stiffer material (E) or a deeper section (I) gives smaller deflections.
- Standard results: cantilever with tip load $\delta = PL^3/(3EI)$; simply supported with center load $PL^3/(48EI)$; with uniform load $5wL^4/(384EI)$.
- Deflections from separate loads add (superposition) while the beam stays linear-elastic.
- An extra support makes the beam indeterminate; remove it, compute the deflection there, and choose the reaction that cancels it.

Key equations

$$EI \frac{d^2v}{dx^2} = M(x)$$

Elastic curve

$$\delta = \frac{PL^3}{3EI}, \quad \delta = \frac{wL^4}{8EI}$$

Cantilever: tip load, uniform load

$$\delta = \frac{PL^3}{48EI}, \quad \delta = \frac{5wL^4}{384EI}$$

Simply supported: center load, uniform load

$$R_B = \frac{3wL}{8}$$

Worked examples, step by step

Example 9.1 A 2 m steel cantilever ($E = 200 \text{ GPa}$, $I = 8 \times 10^{-6} \text{ m}^4$) carries 1 kN at its tip. Find the tip deflection.

$$1. \delta = PL^3/(3EI) = 1000 \times 8/(3 \times 200 \times 10^9 \times 8 \times 10^{-6}).$$

Answer: 1.67 mm.

Example 9.2 A simply supported 6 m steel beam ($I = 1 \times 10^{-4} \text{ m}^4$) carries 10 kN/m. Find the midspan deflection and compare with $L/360$.

$$1. \delta = 5wL^4/(384EI) = 5 \times 10,000 \times 1296/(384 \times 200 \times 10^9 \times 10^{-4}).$$

$$2. L/360 = 16.7 \text{ mm.}$$

Answer: 8.44 mm, within the limit.

Example 9.3 The same beam is fixed at A and propped at B. Find the prop reaction.

$$1. \text{ Remove the prop: cantilever tip deflection } wL^4/(8EI).$$

$$2. \text{ The prop force } R_B \text{ gives } R_B L^3/(3EI) \text{ upward; set them equal.}$$

Answer: $R_B = 3wL/8 = 22.5 \text{ kN}$.

Practice problems

Work each on paper first, then check.

1. A cantilever's length is doubled with the same tip load. By what factor does the tip deflection grow?

$$2^3 = 8 \text{ times.}$$

2. A simply supported wood beam ($E = 10 \text{ GPa}$, $I = 5 \times 10^{-5} \text{ m}^4$, $L = 4 \text{ m}$) carries 5 kN at midspan. Find δ .

$$PL^3/(48EI) = 13.3 \text{ mm.}$$

3. A 3 m steel cantilever ($I = 1 \times 10^{-5} \text{ m}^4$) carries 2 kN/m. Find the tip deflection.

$$wL^4/(8EI) = 10.1 \text{ mm.}$$

4. DOE says rotor swept areas have grown about 670% since 1998–1999. If swept area grows as diameter squared, by what factor did the average diameter grow?

Swept area $\times 7.7$, so diameter $\times \sqrt{7.7} = 2.77$.

Common misconceptions

Myth: Deflection grows in step with length.

Actually: It grows with L^3 or L^4 .

Myth: A stronger steel deflects less.

Actually: Deflection depends on E , which is about the same for all steels.

Myth: If a beam is strong enough, it is good enough.

Actually: It must also be stiff enough: floors that bounce or blades that hit the tower fail in service.

Flashcards

Elastic curve

$EIv'' = M$.

Cantilever tip load

$PL^3/(3EI)$.

Simply supported center load

$PL^3/(48EI)$.

Simply supported uniform load

$5wL^4/(384EI)$.

Superposition

Add deflections from each load.

Serviceability

Limit such as $L/360$.

Self-check quiz

1. Tip deflection of a cantilever with an end load: _____

$PL^3/(3EI)$

$PL^3/(48EI)$

$PL^2/(2EI)$

$5wL^4/(384EI)$

$PL^3/(3EI)$. Standard result.

2. EI is called the...

section modulus

flexural rigidity

polar moment

shear flow

flexural rigidity. Resistance to bending deformation.

3. At a fixed support...

$v = 0$ only

$v = 0$ and $v' = 0$

$M = 0$

$V = 0$

$v = 0$ and $v' = 0$. No deflection and no rotation.

4. Doubling the span of a simply supported beam with uniform load multiplies δ by...

4

8

16

2

16. L^4 .

[Try this quiz again](#)

How this helps save the planet

Longer blades catch more wind, but they also deflect more. The U.S. Department of Energy reports that the average rotor diameter of newly installed U.S. turbines was over 133.8 meters in 2023, that 98% of new turbines that year had rotors of 115 meters or more, and that rotor swept areas have grown around 670% since 1998–1999, letting turbines produce more electricity even where winds are lighter. DOE notes blades cannot be folded or bent once built, which limits trucking, and it is researching more slender and flexible blades. Beam-deflection theory is how designers keep a flexible blade from striking the tower.

On Saving Planets: [Physics II](#)

Sources: [U.S. DOE: Wind turbines, the bigger the better](#) (<https://www.energy.gov/eere/articles/wind-turbines-bigger-better>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on beam deflection.
- [OpenStax Calculus Volume 1, 5.5 Substitution](https://openstax.org/books/calculus-volume-1/pages/5-5-substitution) (<https://openstax.org/books/calculus-volume-1/pages/5-5-substitution>): Free calculus review for integrating the elastic curve.



I finished this unit

Buckling of Columns

Stability and the critical load, Euler's formula for pinned columns, effective length for other end conditions, slenderness ratio and critical stress, inelastic buckling, imperfections, and thin shells with knockdown factors.

Learning objectives

By the end of this unit you can:

- Explain stable, neutral and unstable equilibrium in columns.
- Compute Euler critical loads with effective length factors.
- Use the slenderness ratio to tell buckling from yielding.
- Explain why imperfections and knockdown factors matter for thin shells such as rocket tanks.

Key concepts

- A slender column can fail suddenly by bending sideways at a load far below the crushing load. That load is the critical load P_{cr} .
- Euler: $P_{cr} = \pi^2 EI / (KL)^2$. $K = 1$ for pinned–pinned, 2 for fixed–free, 0.5 for fixed–fixed and about 0.7 for fixed–pinned.
- Buckling uses the smallest I of the section: a column buckles about its weak axis.
- Critical stress $\sigma_{cr} = \pi^2 E / (KL/r)^2$, with radius of gyration $r = \sqrt{I/A}$. If σ_{cr} exceeds the yield strength, the short column yields first; between the two, inelastic buckling applies.
- Thin cylindrical shells, like rocket tanks, are very sensitive to tiny imperfections, so designers multiply the ideal buckling load by a knockdown factor taken from tests.

Key equations

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

Euler critical load

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}, \quad r = \sqrt{I/A}$$

Critical stress

$$K = 1, 2, 0.5, 0.7$$

Pinned, fixed–free, fixed–fixed, fixed–pinned

Worked examples, step by step

Example 10.1 A pinned–pinned steel column ($E = 200 \text{ GPa}$) is 3 m long with $I = 2 \times 10^{-6} \text{ m}^4$. Find P_{cr} .

1. $P_{\text{cr}} = \pi^2 \times 200 \times 10^9 \times 2 \times 10^{-6} / 3^2$.

Answer: 438.6 kN.

Example 10.2 The same column is fixed at the base and free at the top. Find P_{cr} .

1. $K = 2$, so $KL = 6 \text{ m}$ and P_{cr} drops by 4.

Answer: 109.7 kN.

Example 10.3 A solid 50 mm steel rod, 2 m long and pinned at both ends, has $\sigma_Y = 250 \text{ MPa}$. Does it buckle or yield first?

1. $r = d/4 = 12.5 \text{ mm}$; $KL/r = 2000/12.5 = 160$.

2. $\sigma_{\text{cr}} = \pi^2 \times 200,000 / 160^2 \text{ MPa}$.

Answer: $\sigma_{\text{cr}} = 77.1 \text{ MPa} < 250 \text{ MPa}$: it buckles first.

Practice problems

Work each on paper first, then check.

1. A column's length is doubled. What happens to P_{cr} ?

It drops to a quarter.

2. How does a fixed–fixed column compare with a pinned–pinned one of the same length?

$K = 0.5$, so P_{cr} is 4 times larger.

3. NASA describes a test load of up to 750,000 pounds. Convert to meganewtons with NIST's factor.

$750,000 \times 4.448222 \text{ N} = 3.34 \text{ MN}$.

4. Sample values: NASA found removing weld lands can raise load capacity by up to 28%. What would a sample 10 MN cylinder carry?

$10 \times 1.28 = 12.8 \text{ MN}$ (sample cylinder).

Common misconceptions

Myth: Columns fail when the stress reaches yield.

Actually: Slender columns buckle at much lower stress.

Myth: A column buckles about its strong axis.

Actually: It buckles about the axis with the smallest I .

Myth: The Euler load is what a real column will carry.

Actually: Imperfections and eccentric loads lower it, especially for thin shells.

Flashcards

Euler load

$$P_{cr} = \pi^2 EI / (KL)^2.$$

Effective length

$$K = 1, 2, 0.5, 0.7.$$

Radius of gyration

$$r = \sqrt{I/A}.$$

Critical stress

$$\pi^2 E / (KL/r)^2.$$

Weak axis

Smallest I governs.

Knockdown factor

Test-based reduction for thin shells.

Self-check quiz

1. Euler's P_{cr} is proportional to...

☐ L

☐ $1/L$

☐ $1/L^2$

☐ L^2

☐ $1/L^2 \cdot \pi^2 EI / (KL)^2.$

2. K for a fixed-free (flagpole) column: _____

0.5

0.7

1

2

2. Effective length $2L$.

3. Slenderness ratio is... _____

KL/r

r/L

I/A

E/σ_Y

KL/r . Effective length over radius of gyration.

4. A knockdown factor accounts for... _____

temperature

imperfection sensitivity of thin shells

paint

gravity

imperfection sensitivity of thin shells. Real shells buckle below ideal theory.

Try this quiz again

How this helps save the planet

Lighter rockets launch the satellites that watch Earth's climate, and buckling sets how light their tanks can be. NASA's Engineering and Safety Center reports that its Shell Buckling Knockdown

Factor team re-examined knockdown factors that had not been updated since the Apollo era of the 1960s. Eight subscale and two full-scale metallic cylinder tests led to new knockdown factors that are used on the Space Launch System core stage, giving both mass and cost savings. In one test, an 8-foot-diameter cylinder loaded up to 750,000 pounds buckled within 2.5% of the engineer's prediction, and early results showed that removing weld lands can raise load capacity by up to 28%.

On Saving Planets: [Intro to Aerospace Engineering](#)

Sources: [NASA NESC: Shell Buckling Knockdown Factor project](https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/) (<https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/>)

Free learning resources

- [MIT OpenCourseWare 2.001 Mechanics and Materials I](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/) (<https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/>): MIT lectures on buckling.
- [NASA NESC: Shell Buckling Knockdown Factor project](https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/) (<https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/>): How NASA tests buckling of rocket cylinders.



I finished this unit

The lab (Materials lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation or with household items. Stretched rubber bands and snapping spaghetti can fly: wear glasses, keep loads small, and keep faces away from parts under load.

Safety reference: [OSHA: Hand and power tools](https://www.osha.gov/hand-power-tools) (<https://www.osha.gov/hand-power-tools>)

Lab A: Hooke's law and stiffness

WEEK 2

Goal: Load a spring (or rubber band) in steps, show that stretch grows in proportion to force, and find the stiffness k .

$$F = kx, \quad U = \frac{1}{2}kx^2$$

Linear spring and stored energy

Procedure

1. Open PhET Hooke's Law, choose one spring and set its constant; apply forces in equal steps and record the displacement.
2. Home: hang a bag from a rubber band and add coins of known mass ($F = mg$), measuring the stretch with a ruler each time.
3. Plot F against x and fit a line through the origin; the slope is k .
4. Unload step by step and note whether the rubber band returns to its starting length.

Data table

F (N)	x (mm) for $k = 50 \text{ N/m}$
0.5	10
1.0	20
1.5	30
2.0	40

Sample values for a spring with $k = 50 \text{ N/m}$; your own spring sets your slope.

Sample calculation

1. $k = F/x = 0.5 \text{ N}/0.010 \text{ m} = 50 \text{ N/m}$ in every row.
2. Stored energy at 2 N: $U = \frac{1}{2} \times 50 \times 0.04^2 = 0.04 \text{ J}$.

Safety

- Wear glasses; a stretched rubber band can snap back.

Home or virtual alternative

- The rubber-band step is the home version.

PhET: Hooke's Law (<https://phet.colorado.edu/en/simulations/hookes-law>)

Planet link: Stiffness tests like this one, scaled up, are how engineers qualify every bolt, cable and bridge member. ASCE Report Card for America's Infrastructure: Bridges (<https://infrastructurereportcard.org/cat-item/bridges-infrastructure/>)



I completed this lab (wet or virtual)

Lab B: Stress, strain and the modulus of a rubber band

WEEK 3

Goal: Turn force and stretch into stress and strain, estimate a modulus, and see where the response stops being linear.

$$\sigma = \frac{F}{A}, \quad \varepsilon = \frac{\delta}{L_0}, \quad E = \frac{\sigma}{\varepsilon}$$

Engineering stress and strain

Procedure

1. Measure a flat rubber band's width and thickness with a ruler (or count layers of a known thickness) and its unstretched length L_0 .
2. Hang known masses and record the stretch δ for each.
3. Compute σ and ε and plot σ against ε . Fit the straight early part to estimate E .
4. Keep adding small masses and note where the curve bends: rubber is not linear at large strain.

Data table

F (N)	δ (mm)	σ (MPa)	ε
1	8	0.667	0.10
2	16	1.333	0.20
3	24	2.000	0.30

Sample values for a band with $A = 1.5 \text{ mm}^2$ and $L_0 = 80 \text{ mm}$ in its near-linear range.

Sample calculation

1. $\sigma = 1 \text{ N}/1.5 \text{ mm}^2 = 0.667 \text{ MPa}$; $\varepsilon = 8/80 = 0.10$.
2. $E = 0.667/0.10 = 6.67 \text{ MPa}$, about 30,000 times less stiff than steel (200 GPa).

Safety

- Wear glasses and keep the band pointed away from faces; stop before it breaks.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Masses and Springs (<https://phet.colorado.edu/en/simulations/masses-and-springs>)

Planet link: Knowing stiffness and strength per kilogram is how lighter car bodies are chosen, and DOE links a 10% weight cut to 6% to 8% better fuel economy. U.S. DOE: Lightweight materials for cars and trucks (<https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks>)



I completed this lab (wet or virtual)

Lab C: Twisting a foam rod

WEEK 6

Goal: Twist a foam rod with a known torque, measure the angle of twist, and check that ϕ grows in proportion to T as in $\phi = TL/(GJ)$.

$$\phi = \frac{TL}{GJ}, \quad J = \frac{\pi d^4}{32}$$

Angle of twist of a solid rod

Procedure

1. Draw a straight line along a solid foam rod and mark two rings 0.5 m apart.
2. Hold one end fixed and apply a torque with a lever arm and a known weight ($T = F \times \text{arm}$).
3. Photograph the line and read the twist angle between the rings; repeat for three torques.
4. Plot ϕ against T ; the slope is $L/(GJ)$. Note the spiral the straight line becomes: that is shear strain.

Data table

T (N·m)	ϕ (°)
0.1	4.67
0.2	9.34
0.3	14.0

Values computed for a sample rod with $d = 50$ mm, $L = 0.5$ m and $G = 1$ MPa.

Sample calculation

1. $J = \pi \times 0.05^4/32 = 6.136 \times 10^{-7} \text{ m}^4$.
2. $\phi = 0.2 \times 0.5/(10^6 \times 6.136 \times 10^{-7}) = 0.163 \text{ rad} = 9.34^\circ$.
3. Surface shear strain $\gamma = c\phi/L = 0.025 \times 0.163/0.5 = 0.00815$.

Safety

- Keep weights small and low over a soft surface.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Torque (<https://phet.colorado.edu/en/simulations/torque>)

Planet link: Wind turbine shafts carry the same kind of twisting load, millions of times larger. U.S. DOE: How do wind turbines work? (<https://www.energy.gov/eere/wind/how-do-wind-turbines-work>)



I completed this lab (wet or virtual)

Lab D: Shape matters, a ruler flat and on edge

WEEK 8

Goal: Load a ruler as a cantilever lying flat and then on edge and compare deflection and stress with $I = bh^3/12$.

$$I = \frac{bh^3}{12}, \quad \delta = \frac{PL^3}{3EI}, \quad \sigma = \frac{Mc}{I}$$

Section shape and cantilever bending

Procedure

1. Clamp a 30 mm × 3 mm plastic ruler flat to a table with 250 mm sticking out.
2. Hang a 0.5 N load (about 51 g) from the tip and measure the deflection against a second ruler.
3. Turn the ruler on edge (clamp it between two books) and repeat with the same load.
4. Compare the ratio of deflections with the ratio of I values.

Data table

Orientation	I (m ⁴)	δ at 0.5 N (mm)	σ_{max} (MPa)
Flat ($h = 3$ mm)	6.75×10^{-11}	12.9	2.78
On edge ($h = 30$ mm)	6.75×10^{-9}	0.129	0.278

Values computed for a sample plastic ruler with $E = 3$ GPa.

Sample calculation

1. I ratio = $(30/3)^2 = 100$, so the edge deflection is 100 times smaller.
2. $M = 0.5 \times 0.25 = 0.125$ N·m; flat $\sigma = 0.125 \times 0.0015 / 6.75 \times 10^{-11} = 2.78$ MPa.

Safety

- Do not overload the ruler; plastic can snap.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Balancing Act (<https://phet.colorado.edu/en/simulations/balancing-act>)

Planet link: Putting material far from the neutral axis is how blades, beams and towers save material. NREL: Building state-of-the-art wind technology testing facilities (fact sheet) (<https://docs.nrlr.gov/docs/fy12osti/53581.pdf>)



I completed this lab (wet or virtual)

Lab E: Cantilever deflection and Young's modulus

WEEK 12

Goal: Measure tip deflection against load for a flat ruler cantilever and back out Young's modulus from $\delta = PL^3/(3EI)$.

$$E = \frac{L^3}{3I(\delta/P)}$$

From the slope of δ against P

Procedure

1. Use the flat ruler from Lab D with 250 mm overhang.
2. Hang 0.25, 0.5 and 1.0 N at the tip and record the deflection each time.
3. Plot δ against P and fit a line through the origin.
4. Compute E from the slope and compare with a typical value for the plastic.

Data table

P (N)	δ (mm)
0.25	6.43
0.50	12.9
1.00	25.7

Values computed for a sample plastic ruler with $E = 3 \text{ GPa}$, $b = 30 \text{ mm}$, $h = 3 \text{ mm}$.

Sample calculation

1. Slope $\delta/P = 25.7 \text{ mm/N}$.
2. $E = 0.25^3/(3 \times 6.75 \times 10^{-11} \times 0.0257) = 3.0 \text{ GPa}$.

Safety

- Keep loads small and catch them if they slip.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Hooke's Law (<https://phet.colorado.edu/en/simulations/hookes-law>)

Planet link: Longer, more flexible wind blades are designed with the same deflection formula. [U.S. DOE: Wind turbines, the bigger the better](#) (<https://www.energy.gov/eere/articles/wind-turbines-bigger-better>)



I completed this lab (wet or virtual)

Lab F: Buckling spaghetti

WEEK 14

Goal: Find the load that buckles strands of dry spaghetti of different lengths and check that P_{cr} falls as $1/L^2$.

$$P_{cr} = \frac{\pi^2 EI}{L^2}, \quad I = \frac{\pi d^4}{64}$$

Pinned–pinned column

Procedure

1. Cut strands to 100, 150 and 200 mm. Stand each upright on a kitchen scale and press down slowly on top with a flat object.
2. Record the highest scale reading just as the strand bows sideways (convert grams to newtons).
3. Repeat three times per length and average.
4. Plot P_{cr} against $1/L^2$; the line should pass near the origin.

Data table

L (mm)	P_{cr} (N)
100	1.526
150	0.678
200	0.381

Values computed for a sample strand with $d = 1.8$ mm and $E = 3$ GPa; real spaghetti varies.

Sample calculation

1. $I = \pi \times 1.8^4/64 = 0.515 \text{ mm}^4 = 5.153 \times 10^{-13} \text{ m}^4$.
2. P_{cr} at 0.1 m = $\pi^2 \times 3 \times 10^9 \times 5.153 \times 10^{-13}/0.01 = 1.526 \text{ N}$ (about 156 g on the scale).
3. Doubling L from 100 to 200 mm cuts P_{cr} by 4.

Safety

- Wear glasses; spaghetti snaps into sharp pieces.

Home or virtual alternative

- The whole lab is a home lab.

Planet link: NASA tests full-size rocket cylinders to buckling to set knockdown factors that save mass. [NASA NESC: Shell Buckling Knockdown Factor project](https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/) (<https://www.nasa.gov/centers-and-facilities/nesc/proving-management-skills-on-the-shell-buckling-knockdown-factor-project/>)

☐ I completed this lab (wet or virtual)

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. A 16 mm steel rod carries 30 kN in tension. Find the stress.

1. $A = \pi \times 16^2/4 = 201.1 \text{ mm}^2$.

Answer: 149.2 MPa.

2. An aluminum bar ($E = 70 \text{ GPa}$) 1.5 m long with $A = 300 \text{ mm}^2$ carries 12 kN. Find its elongation.

1. $\delta = PL/(AE)$.

Answer: 0.857 mm.

3. A steel bar ($E = 200 \text{ GPa}$, $\alpha = 12 \times 10^{-6}/^\circ\text{C}$) is held between rigid walls and heated 25°C . Find the stress.

1. $\sigma = E\alpha\Delta T$.

Answer: 60 MPa compression.

4. A solid 40 mm shaft carries 800 N·m. Find τ_{max} .

1. $\tau = 16T/(\pi d^3)$.

Answer: 63.7 MPa.

5. A simply supported 5 m beam, 120 mm wide $\times$ 240 mm deep, carries 4 kN/m. Find σ_{max} .

1. $M = wL^2/8 = 12.5 \text{ kN}\cdot\text{m}$; $I = 1.382 \times 10^{-4} \text{ m}^4$.

Answer: 10.9 MPa.

Final practice set (all units)

1. An 80 mm × 160 mm rectangular beam carries $V = 12$ kN. Find $\tau_{\max}$.

1. $\tau = 1.5V/A$.

Answer: 1.41 MPa.

2. Find the principal stresses for $\sigma_x = 50$ MPa, $\sigma_y = 10$ MPa, $\tau_{xy} = 15$ MPa.

1. Center 30 MPa; $R = \sqrt{(20^2 + 15^2)} = 25$ MPa.

Answer: $\sigma_1 = 55$ MPa, $\sigma_2 = 5$ MPa.

3. A thin cylinder has $r = 0.5$ m, $t = 10$ mm and $p = 2$ MPa. Find σ_h and σ_a .

1. pr/t and $pr/(2t)$.

Answer: 100 MPa and 50 MPa.

4. A 1.5 m steel cantilever ($I = 4 \times 10^{-6}$ m⁴) carries 2 kN at its tip. Find the tip deflection.

1. $\delta = PL^3/(3EI)$.

Answer: 2.81 mm.

5. A pinned steel column 4 m long has $I = 5 \times 10^{-6}$ m⁴. Find P_{cr} .

1. $\pi^2 EI/L^2$.

Answer: 616.9 kN.

6. A shaft point has $\sigma = 80$ MPa and $\tau = 30$ MPa. Find the von Mises stress.

1. $\sqrt{(\sigma^2 + 3\tau^2)}$.

Answer: 95.4 MPa.

Formula sheet

$$\sigma = P/A, \tau = V/A, \sigma_b = P/(td)$$

$$\varepsilon = \delta/L, \sigma = E\varepsilon, G = E/(2(1 + \nu))$$

$$\delta = PL/(AE), \delta_T = \alpha\Delta TL$$

$$\tau = T\rho/J, \phi = TL/(GJ), J = \pi d^4/32$$

$$P = T\omega$$

$$dM/dx = V, dV/dx = -w$$

$$\sigma = My/I, I = bh^3/12$$

$$\tau = VQ/(It), q = VQ/I$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{vm} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$\sigma_h = pr/t, \sigma_a = pr/(2t)$$

$$EIv'' = M$$

$$\delta = PL^3/(3EI), PL^3/(48EI), 5wL^4/(384EI)$$

$$P_{cr} = \pi^2 EI/(KL)^2$$

Constants

Constant	Value
1 megapascal (MPa)	10 ⁶ Pa = 1 N/mm ²
1 bar	10 ⁵ Pa = 0.1 MPa
1 psi (lbf/in ²)	6894.757 Pa
1 pound-force (lbf)	4.448222 N
1 revolution per minute	0.1047198 rad/s
Typical steel (textbook values)	E = 200 GPa, G = 80 GPa, ν = 0.3, α = 12 × 10 ⁻⁶ /°C, yield about 250 MPa
Typical aluminum alloy (textbook values)	E = 70 GPa, G = 26 GPa, ν = 0.33, α = 23 × 10 ⁻⁶ /°C

Source: [NIST Guide to the SI, Appendix B.9: conversion factors \(https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9\)](https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free textbooks, lectures and simulations alongside the course.

- [MIT OpenCourseWare 2.001 Mechanics and Materials I \(https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/\)](https://ocw.mit.edu/courses/2-001-mechanics-materials-i-fall-2006/): Free MIT lecture notes, problem sets and exams.
- [MIT OpenCourseWare 2.002 Mechanics and Materials II \(https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/\)](https://ocw.mit.edu/courses/2-002-mechanics-and-materials-ii-spring-2004/): Free MIT follow-on course: plasticity, fracture and fatigue.
- [OpenStax University Physics Volume 1, 12.3 Stress, strain and elastic modulus \(https://openstax.org/book/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus\)](https://openstax.org/book/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook introduction to stress and strain.
- [OpenStax University Physics Volume 1, 12.4 Elasticity and plasticity \(https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity\)](https://openstax.org/books/university-physics-volume-1/pages/12-4-elasticity-and-plasticity): Yield, plastic flow and fracture.
- [PhET: Hooke's Law \(https://phet.colorado.edu/en/simulations/hookes-law\)](https://phet.colorado.edu/en/simulations/hookes-law): Springs, stiffness and energy.
- [PhET: Masses and Springs \(https://phet.colorado.edu/en/simulations/masses-and-springs\)](https://phet.colorado.edu/en/simulations/masses-and-springs): Stiffness and stretch.

Glossary

Bearing stress

Contact pressure between a fastener and a plate, $P/(td)$.

Elastic curve

Deflected shape of a beam's neutral axis.

Factor of safety

Failure load divided by allowable load.

Fatigue

Failure from repeated loading, often below yield.

Flexural rigidity

EI , a beam's resistance to bending deformation.

Hoop stress

Circumferential stress $p r/t$ in a pressurized cylinder.

Knockdown factor

Test-based reduction of the ideal buckling load of thin shells.

Neutral axis

Line of zero bending stress through the centroid.

Poisson's ratio

Ratio of lateral to axial strain, with a minus sign.

Polar moment of inertia

J , a shaft's resistance to twisting.

Principal stresses

Normal stresses on planes with zero shear.

Section modulus

$S = I/c$.

Shear flow

Shear force per unit length, VQ/I .

Slenderness ratio

KL/r of a column.

Strain

Deformation per unit length.

Stress

Internal force per unit area.

Von Mises stress

Equivalent stress compared with the yield strength.

Yield strength

Stress at which permanent deformation begins.

Young's modulus

E, slope of the elastic stress–strain line.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Finds internal loads with a free-body diagram, picks the right deformation model, checks assumptions (elastic, thin-walled, slender), and reports stresses with a factor of safety.
3 · Proficient	Solves the practice problems with occasional unit or sign slips.
2 · Developing	Follows worked examples but mixes up I, J and Q or forgets compatibility.
1 · Beginning	Plugs numbers into formulas without diagrams; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	25%
Labs	20%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.