Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Fall · 🧮 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester junior fluid mechanics course. It is modeled on the topics and depth of MIT 2.06 Fluid Dynamics and MIT 1.060 Engineering Mechanics II (both free on MIT OpenCourseWare), and on courses such as Purdue ME 309 and Georgia Tech CEE 3040, using the free OpenStax University Physics fluids chapter and PhET simulations, which we link where they help. None of those authors or schools endorses this page.
>
> What we add: every unit ties to real work with a cited public source (hydropower, pumped storage, marine energy, household water leaks, ocean circulation and wind turbines), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a safe home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Moving water and air carries most of the world’s renewable energy: rivers through hydropower turbines, wind across turbine blades, tides and waves into marine energy devices. Fluid mechanics also decides how much energy it takes to pump water to cities and farms, and how much is lost to leaks and friction.

Ocean currents move heat around the planet and shape the climate. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (ENGR 330).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Calculus III: partial derivatives, divergence and multiple integrals.
- Differential Equations: first- and second-order ODEs.
- Statics and Dynamics: free-body diagrams, Newton’s laws and momentum.
- Thermodynamics helps with the energy equation but is not required.

How to study

- Start every problem with a sketch: control volume, streamline, or free-body diagram of the fluid.
- Write the governing equation in words first (mass, momentum or energy), then cancel terms you can justify.
- Work in heads (metres of fluid) for liquid systems; they make pumps, pipes and turbines easy to compare.
- Compute the Reynolds number before choosing a friction or drag correlation.
- Check units and limits: zero flow means zero loss; doubling speed roughly quadruples turbulent losses.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free PhET Density, Buoyancy, Under Pressure and Fluid Pressure and Flow simulations.

- A clear plastic bottle, a nail or skewer, a ruler and tape.
- A kitchen scale, a measuring jug, a bucket and a stopwatch.
- A garden hose or a sink tap.
- Spreadsheet software for the pipe and pump labs.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Fluid Properties: Density, Viscosity and Surface Tension	Fluid properties, viscosity, surface tension	OpenStax UP1 14.1, 14.7
2	Fluid Statics, Manometers and Buoyancy	Hydrostatics and manometers	UP1 14.2
3	Fluid Statics, Manometers and Buoyancy	Forces on submerged surfaces, buoyancy, stability; Lab A	UP1 14.4
4	Bernoulli, Heads and the Energy Equation	Continuity, Bernoulli, heads, energy equation; Lab B	UP1 14.5–14.6
5	Integral Momentum: Jets, Vanes, Bends and Turbines	Reynolds transport theorem, momentum, jets, vanes, Pelton	MIT 2.06
6	Differential Analysis: Continuity, Navier–Stokes and Exact Flows	Kinematics, continuity, stream function	MIT 2.06
7	Differential Analysis: Continuity, Navier–Stokes and Exact Flows	Navier–Stokes, Couette and Poiseuille; Lab C; midterm practice	MIT 2.25
8	Dimensional Analysis, Similitude and Model Testing	Dimensional analysis and similitude	MIT 1.060
9	Pipe Flow: Friction, the Moody Chart and Minor Losses	Laminar and turbulent pipe flow, Moody chart	MIT 1.060
10	Pipe Flow: Friction, the Moody Chart and Minor Losses	Minor losses, networks; Lab D	MIT 1.060
11	Pumps, Turbines and Pump Systems	Pumps, system curves, affinity laws, turbines; Lab F	DOE pump systems
12	External Flow: Boundary Layers, Drag and Lift	Boundary layers, drag, Stokes flow	NASA Glenn
13	External Flow: Boundary Layers, Drag and Lift	Lift, wind power, Betz limit; Lab E	NASA Glenn; DOE wind

Week	Unit	Topics	Reading
14	Open-Channel Flow: Rivers, Canals and Hydraulic Jumps	Open channels, Froude number, Manning	MIT 1.060
15	Open-Channel Flow: Rivers, Canals and Hydraulic Jumps	Specific energy, hydraulic jumps, weirs; final practice	MIT 1.060

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Fluid Properties: Density, Viscosity and Surface Tension

What makes a fluid, the continuum assumption, density and specific gravity, pressure, viscosity and Newton’s law of viscosity, kinematic viscosity, non-Newtonian fluids, surface tension and capillary rise, and vapor pressure and cavitation.

Learning objectives

By the end of this unit you can:

- Define a fluid and use the continuum assumption.
- Use Newton’s law of viscosity to find shear stress.
- Compute capillary rise from surface tension.
- Explain how density differences drive flows.

Key concepts

- A fluid deforms continuously under any shear stress, however small. Treating it as a continuum lets us define density, velocity and pressure at every point.
- Density ρ (kg/m³) and specific gravity SG = ρ/ρ_water. Water at 20 °C has ρ = 998 kg/m³; air at 15 °C has 1.225 kg/m³.
- Newton’s law of viscosity: shear stress τ = μ du/dy. Dynamic viscosity μ measures resistance to shearing; kinematic viscosity ν = μ/ρ. Water is about 56 times more viscous than air but has about 15 times lower ν.
- Non-Newtonian fluids (paint, blood, toothpaste) have a viscosity that changes with shear rate.
- Surface tension σ pulls liquid surfaces tight; in a thin tube water rises h = 4σ cos θ/(ρgD). If local pressure drops to the vapor pressure, the liquid boils: cavitation, which damages pumps and propellers.

Key equations

$$\tau = \mu \frac{du}{dy}$$

τ=μdu/dy

Newton’s law of viscosity

$$\nu = \frac{\mu}{\rho}$$

ν=(μ)/(ρ)

Kinematic viscosity

$$SG = \frac{\rho}{\rho_{water}}$$

`SG=ρρ_water`

Specific gravity

$$h = \frac{4\sigma \cos\theta}{\rho g D}$$

`h=(4σθ)/(ρ gD)`

Capillary rise

Worked examples, step by step

Example 1.1 Water (μ = 1.002 × 10⁻³ kg/(m·s)) fills a 1 mm gap between a fixed plate and one moving at 2 m/s. Find the shear stress.

1. du/dy = 2/0.001 = 2000 s⁻¹.
2. τ = 1.002 × 10⁻³ × 2000.

Answer: τ = 2.00 Pa.

Example 1.2 Find the capillary rise of water (σ = 0.0728 N/m, θ = 0) in a 1 mm glass tube.

1. h = 4 × 0.0728/(998 × 9.81 × 0.001).

Answer: h = 29.7 mm.

Example 1.3 Find ν for water at 20 °C.

1. ν = 1.002 × 10⁻³/998.

Answer: ν = 1.004 × 10⁻⁶ m²/s.

Practice problems

Work each on paper first, then check.

1. Find the SG of seawater (1025 kg/m³).

1025/998 = 1.027.

2. Find the capillary rise in a 0.5 mm tube.

   Twice as much: 59.5 mm.

3. Find μ_water/μ_air.

   $1.002 \times 10^{-3}/1.789 \times 10^{-5} = 56.0$.

4. What fraction of an iceberg floats below seawater?

   917/1025 = 89.5%.

Common misconceptions

Myth: Viscosity and density are the same thing.
Actually: Honey is viscous; mercury is dense but not very viscous.

Myth: Fluids resist shear like solids.
Actually: They flow under any shear stress.

Myth: Cavitation needs high temperature.
Actually: Low pressure is enough.

Flashcards

Fluid

Deforms continuously under shear.

Viscosity

$\tau = \mu\, du/dy$.

Kinematic viscosity

$\nu = \mu/\rho$.

Specific gravity

ρ/ρ_{water}.

Capillary rise

$4\sigma \cos\theta/(\rho g D)$.

Cavitation

Boiling at low pressure.

Self-check quiz

1. Shear stress in a Newtonian fluid is...

μ du/dy

ρgh

ρV^2

σ/D

μ du/dy. Newton’s law of viscosity.

2. Kinematic viscosity is...

μρ

μ/ρ

ρ/μ

τ/μ

μ/ρ. ν = μ/ρ.

3. Capillary rise grows when the tube is...

wider

narrower

longer

heavier

narrower. h ∝ 1/D.

4. Cavitation occurs when pressure drops to...

zero gauge

the vapor pressure

atmospheric

stagnation

the vapor pressure. Local boiling.

Try this quiz again

How this helps save the planet

NOAA explains that winds drive ocean currents in the upper 100 meters, but deep-ocean currents thousands of meters down are driven by differences in water density, controlled by temperature and salinity: thermohaline circulation.

In polar regions, sea ice forms and leaves salt behind, so the surrounding water becomes saltier and denser and sinks; surface water flows in to replace it, starting the global conveyor belt. Density, the first property in this unit, moves heat around the whole planet.

On Saving Planets: Oceans · Earth Observation

Sources: NOAA National Ocean Service: Thermohaline circulation (https://oceanservice.noaa.gov/education/tutorial_currents/05conveyor1.html)

Free learning resources

- OpenStax UP1, 14.1 Fluids, density and pressure (https://openstax.org/books/university-physics-volume-1/pages/14-1-fluids-density-and-pressure): Free textbook section.
- OpenStax UP1, 14.7 Viscosity and turbulence (https://openstax.org/books/university-physics-volume-1/pages/14-7-viscosity-and-turbulence): Free textbook section.
- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): MIT fluid properties notes.
- PhET: Density (https://phet.colorado.edu/en/simulations/density): Density interactively.

☐ **I finished this unit**

Fluid Statics, Manometers and Buoyancy

Pressure at a point, the hydrostatic equation, gauge and absolute pressure, manometers, hydrostatic forces on plane submerged surfaces and the center of pressure, forces on curved surfaces, Archimedes' principle, and stability of floating bodies.

Learning objectives

By the end of this unit you can:

- Compute pressure at depth and read manometers.
- Find the resultant force and center of pressure on a submerged gate.
- Apply Archimedes' principle.
- Judge whether a floating body is stable.

Key concepts

- Pressure at a point acts equally in all directions. In a static fluid dp/dz = −ρg, so with constant density p = p_0 + ρgh at depth h. Gauge pressure is measured relative to the atmosphere.
- Manometers balance columns of fluid: move through the tube adding ρgh going down and subtracting going up.
- On a plane surface the resultant force is F = ρg h_c A (h_c = depth of the centroid). It acts at the center of pressure, below the centroid: y_cp = y_c + I_xc/(y_c A).
- On curved surfaces, find the horizontal component as the force on the vertical projection and the vertical component as the weight of fluid above.
- Archimedes: the buoyant force equals the weight of displaced fluid, F_B = ρ_fluid g V_displaced. A floating body is stable if its metacenter lies above its center of gravity.

Key equations

$$p = p_0 + \rho g h$$

`p=p_0+ρ gh`

Hydrostatic pressure

$$F = \rho g h_c A$$

`F=ρ g h_cA`

Force on a plane surface

$$y_{cp} = y_c + \frac{I_{xc}}{y_c A}$$

`y_cp=y_c+_xcy_cA`

Center of pressure

$$F_B = \rho_f g V_{disp}$$

`F_B=ρ_f gV_disp`

Buoyant force

Worked examples, step by step

Example 2.1 Find the gauge pressure 10 m below the surface of fresh water (ρ = 998 kg/m³).

1. p = 998 × 9.81 × 10.

Answer: p = 97.9 kPa (about one atmosphere).

Example 2.2 A vertical rectangular gate 2 m wide and 3 m tall has its top edge at the water surface. Find the force and the depth of the center of pressure.

1. h_c = 1.5 m; A = 6 m².
2. F = 998 × 9.81 × 1.5 × 6 = 88.1 kN.
3. I_xc = 2 × 3³/12 = 4.5 m⁴; y_cp = 1.5 + 4.5/(1.5 × 6).

Answer: F = 88.1 kN acting 2.0 m below the surface (two-thirds of the depth).

Example 2.3 A mercury manometer (ρ = 13,600 kg/m³) connected across a water pipe shows a 0.1 m difference. Find the pressure difference.

1. Δp = (13,600 − 1000) × 9.81 × 0.1 (water above the mercury, ρ_water ≈ 1000 kg/m³).

Answer: Δp = 12.4 kPa.

Practice problems

Work each on paper first, then check.

1. Find the buoyant force on a fully submerged 2 L object in water (ρ ≈ 1000 kg/m³).

   1000 × 9.81 × 0.002 = 19.6 N.

2. What is the absolute pressure 10 m down if p_atm = 101.3 kPa?

   199.2 kPa.

3. Does the force on a dam depend on how long the reservoir is?

   No; only depth and wall area.

4. Why is the center of pressure below the centroid?

   Pressure grows with depth.

Common misconceptions

Myth: Pressure depends on container shape.
Actually: Only on depth (the hydrostatic paradox).

Myth: Buoyancy depends on the object's weight.
Actually: It depends on displaced volume.

Myth: The force on a gate acts at its center.
Actually: It acts at the center of pressure, lower down.

Flashcards

Hydrostatic

$p = p_0 + \rho g h$.

Gauge pressure

Relative to atmosphere.

Plane force

$\rho g\, h_c\, A$.

Center of pressure

$y_c + I/(y_c\, A)$.

Archimedes

$F_B = \rho g V$.

Stability

Metacenter above G.

Self-check quiz

1. Pressure at depth h is...

$\rho g h$

$\rho V^2/2$

$\mu\, du/dy$

mg/A

ρgh. Hydrostatics (gauge).

2. Buoyant force equals...

object weight

displaced fluid weight

object mass

zero

displaced fluid weight. Archimedes.

3. For a vertical gate from the surface, the center of pressure is at...

half depth

two-thirds depth

the surface

the bottom

two-thirds depth. Triangular load.

4. A floating body is stable if...

G above M

M above G

B = G

it is heavy

M above G. Metacenter.

Try this quiz again

How this helps save the planet

DOE notes that the United States has more than 90,000 dams, and that less than 3% of them produce power; the rest serve recreation, farm ponds, flood control, water supply and irrigation. Every one of them is designed with this unit's equations: the water load on a dam grows with the square of the depth and acts low on the wall.

DOE describes the most common hydropower plant, an impoundment facility, as a dam that stores river water in a reservoir; water released through a turbine spins a generator, and releases can also serve flood control, recreation, fish passage and water quality. The stored water's pressure on the dam and its head at the turbine both come straight from $p = \rho g h$.

On Saving Planets: Statics · Oceans

Sources: U.S. DOE: Types of hydropower plants (https://www.energy.gov/eere/water/types-hydropower-plants)

Free learning resources

- OpenStax UP1, 14.2 Measuring pressure (https://openstax.org/books/university-physics-volume-1/pages/14-2-measuring-pressure): Free textbook section.
- OpenStax UP1, 14.4 Archimedes' principle and buoyancy (https://openstax.org/books/university-physics-volume-1/pages/14-4-archimedes-principle-and-buoyancy): Free textbook section.
- PhET: Under Pressure (https://phet.colorado.edu/en/simulations/under-pressure): Hydrostatics interactively.
- PhET: Buoyancy (https://phet.colorado.edu/en/simulations/buoyancy): Buoyancy interactively.

Watch the lecture

Fluid Dynamics and Statics and Bernoulli's Equation (https://oyc.yale.edu/physics/phys-200/lecture-20) (in English)

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☐ **I finished this unit**

Bernoulli, Heads and the Energy Equation

Streamlines and flow visualization, continuity Q = AV, Bernoulli's equation and its limits, pressure, velocity and elevation heads, the energy grade line, Torricelli's law, the steady-flow energy equation with pump, turbine and loss heads, and hydropower output.

Learning objectives

By the end of this unit you can:

- Apply continuity and Bernoulli along a streamline.
- Work in heads: pressure, velocity and elevation.
- Use the energy equation with pump, turbine and loss heads.
- Compute hydropower and pumping power.

Key concepts

- For incompressible steady flow, volume flow Q = AV is the same at every section of a pipe (continuity).
- Bernoulli along a streamline (steady, inviscid, incompressible): p/(ρg) + V²/(2g) + z = constant. Each term is a head in metres.
- Torricelli: a jet leaving a tank at depth h has speed V = √(2gh).
- Real systems add pumps and turbines and lose energy to friction: $p_1/(\rho g) + V_1^2/(2g) + z_1 + h_{pump} = p_2/(\rho g) + V_2^2/(2g) + z_2 + h_{turbine} + h_L$.
- Power from a head: P = ρgQH. Turbines deliver P = ηρgQH; pumps need P = ρgQH/η. DOE notes that hydropower output depends on flow and head.

Key equations

$$Q = A_1 V_1 = A_2 V_2$$

`Q=A_1V_1=A_2V_2`

Continuity

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{const}$$

`(p)/(ρ g)+(V^2)/(2g)+z=const`

Bernoulli in heads

$$V = \sqrt{2gh}$$

`V=2gh`

Torricelli

$$P = \eta \rho g Q H$$

P=ρ gQH

Hydropower

Worked examples, step by step

Example 3.1 Find the jet speed from a hole 5 m below a tank's surface.

1. V = √(2 × 9.81 × 5).

Answer: V = 9.90 m/s.

Example 3.2 A hydropower plant has Q = 50 m³/s, head H = 100 m and overall efficiency 0.9 (sample values; ρ = 1000 kg/m³). Find the power.

1. P = 0.9 × 1000 × 9.81 × 50 × 100.

Answer: P = 44.1 MW, a "large" plant by DOE's 30 MW definition.

Example 3.3 A pump lifts 0.05 m³/s of water 20 m with 5 m of pipe losses at 75% efficiency. Find the power input.

1. h_pump = 20 + 5 = 25 m.
2. P = 1000 × 9.81 × 0.05 × 25/0.75.

Answer: P = 16.4 kW.

Practice problems

Work each on paper first, then check.

1. A hose carries 0.4 L/s through a 20 mm bore into a 10 mm nozzle. Find both speeds.

   1.27 m/s and 5.09 m/s.

2. Convert 200 kPa to metres of water head.

   200,000/(1000 × 9.81) = 20.4 m.

3. Double the head of a hydropower plant at the same Q. Power?

   Doubles.

4. Why does a pitot tube read higher than a static tap?

   It adds the velocity head.

Common misconceptions

Myth: Bernoulli applies across a pump.
Actually: Use the energy equation with h_pump.

Myth: Faster flow means higher pressure.
Actually: Along a streamline, faster means lower static pressure.

Myth: Head is a pressure.
Actually: It is energy per unit weight, in metres.

Flashcards

Continuity

$Q = AV$.

Bernoulli

$p/\rho g + V^2/2g + z = \text{const.}$

Torricelli

$V = \sqrt{(2gh)}$.

Energy equation

Adds h_pump, h_turbine, h_L.

Hydropower

$P = \eta\rho gQH$.

Pump power

$\rho gQH/\eta$.

Self-check quiz

1. Velocity head is...

$V^2/(2g)$

$p/\rho g$

z

ρV^2

$V^2/(2g)$. Kinetic energy per unit weight.

2. Halving pipe area doubles…

pressure

velocity

flow rate

head loss

velocity. Continuity.

3. Hydropower is proportional to…

Q only

H only

Q × H

Q/H

Q × H. $P = \eta\rho gQH$.

4. Torricelli’s jet speed is…

gh

√(2gh)

2gh

√(gh)

√(2gh). Bernoulli tank-to-jet.

Try this quiz again

How this helps save the planet

DOE explains that hydropower is powered by the kinetic energy of flowing water, and that the energy available depends on both the volume of flow and the change in elevation, the head: the greater the flow and the higher the head, the more electricity can be generated. Water flows through a penstock and spins a turbine connected to a generator.

DOE classes plants by size: large hydropower above 30 MW, small hydropower from 100 kW to 10 MW, and micro hydropower up to 100 kW, enough for a single home, farm or village. P = ηρgQH lets you size any of them from a flow measurement and a survey of the drop.

On Saving Planets: Solutions · Thermodynamics

Sources: U.S. DOE: How hydropower works (https://www.energy.gov/eere/water/how-hydropower-works) · U.S. DOE: Types of hydropower plants (https://www.energy.gov/eere/water/types-hydropower-plants)

Free learning resources

- OpenStax UP1, 14.5 Fluid dynamics (https://openstax.org/books/university-physics-volume-1/pages/14-5-fluid-dynamics): Free textbook section.
- OpenStax UP1, 14.6 Bernoulli's equation (https://openstax.org/books/university-physics-volume-1/pages/14-6-bernoullis-equation): Free textbook section.
- PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow): Bernoulli interactively.
- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Energy equation notes.

Watch the lecture

Fluid Dynamics and Statics and Bernoulli's Equation (https://oyc.yale.edu/physics/phys-200/lecture-20) (in English)

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☐ **I finished this unit**

Integral Momentum: Jets, Vanes, Bends and Turbines

Reynolds transport theorem, the linear momentum equation for control volumes, forces of jets on plates and vanes, forces on pipe bends and nozzles, moving control volumes, impulse turbines (Pelton wheels), and the angular momentum equation for turbomachines.

Learning objectives

By the end of this unit you can:

- State the Reynolds transport theorem.
- Apply the momentum equation to jets and vanes.
- Analyze a moving vane and a Pelton wheel.
- Use angular momentum for turbine and pump torque.

Key concepts

- Reynolds transport theorem: the rate of change of a property of a fluid system = rate of change inside a control volume + net outflow across its surface.
- Steady linear momentum: ΣF = Σṁ_out V_out − Σṁ_in V_in (vectors). Include pressure forces on cut sections and the support force.
- A jet of speed V and flow Q hitting a flat plate head-on exerts F = ρQV. A vane turning the jet through angle θ exerts F = ρQV(1 − cos θ) along the jet.
- For a vane moving at speed U, use the relative speed (V − U). A Pelton wheel's buckets turn the jet nearly 180°; power F·U is largest at U = V/2.
- Angular momentum: torque = ṁ(r_2V_θ2 − r_1V_θ1) (Euler turbomachine equation), the basis of pump and turbine design.

Key equations

$$\sum \mathbf{F} = \sum \dot{m}\mathbf{V}_{out} - \sum \dot{m}\mathbf{V}_{in}$$

```
F= m V_out- m V_in
```

Steady momentum

$$F = \rho Q V(1 - \cos\theta)$$

```
F=ρ QV(1-θ)
```

Force of a turned jet

$$F = \rho Q(V - U)(1 - \cos\beta)$$

```
F=ρ Q(V-U)(1-β)
```

Moving vane

$$T = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

```
T= m(r_2V_θ2-r_1V_θ1)
```

Euler turbomachine equation

Worked examples, step by step

Example 4.1 A 50 mm water jet at 20 m/s hits a flat plate head-on. Find the force.

1. $A = \pi \times 0.05^2/4 = 1.963 \times 10^{-3}$ m².
2. $F = \rho A V^2 = 1000 \times 1.963 \times 10^{-3} \times 400$.

Answer: F = 785 N.

Example 4.2 The same jet is turned 120° by a fixed vane. Find the force along the jet.

1. $F = \rho A V^2(1 - \cos 120°) = 785 \times 1.5$.

Answer: F = 1178 N.

Example 4.3 A Pelton bucket moving at 15 m/s receives a 0.01 m³/s jet at 30 m/s and turns it 165°. Find the force and power.

1. $F = 1000 \times 0.01 \times (30 - 15)(1 - \cos 165°) = 294.9$ N.
2. $P = F \times U$.

Answer: F = 294.9 N; P = 4.42 kW (the jet carries $\tfrac{1}{2}\rho Q V^2 = 4.5$ kW).

Practice problems

Work each on paper first, then check.

1. At what bucket speed does a Pelton wheel give the most power?

   Half the jet speed.

2. Double jet speed at the same area. Force on a plate?

4 times.

3. Name the forces in a pipe-bend momentum balance.

   Inlet and outlet pressure forces, weight, and the support force.

4. Why are Pelton buckets split in the middle?

   To turn the jet nearly 180° without it hitting the next bucket.

Common misconceptions

Myth: The plate force depends on the plate size.
Actually: For a jet fully deflected, only on ρQV.

Myth: A moving vane sees the full jet speed.
Actually: It sees V − U.

Myth: Momentum is a scalar.
Actually: It is a vector; resolve components.

Flashcards

RTT

System rate = CV rate + net outflow.

Momentum

$\Sigma F = \Sigma \dot{m}\, V_{out} - \Sigma \dot{m}\, V_{in}$.

Plate

ρQV.

Vane

ρQV(1 − cos θ).

Pelton

Best at U = V/2.

Euler turbine

$T = \dot{m}\, \Delta(rV_\theta)$.

Self-check quiz

1. Force of a jet on a flat plate is...

ρQV

ρgQH

$\frac{1}{2}\rho V^2$

QV

ρQV. Momentum flux.

2. Turning a jet 180° gives a force of...

ρQV

$2\rho QV$

0

$\rho QV/2$

$2\rho QV$. $1 - \cos 180° = 2$.

3. Pelton wheel best U/V is...

0.25

0.5

1

2

0.5. Maximizes $(V - U)U$.

4. The Euler equation gives turbomachine...

torque

drag

buoyancy

viscosity

torque. Angular momentum.

Try this quiz again

How this helps save the planet

DOE explains that pumped storage hydropower works like a giant battery: when demand is low it pumps water to an upper reservoir, and when demand is high the water flows back down through a turbine. DOE reports that pumped storage accounts for 88% of all utility-scale energy storage in the United States, at 43 plants, with room to more than double capacity.

Each pump-turbine is designed with the momentum and angular-momentum equations in this unit: the runner's blade angles set how much torque the water gives up or receives. Storing solar and wind power this way depends on getting those blades right.

On Saving Planets: Solutions · Dynamics

Sources: U.S. DOE: Pumped storage hydropower (https://www.energy.gov/eere/water/pumped-storage-hydropower)

Free learning resources

- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Control-volume momentum.
- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Momentum notes.
- NASA Glenn: Conservation of momentum (https://www.grc.nasa.gov/www/k-12/airplane/conmo.html): Free NASA page.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Deeper treatment.

☐ I finished this unit

Differential Analysis: Continuity, Navier–Stokes and Exact Flows

Fluid kinematics (velocity field, acceleration, vorticity, strain), the differential continuity equation, the stream function, Euler and Navier–Stokes equations, boundary conditions, and exact laminar solutions: Couette flow, plane Poiseuille flow and Hagen–Poiseuille pipe flow.

Learning objectives

By the end of this unit you can:

- Compute the material acceleration of a velocity field.
- Check continuity and find a stream function.
- State the Navier–Stokes equations and their boundary conditions.
- Solve Couette and Hagen–Poiseuille flows.

Key concepts

- In the Eulerian view, acceleration has a local part ∂V/∂t and a convective part (V·∇)V; the sum is the material derivative DV/Dt.
- Incompressible continuity: ∇·V = ∂u/∂x + ∂v/∂y + ∂w/∂z = 0. In 2D, a stream function ψ with u = ∂ψ/∂y, v = −∂ψ/∂x satisfies it automatically.
- Navier–Stokes (incompressible, Newtonian): ρ DV/Dt = −∇p + ρg + μ∇²V. With no-slip at walls, it governs nearly every flow in this course; only a few cases solve exactly.
- Couette flow between a fixed and a moving plate: linear velocity profile, shear τ = μU/h.
- Hagen–Poiseuille flow in a pipe: parabolic profile, u_max = 2V_avg, and Q = πR⁴Δp/(8μL). It holds while the flow stays laminar (Re below about 2300).

Key equations

$$\frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V}$$

```
(D V)/(Dt)=( V)/( t)+( V·) V
```

Material acceleration

$$\nabla \cdot \mathbf{V} = 0$$

Incompressible continuity

$$\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \rho\mathbf{g} + \mu\nabla^2\mathbf{V}$$

ρ(D V)/(Dt)=- p+ρ g+μ^2 V

Navier–Stokes

$$Q = \frac{\pi R^4 \Delta p}{8 \mu L}$$

Q=(π R^4Δ p)/(8μ L)

Hagen–Poiseuille

Worked examples, step by step

Example 5.1 Water flows in a 4 mm diameter tube with Δp/L = 100 Pa/m. Find Q, V_avg, u_max and Re.

1. Q = π × 0.002⁴ × 100/(8 × 1.002 × 10⁻³) = 6.27 × 10⁻⁷ m³/s.
2. V = Q/(π × 0.002²) = 0.0499 m/s; u_max = 2V.
3. Re = 998 × 0.0499 × 0.004/1.002 × 10⁻³.

Answer: Q = 6.27 × 10⁻⁷ m³/s, V = 0.0499 m/s, u_max = 0.0998 m/s, Re = 199 (laminar).

Example 5.2 A plate moves at 1 m/s over a 2 mm water film. Find the shear stress.

1. τ = μU/h = 1.002 × 10⁻³ × 1/0.002.

Answer: τ = 0.501 Pa.

Example 5.3 Is V = (2x, −2y) incompressible, and what is its acceleration at (1, 1)?

1. ∂u/∂x + ∂v/∂y = 2 − 2 = 0.
2. a_x = u ∂u/∂x = 2x × 2 = 4x; a_y = v ∂v/∂y = (−2y)(−2) = 4y.

Answer: Yes; a = (4, 4) m/s².

Practice problems

Work each on paper first, then check.

1. Halve the pipe radius at the same Δp/L. Q?

   1/16 as much.

2. Find ψ for $u = 1$, $v = 0$.

   $\psi = y$.

3. Where is shear largest in Poiseuille flow?

   At the wall.

4. Name the boundary condition at a solid wall.

   No slip: fluid velocity equals wall velocity.

Common misconceptions

Myth: Steady flow has zero acceleration.
Actually: Convective acceleration can be nonzero.

Myth: Navier–Stokes has many exact solutions.
Actually: Only a few simple geometries.

Myth: Poiseuille's law works for any pipe flow.
Actually: Only laminar flow.

Flashcards

Material derivative

$\partial/\partial t + V\cdot\nabla$.

Continuity

$\nabla\cdot V = 0$.

Stream function

$u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$.

Navier–Stokes

$\rho DV/Dt = -\nabla p + \rho g + \mu\nabla^2 V$.

Couette

$\tau = \mu U/h$.

Poiseuille

$Q = \pi R^4 \Delta p/8\mu L$.

Self-check quiz

1. Hagen–Poiseuille Q scales with radius as...

R

R^2

R^3

R^4

R^4. $\pi R^4 \Delta p/(8\mu L)$.

2. In Couette flow the profile is...

parabolic

linear

uniform

logarithmic

linear. No pressure gradient.

3. The convective acceleration is...

$\partial V/\partial t$

$(V\cdot\nabla)V$

∇p

$\mu\nabla^2 V$

$(V\cdot\nabla)V$. Spatial change.

4. u_max/V_avg in laminar pipe flow is...

1

1.5

2

4

2. Parabolic.

Try this quiz again

How this helps save the planet

EPA reports that household leaks can waste nearly 1 trillion gallons of water a year nationwide, that the average household's leaks can waste more than 9,300 gallons a year, and that 9% of homes have leaks wasting 50 gallons or more per day.

Small leaks through worn seals and fittings are often slow, laminar flows through tiny gaps, where the R^4 law above shows why a small widening of a crack makes a big difference. EPA says fixing easily corrected household leaks can save about 10% on water bills.

On Saving Planets: Differential Equations · Calculus III

Sources: U.S. EPA WaterSense: Fix a Leak Week (https://www.epa.gov/watersense/fix-leak-week)

Free learning resources

- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Navier–Stokes lectures.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Exact solutions.
- OpenStax UP1, 14.7 Viscosity and turbulence (https://openstax.org/books/university-physics-volume-1/pages/14-7-viscosity-and-turbulence): Poiseuille's law.
- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Kinematics notes.

☐ **I finished this unit**

Dimensional Analysis, Similitude and Model Testing

Dimensional homogeneity, the Buckingham Pi theorem, common dimensionless groups (Reynolds, Froude, Mach, Weber, Euler), geometric, kinematic and dynamic similarity, model laws and scale effects, and nondimensionalizing the governing equations.

Learning objectives

By the end of this unit you can:

- Find dimensionless groups with the Pi theorem.
- Name what Re, Fr, Ma and We compare.
- Scale model test results to full size with Froude or Reynolds similarity.
- Explain why full similarity is often impossible.

Key concepts

- Every valid equation is dimensionally homogeneous. The Buckingham Pi theorem says a problem with n variables and k basic dimensions has n − k independent dimensionless groups.
- Common groups: Re = ρVL/μ (inertia/viscous), Fr = V/√(gL) (inertia/gravity), Ma = V/a (compressibility), We = ρV²L/σ (inertia/surface tension), Eu = Δp/(ρV²) (pressure).
- Dynamic similarity: if the model and prototype have the same geometry and the same governing dimensionless groups, their dimensionless results (C_D, C_p, efficiency) match.
- Free-surface flows (ships, spillways, waves) use Froude scaling: V_m = V_p√(L_m/L_p); forces scale with the length scale cubed and power with it to the 3.5 power.
- Matching Re and Fr together usually needs an impossible fluid, so engineers match the dominant group and correct for the other (scale effects).

Key equations

$$\Pi \text{ groups} = n - k$$

`groups=n-k`

Buckingham Pi theorem

$$Re = \frac{\rho V L}{\mu},\ Fr = \frac{V}{\sqrt{gL}}$$

`Re=(ρ VL)/(μ),\ Fr=`

Reynolds and Froude numbers

$$V_m = V_p \sqrt{L_m / L_p}$$

```
V_m=V_p_m/L_p
```

Froude scaling

$$\frac{F_p}{F_m} = \left(\frac{L_p}{L_m}\right)^3$$

```
(F_p)/(F_m)=((L_p)/(L_m))^3
```

Force scale (Froude, same fluid)

Worked examples, step by step

Example 6.1 Drag F on a sphere depends on ρ, V, D and μ. How many Pi groups, and what are they?

1. n = 5 variables, k = 3 dimensions (M, L, T).
2. Groups: $F/(\rho V^2 D^2)$ and $\rho VD/\mu$.

Answer: 2 groups: C_D = f(Re).

Example 6.2 A 1:20 wave-energy model is tested with Froude scaling. The full-size wave speed is 10 m/s. Find the model speed.

1. V_m = 10/√20.

Answer: V_m = 2.24 m/s.

Example 6.3 For that model, how do forces and power scale?

1. F_p/F_m = 20^3.
2. P_p/P_m = 20^3.5.

Answer: Forces × 8000; power × 35,777.

Practice problems

Work each on paper first, then check.

1. Which group matters most for a ship’s waves?

   Froude number.

2. Which for flow in a small pipe?

Reynolds number.

3. A 1:10 model in the same fluid must match Re. Model speed vs full size?

   10 times faster.

4. What does We compare?

   Inertia to surface tension.

Common misconceptions

Myth: Model tests use the same speed as full size.
Actually: They use scaled speeds.

Myth: Dimensionless means unimportant.
Actually: Dimensionless groups control the physics.

Myth: Any model gives exact full-scale results.
Actually: Scale effects remain when not all groups match.

Flashcards

Pi theorem

n − k groups.

Reynolds

Inertia/viscous.

Froude

Inertia/gravity.

Weber

Inertia/surface tension.

Similarity

Same groups, same coefficients.

Froude scaling

$V \propto \sqrt{L}$.

Self-check quiz

1. Froude number compares inertia to...

viscosity

gravity

surface tension

compressibility

gravity. V/√(gL).

2. With 6 variables and 3 dimensions there are...

2

3

6

9

3. n − k.

3. Froude-scaled forces (same fluid) scale with length to the...

1st

2nd

3rd

3.5th

3rd. L^3.

4. C_D of a smooth sphere depends mainly on...

Fr

Re

We

Ma

Re. Viscous effects.

Try this quiz again

How this helps save the planet

DOE describes marine energy as power harnessed from waves, tides, and river and ocean currents, and from ocean temperature differences. It says the total U.S. marine energy resource is equivalent to about 57% of all U.S. power generation in 2019, and that these resources are highly predictable.

DOE notes that researchers are testing and deploying new technologies to harness that resource. Wave-energy converters and tidal turbines are developed with scaled models in wave tanks and flumes long before full-size devices go to sea, using the Froude and Reynolds scaling in this unit.

On Saving Planets: Oceans · Solutions

Sources: U.S. DOE: Marine energy basics (https://www.energy.gov/eere/water/marine-energy-basics)

Free learning resources

- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Dimensional analysis lectures.
- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Similitude notes.
- NASA Glenn: Similarity parameters (https://www.grc.nasa.gov/www/k-12/airplane/airsim.html): Free NASA page.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Scaling.

☐ **I finished this unit**

Pipe Flow: Friction, the Moody Chart and Minor Losses

Laminar and turbulent pipe flow, entrance length, the Darcy–Weisbach equation, laminar friction factor 64/Re, the Moody chart and Colebrook and Haaland correlations, minor losses, noncircular ducts and hydraulic diameter, and series, parallel and network piping.

Learning objectives

By the end of this unit you can:

- Decide whether pipe flow is laminar or turbulent.
- Compute friction losses with Darcy–Weisbach and the Haaland formula.
- Add minor losses from fittings and valves.
- Solve simple series and parallel pipe problems.

Key concepts

- In pipes, flow is laminar below Re ≈ 2300, turbulent above about 4000. Turbulent flow mixes momentum strongly and has a flatter profile.
- Darcy–Weisbach: h_L = f(L/D)(V²/2g). For laminar flow f = 64/Re.
- For turbulent flow f depends on Re and relative roughness ε/D (Moody chart). The Haaland formula is explicit: 1/√f = −1.8 $\log_{10}$[(ε/D/3.7)^1.11 + 6.9/Re].
- Fittings, valves, bends and entrances add minor losses h_m = K V²/(2g). In long pipelines friction dominates; in household plumbing fittings matter.
- Noncircular ducts use the hydraulic diameter D_h = 4A/P. Pipes in series share Q and add losses; in parallel they share head loss and add flows.

Key equations

$$Re = \frac{\rho V D}{\mu}$$

`Re=(ρ VD)/(μ)`

Pipe Reynolds number

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

`h_L=fL/D(V^2)/(2g)`

Darcy–Weisbach

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re}\right]$$

`(1)/( f)=-1.8_10[((/D)/(3.7))^1.11+(6.9)/(Re)]`

Haaland

$$h_m = K\frac{V^2}{2g}$$

`h_m=K(V^2)/(2g)`

Minor loss

Worked examples, step by step

Example 7.1 Water flows at 2 m/s in a 50 mm commercial steel pipe (ε = 0.045 mm). Find Re and f.

1. Re = 998 × 2 × 0.05/1.002 × 10⁻³ = 99,600.
2. ε/D = 0.0009; Haaland gives f.

Answer: Re = 9.96 × 10⁴ (turbulent); f = 0.0216.

Example 7.2 Find the friction head loss over 100 m of that pipe.

1. h_L = 0.0216 × (100/0.05) × 2²/(2 × 9.81).

Answer: h_L = 8.82 m.

Example 7.3 Find the laminar friction factor at Re = 1500, and the loss of a sharp entrance (K = 0.5) at 2 m/s.

1. f = 64/1500.
2. h_m = 0.5 × 4/19.62.

Answer: f = 0.0427; h_m = 0.102 m.

Practice problems

Work each on paper first, then check.

1. Find D_h for a 0.2 m × 0.1 m duct.

   $4 \times 0.02/0.6 = 0.133$ m.

2. Double V in fully rough turbulent flow. h_L?

   About 4 times.

3. Two identical pipes in parallel carry total Q. Flow in each?

   Q/2, with the same head loss.

4. Is Re = 3000 laminar or turbulent?

   Transitional.

Common misconceptions

Myth: Friction factor is constant.
Actually: It depends on Re and roughness.

Myth: Minor losses are always minor.
Actually: Not in short pipes with many fittings.

Myth: Laminar flow has higher losses.
Actually: At the same Re range it has lower f, but laminar flow happens only at low speeds.

Flashcards

Transition

Re ≈ 2300–4000.

Darcy–Weisbach

$f(L/D)V^2/2g$.

Laminar f

64/Re.

Haaland

Explicit f.

Minor loss

$KV^2/2g$.

Hydraulic diameter

$4A/P$.

Self-check quiz

1. Laminar friction factor is...

64/Re

0.316/Re^0.25

0.02

Re/64

64/Re. Poiseuille.

2. Darcy–Weisbach loss scales with...

V

V^2

V^3

1/V

V^2. At fixed f.

3. Hydraulic diameter is...

4A/P

A/P

P/A

2A/P

4A/P. Definition.

4. Parallel pipes share the same...

flow

head loss

diameter

velocity

head loss. Common end points.

Try this quiz again

How this helps save the planet

EPA notes that a faucet dripping once per second can waste more than 3,000 gallons a year, enough for more than 180 showers, and that a showerhead leaking 10 drips per minute wastes more than 500 gallons a year. An irrigation leak 1/32 of an inch across can waste about 6,300 gallons a month.

Every gallon delivered also has to be pumped through miles of pipe, against the friction losses in this unit. Fixing leaks saves the water and the pumping energy together; 3,000 gallons is about 11,355 L, or 31.1 L every day.

On Saving Planets: Solutions · Mechanics of Materials

Sources: U.S. EPA WaterSense: Fix a Leak Week (https://www.epa.gov/watersense/fix-leak-week)

Free learning resources

- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Pipe-flow notes.
- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Internal flow lectures.
- OpenStax UP1, 14.7 Viscosity and turbulence (https://openstax.org/books/university-physics-volume-1/pages/14-7-viscosity-and-turbulence): Laminar and turbulent flow.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Turbulence.

☐ **I finished this unit**

Pumps, Turbines and Pump Systems

Centrifugal and axial pumps, pump head, power and efficiency, pump performance curves, system curves and the operating point, pumps in series and parallel, affinity laws and variable-speed drives, cavitation and NPSH, and hydraulic turbines (Pelton, Francis, Kaplan).

Learning objectives

By the end of this unit you can:

- Read a pump curve and find the operating point with a system curve.
- Compute pump power and efficiency.
- Apply the affinity laws to speed changes.
- Explain NPSH and how to avoid cavitation.

Key concepts

- A pump adds head h_p to the flow; water power is ρgQh_p and shaft power is ρgQh_p/η.
- The pump curve (head falling as flow rises) meets the system curve (static lift plus losses rising with Q²) at the operating point.
- Affinity laws for the same pump at a new speed N: Q ∝ N, H ∝ N², P ∝ N³. Slowing a pump with a variable-speed drive instead of throttling a valve saves a lot of energy.
- Cavitation starts when inlet pressure falls near vapor pressure. The available net positive suction head (NPSH_A) must exceed what the pump requires (NPSH_R).
- Hydraulic turbines are pumps in reverse: Pelton (impulse) for high head, Francis for medium head, Kaplan (propeller) for low head and large flow; many pumped-storage units are reversible pump-turbines.

Key equations

$$P_{shaft} = \frac{\rho g Q h_p}{\eta}$$

```
P_shaft=(ρ gQh_p)/()
```

Pump power

$$H_{sys} = H_{static} + kQ^2$$

```
H_sys=H_static+kQ^2
```

System curve

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1},\ \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2,\ \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3$$

```
(Q_2)/(Q_1)=(N_2)/(N_1),\ (H_2)/(H_1)=((N_2)/(N_1))^2,\ (P_2)/(P_1)=((N_2)/(N_1))^3
```

Affinity laws

$$NPSH_A > NPSH_R$$

No cavitation

Worked examples, step by step

Example 8.1 A pump curve is H = 40 − 7000Q² and the system curve is H = 10 + 5000Q² (H in m, Q in m³/s). Find the operating point.

1. 40 − 7000Q² = 10 + 5000Q², so Q² = 30/12,000.
2. Q = 0.05 m³/s; H = 10 + 5000 × 0.0025.

Answer: Q = 0.05 m³/s, H = 22.5 m.

Example 8.2 Find the shaft power at that point with η = 0.75 (ρ = 1000 kg/m³).

1. P = 1000 × 9.81 × 0.05 × 22.5/0.75.

Answer: P = 14.7 kW.

Example 8.3 A pump delivering 0.05 m³/s at 30 m head is slowed from 1800 to 1500 rpm. Find the new flow, head and power ratio.

1. Q = 0.05 × 1500/1800.
2. H = 30 × (1500/1800)².
3. P ratio = (1500/1800)³.

Answer: Q = 0.0417 m³/s, H = 20.8 m, power 57.9% of the original.

Practice problems

Work each on paper first, then check.

1. Two identical pumps in series: what adds?

   Heads at the same flow.

2. Two in parallel?

   Flows at the same head.

3. Cut speed by 20%. Power?

   $0.8^3 = 51.2\%$.

4. Which turbine suits a 500 m head?

   Pelton.

Common misconceptions

Myth: Throttling a valve is an efficient way to cut flow.
Actually: It wastes head; slowing the pump is better.

Myth: A bigger pump is always better.
Actually: Oversized pumps run off their best-efficiency point.

Myth: Cavitation only makes noise.
Actually: It erodes impellers and cuts performance.

Flashcards

Pump power

$\rho g Q h / \eta$.

System curve

$H_{static} + kQ^2$.

Operating point

Curves cross.

Affinity

$Q \propto N$, $H \propto N^2$, $P \propto N^3$.

NPSH

Avoid cavitation.

Turbines

Pelton, Francis, Kaplan.

Self-check quiz

1. Pump power scales with speed as...

N

N^2

N^3

$\sqrt{N}$

N^3. Affinity laws.

2. The operating point is where...

pump curve meets system curve

efficiency is zero

Q = 0

H = 0

pump curve meets system curve. Balance.

3. To avoid cavitation...

NPSH_A > NPSH_R

NPSH_A < NPSH_R

raise speed

lower inlet pressure

NPSH_A > NPSH_R. Margin.

4. Low head, high flow suits a...

Pelton

Francis

Kaplan

piston pump

Kaplan. Propeller turbine.

Try this quiz again

How this helps save the planet

DOE's industrial efficiency office states that dramatic energy and cost savings can be achieved in pump systems by applying best energy-management practices and buying energy-efficient equipment, and it offers the free Pumping System Assessment Tool (PSAT) and MEASUR software to find them.

The affinity laws show where much of that saving comes from: a pump slowed by 20% with a variable-speed drive needs only about half the power. DOE's pumped storage page adds that pump-turbines move water uphill when power is cheap and generate when it is needed.

On Saving Planets: Solutions · Circuits

Sources: U.S. DOE: Pump systems (https://www.energy.gov/eere/iedo/pump-systems) · U.S. DOE: Pumped storage hydropower (https://www.energy.gov/eere/water/pumped-storage-hydropower)

Free learning resources

- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Pump notes.
- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Turbomachinery lectures.
- U.S. DOE: Pump systems (https://www.energy.gov/eere/iedo/pump-systems): Free DOE pump tools.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Deeper treatment.

☐ **I finished this unit**

External Flow: Boundary Layers, Drag and Lift

Flow past bodies, boundary layers on flat plates, friction and pressure drag, drag coefficients of spheres, cylinders and streamlined bodies, the drag crisis, Stokes flow and settling, lift, and the power available in a wind or water stream.

Learning objectives

By the end of this unit you can:

- Compute drag from a drag coefficient.
- Explain the roles of friction and pressure drag.
- Use Stokes' law for small particles.
- Find the power in a wind stream and the Betz limit.

Key concepts

- Drag $F_D = C_D \tfrac{1}{2}\rho V^2 A$ and lift $F_L = C_L \tfrac{1}{2}\rho V^2 A$, with A a reference area. C_D depends on shape and Re.
- Streamlined bodies have mostly friction drag; bluff bodies (spheres, cylinders, trucks) have mostly pressure drag from a separated wake.
- For a smooth sphere $C_D \approx 0.47$ for $10^3 < Re < 2 \times 10^5$; around $Re \approx 3 \times 10^5$ the boundary layer turns turbulent, separates later, and C_D drops sharply (the drag crisis).
- At very low Re ($Re < 1$), Stokes drag $F = 3\pi\mu V D$; a small particle settles at $V = (\rho_p - \rho)gD^2/(18\mu)$.
- A stream of area A carries power $\tfrac{1}{2}\rho A V^3$. No turbine can capture more than 16/27 ≈ 59.3% of it (the Betz limit).

Key equations

$$F_D = C_D \tfrac{1}{2}\rho V^2 A$$

`F_D=C_D12ρ V^2A`

Drag

$$F = 3\pi\mu V D$$

`F=3πμ VD`

Stokes drag

$$V_s = \frac{(\rho_p - \rho)gD^2}{18\mu}$$

`V_s=((ρ_p-ρ)gD^2)/(18μ)`

Settling speed

$$P = \tfrac{1}{2}\rho A V^3, \;\; C_{P,max} = \frac{16}{27}$$

P=12ρ AV^3,\ \ C_P,max=16/27

Wind power and Betz limit

Worked examples, step by step

Example 9.1 A 0.22 m ball moves at 20 m/s in air (C_D = 0.47, sample value). Find Re and drag.

1. Re = 1.225 × 20 × 0.22/1.789 × 10^{-5} = 3.01 × 10^{5}.
2. F = 0.47 × ½ × 1.225 × 400 × π × 0.11^{2}.

Answer: Re = 3.01 × 10^{5} (near the drag crisis); F = 4.38 N.

Example 9.2 Find the settling speed of a 50 μm sand grain (ρ_p = 2500 kg/m^{3}) in water.

1. V = (2500 − 998) × 9.81 × $(50 \times 10^{-6})^{2}$/(18 × 1.002 × 10^{-3}).
2. Check Re = 998 × 0.00204 × 50 × 10^{-6}/1.002 × 10^{-3} = 0.102.

Answer: V = 2.04 mm/s; Re < 1, so Stokes' law holds.

Example 9.3 Find the wind power through a 100 m diameter rotor at 10 m/s, and the Betz maximum.

1. A = π × 50^{2} = 7854 m^{2}.
2. P = ½ × 1.225 × 7854 × 1000.
3. P_max = P × 16/27.

Answer: P = 4.81 MW; Betz maximum 2.85 MW.

Practice problems

Work each on paper first, then check.

1. Double wind speed. Power in the wind?

   8 times.

2. Why do trucks use fairings?

   To cut pressure drag from the wake.

3. Halve a particle's diameter. Stokes settling speed?

   1/4.

4. What is the drag crisis?

   The sudden fall in C_D when the boundary layer turns turbulent.

Common misconceptions

Myth: Drag is always proportional to V^2.
Actually: At very low Re it is proportional to V.

Myth: A turbine can extract all the wind's power.
Actually: Betz limits it to 59.3%.

Myth: Rough surfaces always increase drag.
Actually: On spheres they can trigger the drag crisis earlier.

Flashcards

Drag

C_D $\frac{1}{2}\rho V^2 A$.

Pressure drag

Wake behind bluff bodies.

Drag crisis

C_D drops near Re 3×10^5.

Stokes

$F = 3\pi\mu VD$.

Wind power

$\frac{1}{2}\rho A V^3$.

Betz

59.3%.

Self-check quiz

1. Wind power scales with V as...

V

V^2

V^3

V^4

V^3. $\frac{1}{2}\rho A V^3$.

2. The Betz limit is about…

30%

45%

59%

100%

59%. 16/27.

3. Stokes drag applies when Re is…

< 1

$\approx 10^3$

$> 10^5$

any

< 1. Creeping flow.

4. Bluff-body drag is mostly…

friction

pressure

induced

wave

pressure. Separated wake.

Try this quiz again

How this helps save the planet

DOE explains that wind turbine blades work like an airplane wing or helicopter rotor: the pressure difference across a blade creates lift and drag, and because the lift is stronger than the drag the rotor spins a generator. DOE notes land-based turbines range from 100 kilowatts to several megawatts.

The $\frac{1}{2}\rho AV^3$ law explains why turbines keep getting taller and wider: doubling the rotor diameter quadruples the area, and higher hub heights reach faster winds, with power rising as the cube of speed. The Betz limit sets the target every blade designer chases.

On Saving Planets: Aerodynamics I · Solutions

Sources: U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work)

Free learning resources

- NASA Glenn: Drag of a sphere (https://www.grc.nasa.gov/www/k-12/airplane/dragsphere.html): Free NASA page.
- NASA Glenn: Boundary layer (https://www.grc.nasa.gov/www/k-12/airplane/boundlay.html): Free NASA page.
- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): External flow lectures.
- OpenStax UP1, 14.7 Viscosity and turbulence (https://openstax.org/books/university-physics-volume-1/pages/14-7-viscosity-and-turbulence): Free textbook section.

☐ **I finished this unit**

UNIT 10 · WEEKS 14–15

Open-Channel Flow: Rivers, Canals and Hydraulic Jumps

Open channels, hydraulic radius, the Froude number and subcritical, critical and supercritical flow, specific energy and critical depth, uniform flow and Manning's equation, hydraulic jumps and energy dissipation, and weirs for flow measurement.

Learning objectives

By the end of this unit you can:

- Classify channel flow with the Froude number.
- Find critical depth and specific energy.
- Use Manning's equation for uniform flow.
- Compute the depth after and energy lost in a hydraulic jump.

Key concepts

- In open channels gravity drives the flow and the free surface is at atmospheric pressure. The hydraulic radius is R_h = A/P (wetted perimeter).
- Froude number Fr = V/√(gy): Fr < 1 subcritical (slow, deep), Fr = 1 critical, Fr > 1 supercritical (fast, shallow). Surface waves can travel upstream only in subcritical flow.
- Specific energy E = y + V²/(2g). For flow per unit width q, the minimum E occurs at critical depth y_c = (q²/g)^(1/3).
- Uniform flow (Manning, SI): V = (1/n) R_h^(2/3) S^(1/2), with roughness n (about 0.013 for finished concrete) and bed slope S.
- A hydraulic jump turns supercritical flow subcritical: $y_2/y_1 = \tfrac{1}{2}(\sqrt{(1 + 8Fr_1^2)} - 1)$, and dissipates head h_L = $(y_2 - y_1)^3/(4y_1y_2)$. Spillways use jumps to protect riverbeds below dams.

Key equations

$$Fr = \frac{V}{\sqrt{gy}}$$

Froude number

$$E = y + \frac{V^2}{2g}, \;\; y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

`E=y+(V^2)/(2g),\ \ y_c=((q^2)/(g))^1/3`

Specific energy and critical depth

$$V = \frac{1}{n} R_h^{2/3} S^{1/2}$$

`V=1n R_h^2/3S^1/2`

Manning's equation (SI)

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8Fr_1^2} - 1\right)$$

`(y_2)/(y_1)=12(1+8Fr_1^2-1)`

Hydraulic jump

Worked examples, step by step

Example 10.1 A 2 m wide concrete channel (n = 0.013) flows 1 m deep on a slope of 0.001. Find V and Q.

1. A = 2 m², P = 4 m, R_h = 0.5 m.
2. V = (1/0.013) × 0.5^(2/3) × 0.001^0.5.
3. Q = VA.

Answer: V = 1.53 m/s; Q = 3.06 m³/s.

Example 10.2 Water flows 0.5 m deep at 6 m/s below a spillway. Find Fr, the depth after a hydraulic jump and the head lost.

1. Fr = 6/√(9.81 × 0.5) = 2.71.
2. y_2 = 0.25 × (√(1 + 8 × 2.71²) − 1).
3. h_L = $(y_2 - y_1)^3/(4y_1y_2)$.

Answer: Fr = 2.71; y_2 = 1.68 m; h_L = 0.491 m.

Example 10.3 Find the critical depth for q = 2 m²/s.

1. y_c = (4/9.81)^(1/3).

Answer: y_c = 0.742 m.

Practice problems

Work each on paper first, then check.

1. Is V = 3 m/s at 1 m depth sub- or supercritical?

Fr = 0.958, subcritical.

2. Find E for y = 1 m, V = 3 m/s.

   1 + 9/19.62 = 1.459 m.

3. What depth would carry q = 2 m²/s at V = 1.53 m/s?

   2/1.532 = 1.31 m.

4. Why build a stilling basin below a dam?

   To force a jump that dissipates energy.

Common misconceptions

Myth: Fast flow is always supercritical.
Actually: Compare V with √(gy).

Myth: A jump gains energy.
Actually: It loses energy.

Myth: Manning's n is the same everywhere.
Actually: It depends on the surface.

Flashcards

Froude

V/√(gy).

Hydraulic radius

A/P.

Specific energy

y + V²/2g.

Critical depth

(q²/g)^(1/3).

Manning

(1/n)R^(2/3)S^(1/2).

Jump

$y_2/y_1 = ½(\sqrt{(1 + 8Fr^2)} - 1)$.

Self-check quiz

1. Supercritical flow has...

Fr < 1

Fr = 1

Fr > 1

Fr = 0

Fr > 1. Fast, shallow.

2. Hydraulic radius is...

A/P

P/A

4A/P

y

A/P. Definition.

3. Specific energy is minimum at...

critical depth

zero depth

normal depth

any depth

critical depth. dE/dy = 0.

4. A hydraulic jump...

dissipates energy

adds energy

is reversible

speeds up flow

dissipates energy. Turbulent mixing.

Try this quiz again

How this helps save the planet

DOE describes a diversion, or run-of-river, hydropower facility as one that channels part of a river through a canal and/or a penstock, using the natural decline of the riverbed to produce energy, sometimes without a dam. Designing that canal is an open-channel problem: Manning's equation sizes it for the flow, and the drop sets the head.

DOE adds that impoundment releases can serve fish passage and water-quality needs. Engineers use the Froude number and hydraulic jumps to shape spillways and channels that protect riverbeds and keep flows safe for wildlife.

On Saving Planets: Oceans · Earth Observation

Sources: U.S. DOE: Types of hydropower plants (https://www.energy.gov/eere/water/types-hydropower-plants) · U.S. DOE: How hydropower works (https://www.energy.gov/eere/water/how-hydropower-works)

Free learning resources

- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Open-channel notes.
- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Free-surface flows.
- NOAA: Currents tutorial (https://oceanservice.noaa.gov/education/tutorial_currents/05conveyor1.html): Free NOAA tutorial.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Deeper treatment.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Fluid Properties: Density, Viscosity and Surface Tension
   4 problems
2. Fluid Statics, Manometers and Buoyancy
   4 problems
3. Bernoulli, Heads and the Energy Equation
   4 problems

4. Integral Momentum: Jets, Vanes, Bends and Turbines
   4 problems
5. Differential Analysis: Continuity, Navier–Stokes and Exact Flows
   4 problems
6. Dimensional Analysis, Similitude and Model Testing
   4 problems
7. Pipe Flow: Friction, the Moody Chart and Minor Losses
   4 problems
8. Pumps, Turbines and Pump Systems
   4 problems
9. External Flow: Boundary Layers, Drag and Lift
   4 problems
10. Open-Channel Flow: Rivers, Canals and Hydraulic Jumps
    4 problems

The lab (Fluid mechanics lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation or with kitchen and garden items. Work over a sink or outdoors, wipe up spills at once to avoid slips, use cold tap water only, and keep water away from electrical outlets and devices.

Safety reference: OSHA: Walking-working surfaces (slip, trip and fall hazards) (https://www.osha.gov/walking-working-surfaces)

Lab A: Density and buoyancy by weighing

WEEK 3

Goal: Find an object's volume and density from its weight in air and its apparent weight in water, using Archimedes' principle.

$$V = \frac{m_{air} - m_{water}}{\rho_w}, \quad \rho = \frac{m_{air}}{V}$$

V=_air-m_waterρ_w,\ \ ρ=_airV

Archimedes' principle

Procedure

1. Explore PhET Buoyancy: change object density and fluid density and watch the forces.
2. Home: weigh a small dense object (a stone or metal spoon) on a kitchen scale.
3. Hang it by a thread fully under water in a jug resting on the scale (tare first), or use a hanging luggage scale; record the apparent mass.
4. Compute volume and density.

Data table

Quantity	Value
Mass in air	200 g
Apparent mass in water	120 g
Volume	80 cm³
Density	2500 kg/m³

Sample values (typical of stone or glass).

Sample calculation

1. V = (200 − 120)/1.00 = 80 cm³.
2. ρ = 200/80 = 2.50 g/cm³ = 2500 kg/m³.

> **Safety**
>
> - Wipe up spills at once.

Home or virtual alternative

- This is a home lab.

PhET: Buoyancy (https://phet.colorado.edu/en/simulations/buoyancy) · PhET: Density (https://phet.colorado.edu/en/simulations/density)

Planet link: NOAA explains that density differences from temperature and salinity drive the deep-ocean conveyor belt. NOAA National Ocean Service: Thermohaline circulation (https://oceanservice.noaa.gov/education/tutorial_currents/05conveyor1.html)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Torricelli’s bottle

WEEK 4

Goal: Check Torricelli’s law by measuring how far a water jet travels from a hole in a bottle.

$$V = \sqrt{2gh}, \;\; x = 2\sqrt{hY}$$

V=2gh,\ \ x=2

Jet speed and range from height Y

Procedure

1. Poke a small hole in the side of a plastic bottle near the bottom; tape over it.
2. Fill the bottle and set it on a box over a sink or outdoors, with the hole Y = 0.3 m above the landing surface.
3. Remove the tape; when the water level is h = 0.2 m above the hole, mark where the jet lands.
4. Compare with the prediction; repeat at other depths.

Data table

Quantity	Value
Head h	0.20 m
Jet speed	1.98 m/s
Drop height Y	0.30 m
Predicted range	0.49 m

Predicted values; real jets land a little short because of losses at the hole.

Sample calculation

1. V = √(2 × 9.81 × 0.2) = 1.98 m/s.
2. x = 2√(0.2 × 0.3) = 0.49 m.

> **Safety**
>
> - Do it over a sink or outdoors; wipe up water to avoid slips.
> - Use a skewer carefully, pointing away from your hand.

Home or virtual alternative

- This is a home lab.

PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow) · PhET: Under Pressure (https://phet.colorado.edu/en/simulations/under-pressure)

Planet link: DOE notes hydropower output grows with head; the bottle's jet speed grows with √h for the same reason. U.S. DOE: How hydropower works (https://www.energy.gov/eere/water/how-hydropower-works)

- [ ] **I completed this lab (wet or virtual)**

Lab C: Measuring flow rate and Reynolds number

WEEK 7

Goal: Measure a tap or hose flow rate with a bucket and stopwatch and decide whether the flow inside is laminar or turbulent.

$$Q = \frac{V_{bucket}}{t}, \quad Re = \frac{\rho V D}{\mu}$$

```
Q=_buckett,\ \ Re=(ρ VD)/(μ)
```

Flow rate and Reynolds number

Procedure

1. Time how long a hose or tap takes to fill 10 L.
2. Measure the inner diameter of the hose or tap outlet.
3. Compute Q, the mean speed and Re.
4. Turn the tap down until the stream looks smooth and glassy, and repeat.

Data table

Quantity	Value
10 L in	25 s
Q	0.4 L/s
Hose bore	16 mm
V	1.99 m/s
Re	31,700

Sample values; Re far above 4000 means turbulent flow inside the hose.

Sample calculation

1. V = 0.0004/(π × 0.008²) = 1.99 m/s.
2. Re = 998 × 1.99 × 0.016/1.002 × 10^{-3} = 31,700.

> **Safety**
>
> - Turn off the tap when done; watch for slippery ground.

Home or virtual alternative

- This is a home lab.

PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow)

Planet link: EPA suggests checking your water meter over two hours with no water in use; any change means a leak. U.S. EPA WaterSense: Fix a Leak Week (https://www.epa.gov/watersense/fix-leak-week)

- [ ] **I completed this lab (wet or virtual)**

Lab D: Pipe friction spreadsheet

WEEK 10

Goal: Build a spreadsheet of Reynolds number, friction factor and head loss against speed, and see how losses grow.

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

`h_L=fL/D(V^2)/(2g)`

Darcy–Weisbach

Procedure

1. Set D = 50 mm, L = 100 m, ε = 0.045 mm, water at 20 °C.
2. Compute Re and the Haaland f for V from 0.5 to 3 m/s.
3. Compute h_L and plot it against V.
4. Fit h_L ∝ V^n and find n.

Data table

V (m/s)	Re	f	h_L (m)
1	49,800	0.0235	2.39
2	99,600	0.0216	8.82
3	149,400	0.0209	19.2

Computed values; n comes out a little under 2 because f falls slowly as Re rises.

Sample calculation

1. h_L(2 m/s) = 0.0216 × 2000 × 4/19.62 = 8.82 m.
2. Tripling V raises h_L by 19.2/2.39 = 8.0 times.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/)

Planet link: DOE says best practices in pump systems save dramatic energy; lower losses mean smaller pumps. U.S. DOE: Pump systems (https://www.energy.gov/eere/iedo/pump-systems)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Settling of sand in water

WEEK 13

Goal: Watch fine and coarse sand settle in a tall glass of water and compare with Stokes' law.

$$V_s = \frac{(\rho_p - \rho) g D^2}{18 \mu}$$

`V_s=((ρ_p-ρ)gD^2)/(18μ)`

Stokes settling

Procedure

1. Fill a tall clear glass or bottle with water.
2. Drop a pinch of fine sand and time how long the grains take to fall 0.2 m.
3. Repeat with coarser sand.
4. Compare the speeds with Stokes' law and note when it stops applying.

Data table

Grain diameter	Stokes speed	Re
50 µm	2.04 mm/s	0.102
100 µm	8.17 mm/s	0.81

Computed for quartz-like grains (ρ = 2500 kg/m³); coarser grains with Re above about 1 fall slower than Stokes predicts.

Sample calculation

1. V(50 µm) = 1502 × 9.81 × 2.5 × 10^{-9}/(18 × 1.002 × 10^{-3}) = 2.04 mm/s.
2. Doubling D gives 4 times the speed.

> **Safety**
>
> - Use clean play sand; wash hands afterward.

Home or virtual alternative

- This is a home lab.

NASA Glenn: Drag of a sphere (https://www.grc.nasa.gov/www/k-12/airplane/dragsphere.html)

Planet link: DOE notes impoundment releases serve water-quality needs; sediment settling decides how reservoirs fill and rivers carry silt. U.S. DOE: Types of hydropower plants (https://www.energ

y.gov/eere/water/types-hydropower-plants)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Pump system and variable speed

WEEK 11

Goal: Find a pump's operating point and use the affinity laws to compare throttling with slowing the pump.

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3$$

```
(P_2)/(P_1)=((N_2)/(N_1))^3
```

Affinity law for power

Procedure

1. Enter the pump curve H = 40 − 7000Q² and system curve H = 10 + 5000Q².
2. Find the operating point and shaft power.
3. Apply the affinity laws at 1500 and 1200 rpm to the 1800 rpm point (0.05 m³/s, 30 m).
4. Compare power needs.

Data table

N (rpm)	Q (m³/s)	H (m)	Power ratio
1800	0.050	30.0	1.000
1500	0.0417	20.8	0.579
1200	0.0333	13.3	0.296

Ideal affinity scaling; static lift in the system limits how far a real pump can be slowed.

Sample calculation

1. Q(1200) = 0.05 × 2/3 = 0.0333 m³/s.
2. Power ratio = (2/3)³ = 0.296.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

U.S. DOE: Pump systems (https://www.energy.gov/eere/iedo/pump-systems)

Planet link: DOE's free PSAT and MEASUR tools help plants find exactly these pump savings. U.S. DOE: Pump systems (https://www.energy.gov/eere/iedo/pump-systems)

- [ ] **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. Find the shear stress in water between plates 0.5 mm apart when one moves at 1 m/s.
    1. $\tau = 1.002 \times 10^{-3} \times 1/0.0005$.

    Answer: 2.00 Pa.
2. Find the gauge pressure 25 m deep in seawater (1025 kg/m³).
    1. $1025 \times 9.81 \times 25$.

    Answer: 251.4 kPa.
3. Find the power of a hydro plant with Q = 20 m³/s, H = 40 m, $\eta = 0.88$ ($\rho = 1000$).
    1. $0.88 \times 1000 \times 9.81 \times 20 \times 40$.

    Answer: 6.91 MW.
4. A 0.01 m³/s water jet at 25 m/s is turned 180° by a fixed vane. Force?
    1. $2\rho QV = 2 \times 1000 \times 0.01 \times 25$.

    Answer: 500 N.
5. Find laminar Q in a 2 mm radius tube, $\Delta p/L = 200$ Pa/m, water.
    1. $\pi \times 0.002^4 \times 200/(8 \times 1.002 \times 10^{-3})$.

    Answer: 1.25×10^{-6} m³/s.

Final practice set (all units)

1. A 1:25 ship model is Froude-scaled. Full speed 10 m/s. Model speed?
    1. 10/5.

    Answer: 2 m/s.

2. Find h_L in 200 m of 0.1 m pipe with f = 0.02 at 1.5 m/s.

    1. 0.02 × 2000 × 2.25/19.62.

    Answer: 4.59 m.

3. A pump at 1200 rpm gives 25 m head. Head at 1500 rpm?

    1. 25 × (1.25)².

    Answer: 39.1 m.

4. Find the wind power through a 80 m rotor at 8 m/s (ρ = 1.225).

    1. ½ × 1.225 × π × 40² × 512.

    Answer: 1.58 MW.

5. Find Fr for 4 m/s flow 0.4 m deep.

    1. 4/√(9.81 × 0.4).

    Answer: 2.02, supercritical.

6. Find y_c for q = 3 m²/s.

    1. (9/9.81)^(1/3).

    Answer: 0.972 m.

Formula sheet

$$\tau = \mu\, du/dy,\ \nu = \mu/\rho$$

$$p = p_0 + \rho g h,\ F = \rho g h_c A,\ F_B = \rho g V$$

$$Q = AV,\ p/\rho g + V^2/2g + z = \text{const}$$

$$P = \eta \rho g Q H,\ V = \sqrt{2gh}$$

$$\sum F = \sum \dot{m} V_{out} - \sum \dot{m} V_{in},\ F = \rho Q V (1 - \cos$$

$$\nabla \cdot \mathbf{V} = 0,\ Q = \pi R^4 \Delta p/(8\mu L)$$

$$Re = \rho V L/\mu,\ Fr = V/\sqrt{gL}$$

$$h_L = f(L/D)V^2/2g,\ f_{lam} = 64/Re$$

$$h_m = K V^2/2g,\ D_h = 4A/P$$

$$P = \rho g Q h/\eta,\ Q \propto N,\ H \propto N^2,\ P \propto N^3$$

$$F_D = C_D \tfrac{1}{2}\rho V^2 A,\ P_{wind} = \tfrac{1}{2}\rho A V^3$$

$$V = (1/n) R_h^{2/3} S^{1/2},\ y_c = (q^2/g)^{1/3}$$

$$y_2/y_1 = \frac{1}{2}(\sqrt{1 + 8Fr_1^2} - 1)$$

Constants

Constant	Value
Water at 20 °C (textbook values)	ρ = 998 kg/m³, μ = 1.002 × 10⁻³ kg/(m·s), ν = 1.004 × 10⁻⁶ m²/s, surface tension σ = 0.0728 N/m
Air at 15 °C (textbook values)	ρ = 1.225 kg/m³, μ = 1.789 × 10⁻⁵ kg/(m·s)
Seawater and ice (textbook values)	ρ_sea = 1025 kg/m³, ρ_ice = 917 kg/m³
Mercury (textbook value)	ρ = 13,600 kg/m³
Gravity	g = 9.81 m/s²
1 U.S. gallon	3.785 L

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, textbook sections and simulations alongside the course.

- MIT OpenCourseWare 2.06 Fluid Dynamics (Spring 2013) (https://ocw.mit.edu/courses/2-06-fluid-dynamics-spring-2013/): Free MIT lecture notes and problem sets.
- MIT OpenCourseWare 1.060 Engineering Mechanics II (Spring 2006) (https://ocw.mit.edu/courses/1-060-engineering-mechanics-ii-spring-2006/): Free MIT civil-engineering fluids course.
- MIT OpenCourseWare 2.25 Advanced Fluid Mechanics (Fall 2013) (https://ocw.mit.edu/courses/2-25-advanced-fluid-mechanics-fall-2013/): Deeper MIT treatment.
- OpenStax University Physics Volume 1, Chapter 14: Fluid mechanics (https://openstax.org/books/university-physics-volume-1/pages/14-1-fluids-density-and-pressure): Free textbook chapter.
- PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow): Pressure and flow interactively.
- PhET: Buoyancy (https://phet.colorado.edu/en/simulations/buoyancy): Buoyancy interactively.

Glossary

Boundary layer
Thin layer near a wall where viscosity matters.

Buoyancy
Upward force equal to the weight of displaced fluid.

Cavitation
Vapor bubbles forming where pressure falls to vapor pressure.

Continuity
Conservation of mass in a flow.

Drag coefficient
Drag divided by $\frac{1}{2}\rho V^2 A$.

Froude number
$V/\sqrt{(gL)}$, inertia over gravity.

Head
Energy per unit weight of fluid, in metres.

Head loss
Mechanical energy lost to friction, per unit weight.

Hydraulic jump
Sudden change from supercritical to subcritical flow.

Hydraulic radius
Flow area over wetted perimeter.

Laminar flow
Smooth, layered flow at low Reynolds number.

Manometer
Tube of liquid used to measure pressure differences.

Navier–Stokes equations
Momentum equations for a viscous fluid.

Operating point
Where a pump curve meets the system curve.

Reynolds number
$\rho VL/\mu$, inertia over viscous forces.

Specific gravity
Density relative to water.

Stokes flow
Creeping flow at Re much less than 1.

Surface tension
Force per length along a liquid surface.

Turbulent flow
Chaotic, mixing flow at high Reynolds number.

Viscosity
Resistance of a fluid to shearing.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Sketches control volumes, picks the right law (hydrostatics, Bernoulli, energy, momentum, Navier–Stokes), checks Re and Fr, keeps units straight, and runs labs safely.
3 · Proficient	Solves the practice problems with occasional unit or sign slips.
2 · Developing	Follows worked examples but applies Bernoulli across pumps or ignores losses.
1 · Beginning	Plugs numbers into formulas without a sketch; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Thermodynamics ENGR 320
- Aircraft Performance AERO 320
- Aerodynamics I AERO 310
- All courses