Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Spring · 🧮 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester senior flight dynamics and controls course for aerospace engineers. It is modeled on the topics and depth of MIT 16.333 Aircraft Stability and Control, MIT 16.06 Principles of Automatic Control and MIT 16.30 Feedback Control Systems (free on MIT OpenCourseWare), using NASA Glenn's free Beginner's Guide to Aeronautics, which we link where it helps. None of those authors, schools or agencies endorses this page.
>
> What we add: every unit ties to real work with a cited public source (NASA's X-59 flight tests, Hubble's pointing system and the Dragonfly rotorcraft), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a spreadsheet.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Stable, well-controlled vehicles fly efficient paths, and the same control ideas point Earth-observing satellites and telescopes precisely enough to measure our planet and universe.

Control engineering also makes new, quieter and more autonomous aircraft possible. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample or textbook values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (AERO 420).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Dynamics: rigid-body rotation, moments of inertia and angular momentum.
- Differential Equations: linear ODEs and the Laplace transform.
- Linear Algebra: matrices and eigenvalues.
- Aerodynamics I and Aircraft Performance: lift, drag, moments and turning flight.

How to study

- Draw the body axes and the center of gravity before writing any moment equation.
- Keep angles in radians inside formulas; convert to degrees only for the answer.
- Separate static stability (the first tendency after a disturbance) from dynamic stability (what happens over time).
- Sketch the poles in the s-plane for every system you meet.
- Check every answer against physical sense: stable systems settle, unstable ones grow.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free PhET Balancing Act and Masses and Springs simulations.
- Spreadsheet software (any free one works), or Python.
- NASA Glenn’s Beginner’s Guide to Aeronautics pages in a web browser.
- A calculator.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Axes, Rotations and Moments	Axes, cg and moments; Lab A	NASA Glenn
2	Longitudinal Static Stability	Longitudinal static stability	MIT 16.333
3	Trim and Elevator Control	Trim and elevator; Lab B	NASA Glenn
4	Lateral-Directional Stability and Turns	Lateral-directional stability and turns	MIT 16.333
5	Equations of Motion and Dynamic Modes	Equations of motion and linearization	MIT 16.333
6	Equations of Motion and Dynamic Modes	Dynamic modes; Lab C	MIT 16.333
7	Laplace Transforms and Transfer Functions	Transfer functions; midterm practice	MIT 16.06
8	Second-Order Time Response	Second-order response; Lab D	MIT 16.06
9	Feedback and PID Control	Feedback and PID	MIT 16.06
10	Feedback and PID Control	Autopilot design; Lab E	MIT 16.30
11	Stability Analysis: Routh, Root Locus and Margins	Routh, root locus and margins	MIT 16.06
12	Spacecraft Attitude Determination and Control	Attitude dynamics and sensors	MIT 16.30
13	Spacecraft Attitude Determination and Control	Reaction wheels and slews; Lab F	NASA Hubble
14	Spacecraft Attitude Determination and Control	Autonomy and fault response	NASA Dragonfly
15	Spacecraft Attitude Determination and Control	Review; final practice	MIT 16.06

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Axes, Rotations and Moments

Body axes through the center of gravity, roll, pitch and yaw, control surfaces and the moments they make, finding the center of gravity, and Euler angles.

Learning objectives

By the end of this unit you can:

- Define the roll, pitch and yaw axes.
- Locate a center of gravity from component masses.
- Compute the moment of a control force.
- Match each control surface to its moment.

Key concepts

- In flight an aircraft rotates about its center of gravity (cg), the average location of its mass.
- Body axes: roll toward the nose, pitch toward the right wing tip, yaw toward the bottom of the aircraft.
- Control surfaces make aerodynamic forces at a distance from the cg, producing moments: elevators pitch, the rudder yaws, ailerons roll.
- The cg of several masses is x_cg = Σm_ix_i/Σm_i; moving it changes stability and control.
- Euler angles (yaw ψ, pitch θ, roll φ, applied in that order) describe attitude relative to the Earth.

Key equations

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

`x_cg=( m_ix_i)/( m_i)`

Center of gravity

$$M = Fd$$

`M=Fd`

Moment of a force

$$L,\ M,\ N$$

Roll, pitch and yaw moments

$$\psi \to \theta \to \phi$$

Worked examples, step by step

Example 1.1 A tail force of 2000 N acts 6 m behind the cg (sample values). Find the pitching moment.

1. M = 2000 × 6.

Answer: 12,000 N·m.

Example 1.2 Masses of 1000 kg at 2 m, 500 kg at 6 m and 300 kg at 10 m from the nose (sample values). Find the cg.

1. Σm = 1800 kg.
2. Σmx = 2000 + 3000 + 3000 = 8000 kg·m.

Answer: x_cg = 4.44 m.

Example 1.3 Which surface rolls the aircraft?

1. Ailerons deflect in opposite directions on each wing.

Answer: Ailerons produce a rolling moment.

Practice problems

Work each on paper first, then check.

1. Moment of 500 N at 4 m?

   2000 N·m.

2. Which surface yaws the nose?

   The rudder.

3. Add 200 kg at 10 m to the example. New cg?

   10,000/2000 = 5.0 m.

4. About which point does a kite rotate?

Its bridle point.

Common misconceptions

Myth: Aircraft rotate about their nose.

Actually: They rotate about the cg.

Myth: Elevators roll the aircraft.

Actually: They pitch it.

Myth: Euler angle order does not matter.

Actually: Rotations do not commute.

Flashcards

cg

Average location of mass.

Roll

Rotation about the nose axis.

Pitch

Nose up or down.

Yaw

Nose side to side.

Moment

Force × distance.

Euler angles

ψ, θ, φ.

Self-check quiz

1. Aircraft rotate about...

the cg

the nose

the tail

the wing tip

the cg. NASA Glenn.

2. Rudder makes...

yaw

roll

pitch

lift only

yaw. Yawing moment.

3. Example cg...

4.44 m

6 m

2 m

10 m

4.44 m. 8000/1800.

4. Pitch axis points to...

the right wing tip

the nose

down

up

the right wing tip. Definition.

Try this quiz again

How this helps save the planet

NASA Glenn explains that control surfaces produce forces at their centers of pressure, some distance from the cg, creating torques about the principal axes: elevators pitch, the rudder yaws

and ailerons roll. The first aircraft to show active control about all three axes was the Wrights' 1902 glider.

Every modern efficient airliner and Earth-observing aircraft still uses those three moments, now often commanded through flight-control computers.

On Saving Planets: Aerodynamics I · Dynamics

Sources: NASA Glenn Beginner's Guide: Aircraft rotations (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-rotations/) · NASA Glenn Beginner's Guide: Center of gravity (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/center-of-gravity/)

Free learning resources

- NASA Glenn Beginner's Guide: Aircraft rotations (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-rotations/): Free NASA explainer.
- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT lectures.

☐ **I finished this unit**

Longitudinal Static Stability

Pitching-moment coefficient, the stability derivative C_mα, the neutral point, static margin, and how cg position and tail size set stability.

Learning objectives

By the end of this unit you can:

- State the condition for static pitch stability.
- Compute static margin.
- Relate static margin to C_mα.
- Explain the effect of moving the cg.

Key concepts

- Static stability is the first tendency after a disturbance: a statically stable aircraft makes a moment that pushes it back.
- In pitch this needs C_mα < 0: more angle of attack makes a nose-down moment.
- The neutral point is the cg location where C_mα = 0.
- Static margin SM = (x_np − x_cg)/c̄; positive means stable. Approximately C_mα = −C_Lα × SM.
- Moving the cg aft reduces stability and control forces; too far aft makes the aircraft unstable unless a computer stabilizes it.

Key equations

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} < 0$$

`C_m_α=( C_m)/(α)<0`

Pitch stability

$$SM = \frac{x_{np} - x_{cg}}{\bar{c}}$$

`SM=_np-x_cg c`

Static margin

$$C_{m_\alpha} = -C_{L_\alpha} SM$$

`C_m_α=-C_L_αSM`

Stability derivative

$$M = \tfrac{1}{2}\rho V^2 S \bar{c} C_m$$

`M=12ρ V^2S cC_m`

Pitching moment

Worked examples, step by step

Example 2.1 The neutral point is at 0.35c̄ and the cg at 0.25c̄ (sample values). Find the static margin.

1. SM = 0.35 − 0.25.

Answer: SM = 0.10 (10%).

Example 2.2 With C_Lα = 5.5 per radian, find C_mα.

1. C_mα = −5.5 × 0.10.

Answer: C_mα = −0.55 per radian: stable.

Example 2.3 Move the cg to 0.30c̄. Find the new C_mα.

1. SM = 0.05; −5.5 × 0.05.

Answer: −0.275 per radian: half as stiff.

Practice problems

Work each on paper first, then check.

1. SM with cg at 0.35c̄?

   0: neutrally stable.

2. Sign of C_mα for cg at 0.40c̄?

   Positive: unstable.

3. Does a bigger tail move the neutral point fore or aft?

   Aft, raising static margin.

4. Why fly with small margins?

Less trim drag and lighter controls.

Common misconceptions

Myth: Static stability guarantees dynamic stability.

Actually: It is only the first tendency.

Myth: Aft cg is always safer.

Actually: It reduces stability.

Myth: $C_{m\alpha}$ should be positive.

Actually: Negative is stable.

Flashcards

Static stability

First tendency to return.

$C_{m\alpha}$

Pitch stiffness.

Neutral point

cg where $C_{m\alpha} = 0$.

Static margin

$(x_{np} - x_{cg})/\bar{c}$.

Mean chord $\bar{c}$

Reference length.

Relaxed stability

Small or negative margin with computer help.

Self-check quiz

1. Pitch stable if $C_{m\alpha}$...

< 0

> 0

$= 1$

$= 0$

< 0. Restoring.

2. SM example...

0.10

0.35

0.25

0.60

0.10. 0.35 − 0.25.

3. Neutral point: C_mα = ...

0

1

−1

∞

0. Definition.

4. Moving cg aft...

reduces SM

raises SM

no change

adds lift

reduces SM. Margin shrinks.

Try this quiz again

How this helps save the planet

NASA reports that after 25 flights the X-59's real-world flight characteristics closely match simulations, including its flying qualities and its stability margin, which NASA describes as how

close the aircraft is to becoming unstable under certain conditions.

The X-59 is shaped to turn a loud sonic boom into a quiet thump, and confirming its stability margin in flight is part of proving that quieter supersonic flight is practical.

On Saving Planets: Aerodynamics I · Aircraft Performance

Sources: NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/questt/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

Free learning resources

- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT lectures.
- NASA Glenn Beginner's Guide: Center of gravity (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/center-of-gravity/): Free NASA explainer.

☐ **I finished this unit**

Trim and Elevator Control

Trim as zero net pitching moment, elevator effectiveness, the elevator angle to trim, tail download and trim drag, and stick-fixed versus stick-free stability.

Learning objectives

By the end of this unit you can:

- Write the trim condition.
- Solve for elevator angle to trim.
- Explain why tails often push down.
- Describe trim drag.

Key concepts

- Trim means zero net pitching moment: C_m = C_m0 + C_mα α + C_mδe δ_e = 0.
- Elevator effectiveness C_mδe is negative for a tail elevator: trailing edge down gives nose down.
- The elevator lift acts some distance from the cg, so it makes a torque and the aircraft rotates about its cg.
- Modern tails often carry negative lift to keep the nose up, which the wing must offset with extra lift.
- That extra lift costs induced drag, called trim drag; a smaller static margin reduces it.

Key equations

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e = 0$$

`C_m=C_m_0+C_m_αα+C_m__e_e=0`

Trim condition

$$\delta_e = -\frac{C_{m_0} + C_{m_\alpha}\alpha}{C_{m_{\delta_e}}}$$

`_e=-_m_0+C_m_ααC_m__e`

Elevator to trim

$$C_L = C_{L_\alpha}\alpha$$

`C_L=C_L_αα`

Lift slope

$$L = \tfrac{1}{2}\rho V^2 S C_L$$

L=12ρ V^2SC_L

Lift

Worked examples, step by step

Example 3.1 C_m0 = 0.05, C_mα = −0.55, C_mδe = −1.2 per radian (sample values). Find δ_e to trim at α = 0.05 rad.

1. 0.05 − 0.0275 − 1.2δ_e = 0.
2. δ_e = 0.0225/1.2.

Answer: δ_e = 0.01875 rad = 1.07°.

Example 3.2 What angle of attack trims with zero elevator?

1. α = 0.05/0.55.

Answer: α = 0.0909 rad = 5.21°.

Example 3.3 Find C_L at that trim with C_Lα = 5.5.

1. 5.5 × 0.0909.

Answer: C_L = 0.50.

Practice problems

Work each on paper first, then check.

1. δ_e to trim at α = 0.1 rad?

   −0.00417 rad (−0.24°).

2. Sign convention for δ_e?

   Trailing edge down positive.

3. Why does tail download cost fuel?

The wing makes more lift, adding induced drag.

4. Where was the Wright Flyer’s elevator?

   At the front.

Common misconceptions

Myth: Trim means level flight.

Actually: It means zero net moment at any steady condition.

Myth: Tails always lift upward.

Actually: Many push down.

Myth: Elevator only changes pitch angle.

Actually: It sets the trimmed angle of attack and speed.

Flashcards

Trim

Zero net moment.

$C_{m\delta e}$

Elevator effectiveness.

Elevator

Pitch control surface.

Tail download

Negative tail lift.

Trim drag

Drag from trimming.

Canard

Forward elevator surface.

Self-check quiz

1. Trim: C_m = ...

0

1

C_L

$C_{m\alpha}$

0. No net moment.

2. Example δ_e...

1.07°

10°

0°

−5°

1.07°. Worked.

3. Trim drag comes from...

extra wing lift

friction only

engines

gear

extra wing lift. Induced.

4. $C_{m\delta e}$ for a tail is...

negative

positive

zero

infinite

negative. Nose down.

Try this quiz again

How this helps save the planet

NASA Glenn explains that the horizontal stabilizer on modern aircraft is usually set at a negative angle, making a downward force that keeps the tail down and the airplane trimmed, and that this negative force must be overcome with greater lift from the wing.

Every bit of extra lift adds drag and fuel burn, so careful trim and cg management are small but real ways to cut aviation emissions.

On Saving Planets: Aircraft Performance · Solutions

Sources: NASA Glenn Beginner's Guide: Aircraft elevator interactive (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-elevator-interactive/)

Free learning resources

- NASA Glenn Beginner's Guide: Aircraft elevator interactive (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-elevator-interactive/): Free NASA interactive.
- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT lectures.

☐ **I finished this unit**

Lateral-Directional Stability and Turns

Weathercock (directional) stability, dihedral effect, aileron roll control, roll damping and the roll mode, adverse yaw, and the coordinated turn.

Learning objectives

By the end of this unit you can:

- State directional and roll stability conditions.
- Compute roll-mode time constant and steady roll rate.
- Find bank angle and load factor for a coordinated turn.
- Explain adverse yaw.

Key concepts

- Directional (weathercock) stability needs C_nβ > 0: a sideslip yaws the nose back into the wind.
- Dihedral effect needs C_lβ < 0: a sideslip rolls the aircraft away from the slip.
- Ailerons deflect oppositely to make a rolling moment; roll damping L_p resists roll rate, giving the roll mode with τ = 1/|L_p|.
- The down-going aileron makes more lift and drag, yawing the nose away from the turn (adverse yaw); the rudder fixes it.
- In a coordinated level turn tan φ = V²/(gR) and load factor n = 1/cos φ.

Key equations

$$C_{n_\beta} > 0,\ C_{l_\beta} < 0$$

Lateral-directional stability

$$p_{ss} = -\frac{L_{\delta_a}}{L_p}\delta_a$$

```
p_ss=-__aL_p_a
```

Steady roll rate

$$\tan\phi = \frac{V^2}{gR}$$

```
=(V^2)/(gR)
```

Coordinated turn

$$n = \frac{1}{\cos \phi}$$

Load factor

Worked examples, step by step

Example 4.1 A turn at 100 m/s with radius 2000 m (sample values). Find the bank angle.

1. tan φ = 100²/(9.80665 × 2000) = 0.510.

Answer: φ = 27.0°.

Example 4.2 Find the load factor.

1. n = 1/cos 27.0°.

Answer: n = 1.122.

Example 4.3 L_δa = 2.0 s⁻², L_p = −4.0 s⁻¹ (sample values), δ_a = 0.1 rad. Find τ and steady roll rate.

1. τ = 1/4.
2. p = 2.0 × 0.1/4.0.

Answer: τ = 0.25 s; p = 0.05 rad/s.

Practice problems

Work each on paper first, then check.

1. Load factor at 60° bank?

   2.0.

2. Radius at 100 m/s and 45° bank?

   1020 m.

3. What does dihedral do?

   Rolls the aircraft level after a sideslip.

4. Why use the rudder in a turn?

   To cancel adverse yaw.

Common misconceptions

Myth: Ailerons alone make a clean turn.

Actually: Adverse yaw needs rudder.

Myth: Bank angle is independent of speed.

Actually: It grows with V^2 for a fixed radius.

Myth: $C_{n\beta}$ should be negative.

Actually: Positive is stable.

Flashcards

Sideslip β

Wind angle from the side.

$C_{n\beta}$

Directional stability.

$C_{l\beta}$

Dihedral effect.

Roll damping

L_p.

Adverse yaw

Yaw opposite the roll.

Load factor

Lift/weight.

Self-check quiz

1. Weathercock stable if...

$C_{n\beta} > 0$

$C_{n\beta} < 0$

$C_{l\beta} > 0$

$L_p > 0$

C_nβ > 0. Restoring yaw.

2. Example bank...

27.0°

45°

10°

60°

27.0°. $\tan^{-1}$ 0.51.

3. n at 60° bank...

2

1

1.5

3

2. 1/cos.

4. Roll mode τ...

1/|L_p|

L_p

L_δa

2π

1/|L_p|. First order.

Try this quiz again

How this helps save the planet

NASA Glenn's guide explains that ailerons usually work in opposition, one up and one down, to change lift on each wing, and that airplanes turn because of the banking created by the ailerons, not because of a rudder input: the bank tilts the wing's lift and curves the flight path.

This unit's turn equation is that idea in numbers. Coordinated, efficient turns keep extra drag, and so extra fuel burn, small.

On Saving Planets: Aircraft Performance · Aerodynamics I

Sources: NASA Glenn Beginner's Guide: Ailerons (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/ailerons/) · NASA Glenn Beginner's Guide: Aircraft rotations (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-rotations/)

Free learning resources

- NASA Glenn Beginner's Guide: Ailerons (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/ailerons/): Free NASA explainer.
- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT lectures.

☐ **I finished this unit**

Equations of Motion and Dynamic Modes

Six-degree-of-freedom equations, small-disturbance linearization, state-space form, eigenvalues, and the short-period, phugoid, roll, spiral and Dutch-roll modes.

Learning objectives

By the end of this unit you can:

- Linearize equations about trim.
- Write state-space form.
- Find natural frequency and damping from a characteristic equation.
- Estimate the phugoid period.

Key concepts

- Newton’s laws in body axes give six coupled nonlinear equations; for small disturbances about trim they become linear.
- State-space form ẋ = Ax + Bu; the eigenvalues of A are the modes.
- Longitudinal modes: a fast, well-damped short period (pitch) and a slow, lightly damped phugoid (speed–altitude exchange).
- Lateral modes: a fast roll mode, a slow spiral mode and the oscillating Dutch roll.
- A second-order mode s² + 2ζω_n s + ω_n² has natural frequency ω_n and damping ratio ζ. The phugoid approximation is ω_n ≈ √2 g/V.

Key equations

$$\dot{x} = Ax + Bu$$

`x=Ax+Bu`

State space

$$\det(sI - A) = 0$$

`(sI-A)=0`

Characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

`s^2+2ω_ns+ω_n^2=0`

Second-order mode

$$\omega_{ph} \approx \frac{\sqrt{2}g}{V}$$

Phugoid approximation

Worked examples, step by step

Example 5.1 A short-period mode has $s^2 + 2.8s + 16 = 0$ (sample values). Find ω_n and ζ.

1. $\omega_n = \sqrt{16}$.
2. $\zeta = 2.8/(2 \times 4)$.

Answer: $\omega_n = 4$ rad/s; $\zeta = 0.35$.

Example 5.2 Estimate the phugoid frequency and period at 100 m/s.

1. $\omega = 1.4142 \times 9.80665/100$.
2. $T = 2\pi/\omega$.

Answer: $\omega = 0.1387$ rad/s; $T = 45.3$ s.

Example 5.3 Find the damped frequency of the short period.

1. $\omega_d = 4\sqrt{(1 - 0.35^2)}$.

Answer: $\omega_d = 3.75$ rad/s.

Practice problems

Work each on paper first, then check.

1. Phugoid period at 200 m/s?

   90.6 s.

2. ζ for $s^2 + 4s + 16$?

   0.5.

3. Which mode trades speed for height?

   Phugoid.

4. Which lateral mode oscillates?

   Dutch roll.

Common misconceptions

Myth: Linear models work at any condition.
Actually: Only near the trim point.

Myth: All modes oscillate.
Actually: Roll and spiral are not oscillatory.

Myth: Phugoid is fast.
Actually: It takes tens of seconds.

Flashcards

Short period

Fast pitch mode.

Phugoid

Slow speed–height mode.

Dutch roll

Yaw–roll oscillation.

Spiral mode

Slow bank divergence or convergence.

ω_n

Natural frequency.

ζ

Damping ratio.

Self-check quiz

1. Example ζ...

0.35

0.7

4

2.8

0.35. 2.8/8.

2. Phugoid period at 100 m/s...

45.3 s

4.5 s

453 s

1 s

45.3 s. Approx.

3. Modes are...

eigenvalues of A

entries of B

inputs

gains

eigenvalues of A. State space.

4. Short period is...

fast pitch

slow speed

roll

yaw

fast pitch. Definition.

Try this quiz again

How this helps save the planet

NASA says the X-59 team uses a real-time digital twin that compares flight data to simulated predictions, and that the aircraft has shown no unexpected phenomena such as instability or excessive vibration in response to aerodynamic forces.

NASA reports the X-59 reached its target cruise of Mach 1.4 (924 mph) at 55,000 feet relatively early in testing. Models built from this unit's linear equations are what a digital twin compares against.

On Saving Planets: Differential Equations · Linear Algebra

Sources: NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/qusst/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

Free learning resources

- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT lectures.
- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT course.

☐ **I finished this unit**

Laplace Transforms and Transfer Functions

Transfer functions, poles and zeros, block diagrams, first-order systems, time constants, DC gain and the final value theorem.

Learning objectives

By the end of this unit you can:

- Write a transfer function from an ODE.
- Find poles, DC gain and time constant.
- Use the final value theorem.
- Reduce simple block diagrams.

Key concepts

- With zero initial conditions, the Laplace transform turns a linear ODE into algebra: Y(s) = G(s)U(s).
- Poles (roots of the denominator) set the natural response; left-half-plane poles decay, right-half-plane poles grow.
- A first-order system K/(τs + 1) reaches 63.2% of its final value at t = τ and about 98% at 4τ.
- The final value theorem: y(∞) = lim s→0 of sY(s), if the system is stable. For a unit step, y(∞) = G(0).
- Blocks in series multiply; a negative feedback loop gives G/(1 + GH).

Key equations

$$G(s) = \frac{Y(s)}{U(s)}$$

`G(s)=(Y(s))/(U(s))`

Transfer function

$$y(t) = K(1 - e^{-t/\tau})$$

`y(t)=K(1-e^-t/τ)`

First-order step response

$$y(\infty) = \lim_{s \to 0} sY(s)$$

`y(∞)=_s0sY(s)`

Final value theorem

$$T = \frac{G}{1 + GH}$$

`T=(G)/(1+GH)`

Closed loop

Worked examples, step by step

Example 6.1 Find the pole, time constant and DC gain of G(s) = 5/(s + 2).

1. Pole at s = −2.
2. τ = 1/2.
3. G(0) = 5/2.

Answer: Pole −2; τ = 0.5 s; gain 2.5.

Example 6.2 Find its unit-step response at t = 0.5 s and 1 s.

1. y = 2.5(1 − e^(−2t)).

Answer: y(0.5) = 1.580; y(1) = 2.162.

Example 6.3 Close a unity feedback loop around G. Find the closed-loop DC gain.

1. T = 5/(s + 7).

Answer: T(0) = 0.714.

Practice problems

Work each on paper first, then check.

1. 2% settling time of G?

   About 4τ = 2 s.

2. Fraction reached at t = τ?

   63.2%.

3. Is a pole at +1 stable?

No; it grows as e^t.

4. Two blocks 2 and 3/(s+1) in series?

   6/(s+1).

Common misconceptions

Myth: Final value theorem always applies.
Actually: Only for stable systems.

Myth: Zeros set stability.
Actually: Poles do.

Myth: τ is the settling time.
Actually: Settling is about 4τ.

Flashcards

Transfer function

Output/input in s.

Pole

Denominator root.

Zero

Numerator root.

Time constant

τ.

DC gain

G(0).

Final value theorem

lim sY(s).

Self-check quiz

1. G(0) for 5/(s+2)...

2.5

5

2

0.4

2.5. DC gain.

2. At t = τ...

63.2%

50%

100%

98%

63.2%. $1 - e^{-1}$.

3. Stable poles lie...

in the left half plane

right half

on the real axis only

at infinity

in the left half plane. Decay.

4. Closed loop...

G/(1+GH)

GH

G − H

1/G

G/(1+GH). Feedback.

Try this quiz again

How this helps save the planet

NASA explains that Hubble's gyroscopes measure the telescope's slew rates and are part of the system that determines and controls the direction the telescope points.

Sensors like these are the measurement blocks in this unit's diagrams: a controller can only be as good as the rates and angles it measures.

On Saving Planets: Differential Equations · Earth Observation

Sources: NASA Science: NASA to change how it points Hubble Space Telescope (June 4, 2024) (https://science.nasa.gov/missions/hubble/nasa-to-change-how-it-points-hubble-space-telescope/)

Free learning resources

- MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/): Free MIT lectures.
- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT course.

Watch the lecture

Lecture 6 laplace transform (https://ocw.mit.edu/courses/6-003-signals-and-systems-fall-2011/resources/lecture-6-laplace-transform/) (in English)

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☐ **I finished this unit**

Second-Order Time Response

Underdamped step response, overshoot, peak time, settling time, damping ratio and natural frequency, and handling-quality targets.

Learning objectives

By the end of this unit you can:

- Compute percent overshoot from ζ.
- Compute peak and settling times.
- Read ζ and ω_n from pole locations.
- Choose damping for good response.

Key concepts

- Poles at −ζω_n ± jω_n√(1 − ζ²) for 0 < ζ < 1 give an oscillating response that decays.
- Percent overshoot depends only on ζ: M_p = exp(−ζπ/√(1 − ζ²)).
- Peak time t_p = π/ω_d, with ω_d = ω_n√(1 − ζ²).
- Settling time (2%) is about t_s = 4/(ζω_n), set by the real part of the poles.
- Damping near 0.7 gives small overshoot (about 4.6%) with a quick rise; pilots dislike both sluggish and oscillating responses.

Key equations

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

`M_p=e^-π/1-^2`

Overshoot

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

`t_p=πω_n1-^2`

Peak time

$$t_s \approx \frac{4}{\zeta\omega_n}$$

Settling time

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

s=-ω_n jω_n1-^2

Poles

Worked examples, step by step

Example 7.1 For the Unit 5 short period (ω_n = 4, ζ = 0.35), find the overshoot.

1. M_p = exp(−0.35π/√(1 − 0.1225)).

Answer: 30.9%.

Example 7.2 Find its peak and settling times.

1. t_p = π/3.747.
2. t_s = 4/(0.35 × 4).

Answer: t_p = 0.838 s; t_s = 2.86 s.

Example 7.3 Find the overshoot for ζ = 0.7.

1. exp(−0.7π/√0.51).

Answer: 4.6%.

Practice problems

Work each on paper first, then check.

1. Overshoot at ζ = 0.25?

   44.4%.

2. Poles for ω_n = 4, ζ = 0.35?

   −1.4 ± 3.75j.

3. Overshoot at ζ = 1?

   None (critically damped).

4. What sets settling time?

   The real part ζω_n.

Common misconceptions

Myth: More damping is always better.

Actually: Too much makes the response sluggish.

Myth: Overshoot depends on ω_n.

Actually: Only on ζ.

Myth: Peak time uses ω_n.

Actually: It uses ω_d.

Flashcards

Overshoot

Peak above final value.

Peak time

π/ω_d.

Settling time

≈4/(ζω_n).

ω_d

Damped frequency.

Critical damping

ζ = 1.

Handling qualities

How easily pilots fly a vehicle.

Self-check quiz

1. M_p at ζ = 0.35...

30.9%

4.6%

50%

0%

30.9%. Formula.

2. $t_s \approx ...$

$4/(\zeta\omega_n)$

π/ω_n

ζ/ω_n

$1/\omega_n$

$4/(\zeta\omega_n)$. 2% rule.

3. $\zeta = 0.7$ overshoot...

4.6%

30%

0%

70%

4.6%. Formula.

4. Overshoot depends on...

ζ only

ω_n only

gain

input

ζ only. Formula.

Try this quiz again

How this helps save the planet

NASA reports that between X-59 test flights, engineers made minor adjustments to the aircraft's control software, and that its flying qualities, or controllability, matched simulations closely.

Tuning response speed and damping, the quantities in this unit, is how control software turns a radical quiet-supersonic shape into an aircraft pilots describe as “exciting but uneventful.”

On Saving Planets: Differential Equations · Aircraft Performance

Sources: NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/questt/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

Free learning resources

- MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/): Free MIT lectures.
- PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs): Damped oscillation.

Watch the lecture

Lecture 11 continuous time ct frequency response and bode plot (https://ocw.mit.edu/courses/6-003-signals-and-systems-fall-2011/resources/lecture-11-continuous-time-ct-frequency-response-and-bode-plot/) (in English)

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☐ **I finished this unit**

Feedback and PID Control

Why feedback, proportional, integral and derivative action, steady-state error and system type, gain selection for damping, and a pitch-attitude autopilot example.

Learning objectives

By the end of this unit you can:

- Close a loop and find the characteristic equation.
- Choose a gain for a target damping.
- Compute steady-state error.
- Explain what D and I terms add.

Key concepts

- Feedback compares the measured output to the command and acts on the error, reducing sensitivity to disturbances and model errors.
- Proportional gain speeds response but can reduce damping; derivative action adds damping; integral action removes steady-state error.
- For a plant 1/(s(s + 2)) with gain K, the closed loop is s² + 2s + K = 0, so ω_n = √K and ζ = 1/√K.
- System type (integrators in the loop) sets steady-state error: a type-1 loop has zero step error and ramp error a/K for this plant.
- PID: u = K_p e + K_i∫e dt + K_d de/dt.

Key equations

$$u = K_p e + K_i \int e\,dt + K_d \dot{e}$$

`u=K_pe+K_i e dt+K_d e`

PID law

$$1 + KG(s) = 0$$

`1+KG(s)=0`

Closed-loop characteristic equation

$$s^2 + (2 + K_d)s + K_p = 0$$

`s^2+(2+K_d)s+K_p=0`

PD on 1/(s(s+2))

$$e_{ss,ramp} = \frac{2}{K}$$

`e_ss,ramp=2/K`

Ramp error (this plant)

Worked examples, step by step

Example 8.1 With K = 16 on 1/(s(s + 2)), find ω_n and ζ.

1. $s^2 + 2s + 16$.
2. ω_n = 4; ζ = 2/8.

Answer: ω_n = 4 rad/s; ζ = 0.25 (44% overshoot).

Example 8.2 What K gives ζ = 0.7?

1. 2ζω_n = 2 ⇒ ω_n = 1/0.7 = 1.429.
2. $K = \omega_n^2$.

Answer: K = 2.04 (slower response).

Example 8.3 Keep K_p = 16 and add derivative gain for ζ = 0.7.

1. 2 + K_d = 2 × 0.7 × 4.

Answer: K_d = 3.6: fast and well damped.

Practice problems

Work each on paper first, then check.

1. Ramp error with K = 16?

   0.125.

2. Step error for this type-1 loop?

   Zero.

3. What does integral action fix?

Steady-state error.

4. Risk of too much derivative gain?

   Noise amplification.

Common misconceptions

Myth: Higher gain always helps.
Actually: It can cut damping and margins.

Myth: Derivative removes error.
Actually: Integral does.

Myth: Feedback needs a perfect model.
Actually: It tolerates model error.

Flashcards

Feedback

Act on measured error.

K_p

Proportional gain.

K_i

Integral gain.

K_d

Derivative gain.

System type

Number of loop integrators.

Steady-state error

Error after transients.

Self-check quiz

1. ζ with K = 16...

0.25

0.7

4

1

0.25. 1/√K.

2. Integral action...

removes steady error

adds damping

adds noise only

nothing

removes steady error. PID.

3. K_d for ζ = 0.7 at K_p = 16...

3.6

16

0.7

2

3.6. Worked.

4. Ramp error K = 16...

0.125

0

1

2

0.125. 2/K.

Try this quiz again

How this helps save the planet

NASA reports that Dragonfly recently completed its most comprehensive simulated “first flight” on flight-like avionics at Johns Hopkins APL, with flight software using simulated navigation-camera, inertial-measurement-unit and lidar data; the rotorcraft is due at Saturn’s moon Titan in 2034.

NASA’s mobility lead notes that everything Dragonfly does on Titan has to happen autonomously, without real-time help from Earth. Closed-loop feedback like this unit’s is what makes that possible.

On Saving Planets: Solutions · Orbital Mechanics

Sources: NASA Science: Dragonfly completes first simulated flight (Oct. 23, 2026) (https://science.nasa.gov/blogs/dragonfly/2026/10/23/nasas-dragonfly-completes-first-simulated-flight/)

Free learning resources

- MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/): Free MIT lectures.
- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT course.

Watch the lecture

Lecture 10 feedback and control (https://ocw.mit.edu/courses/6-003-signals-and-systems-fall-2011/resources/lecture-10-feedback-and-control/) (in English)

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☐ **I finished this unit**

Stability Analysis: Routh, Root Locus and Margins

The Routh–Hurwitz test, root-locus sketching, frequency response and Bode plots, gain and phase margin, and the link between phase margin and damping.

Learning objectives

By the end of this unit you can:

- Apply the Routh test to find a gain limit.
- Sketch how poles move with gain.
- Compute a phase margin.
- Relate phase margin to damping.

Key concepts

- Routh–Hurwitz finds how many poles are in the right half plane without solving the polynomial.
- The root locus shows closed-loop pole paths as gain rises from 0 to ∞.
- Bode plots show loop gain and phase versus frequency; gain margin and phase margin measure distance from instability.
- Phase margin PM = 180° + ∠G(jω_c) at the crossover frequency where |G| = 1.
- A rule of thumb for second-order loops: ζ ≈ PM/100 (PM in degrees).

Key equations

$$s^3 + 6s^2 + 11s + (6 + K) = 0$$

```
s^3+6s^2+11s+(6+K)=0
```

Example characteristic equation

$$66 > 6 + K$$

Routh condition

$$PM = 180^\circ + \angle G(j\omega_c)$$

```
PM=180^+ G(jω_c)
```

Phase margin

$$\zeta \approx PM/100$$

Rule of thumb

Worked examples, step by step

Example 9.1 Unity feedback around $K/((s + 1)(s + 2)(s + 3))$. Find the K limit.

1. Characteristic: $s^3 + 6s^2 + 11s + 6 + K$.
2. Routh: $6 \times 11 > 6 + K$.

Answer: Stable for $K < 60$.

Example 9.2 At $K = 60$, at what frequency does it oscillate?

1. Auxiliary $6s^2 + 66 = 0$.

Answer: $\omega = \sqrt{11} = 3.317$ rad/s.

Example 9.3 Find the phase margin of $16/(s(s + 2))$.

1. $|G| = 1$ at $\omega_c = 3.758$ rad/s.
2. $PM = 90° - \tan^{-1}(3.758/2)$.

Answer: PM = 28.0°, so $\zeta \approx 0.28$ (exact 0.25).

Practice problems

Work each on paper first, then check.

1. Is $K = 30$ stable in the example?

   Yes.

2. Gain margin at $K = 30$?

   2 (60/30).

3. What happens to the locus as $K \to \infty$ here?

   Two branches cross into the right half plane.

4. Typical PM target?

   About 45°–60°.

Common misconceptions

Myth: Stable at one gain means stable at all.

Actually: Gain limits are common.

Myth: Phase margin is measured at 180° crossing.

Actually: At gain crossover.

Myth: Routh gives pole locations.

Actually: Only counts.

Flashcards

Routh test

Counts RHP poles.

Root locus

Pole paths with gain.

Bode plot

Gain and phase vs frequency.

Gain margin

Gain factor to instability.

Phase margin

Phase to instability.

Crossover

|G| = 1.

Self-check quiz

1. K limit example...

60

6

11

66

60. Routh.

2. Oscillation at K = 60...

3.317 rad/s

6 rad/s

1 rad/s

60 rad/s

3.317 rad/s. √11.

3. PM example...

28°

45°

90°

0°

28°. Worked.

4. ζ ≈ ...

PM/100

PM

1/PM

GM

PM/100. Rule.

Try this quiz again

How this helps save the planet

NASA says Dragonfly engineers will test other flight profiles and deliberately introduce faults to make sure the rotorcraft can respond autonomously if something goes wrong.

Margins are the engineering answer to “what if”: a loop with healthy gain and phase margins keeps working when parts age, sensors drift or the environment surprises it.

On Saving Planets: Linear Algebra · Solutions

Sources: NASA Science: Dragonfly completes first simulated flight (Oct. 23, 2026) (https://science.nasa.gov/blogs/dragonfly/2026/10/23/nasas-dragonfly-completes-first-simulated-flight/)

Free learning resources

- MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/): Free MIT lectures.
- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT course.

☐ **I finished this unit**

UNIT 10 · WEEKS 12–13

Spacecraft Attitude Determination and Control

Rigid-body rotation, attitude sensors (gyros, star trackers), reaction wheels and momentum, bang-bang slews, safe modes, and pointing for Earth and space science.

Learning objectives

By the end of this unit you can:

- Use τ = Iα for single-axis slews.
- Compute a minimum-time slew.
- Compute wheel momentum.
- Explain gyro roles and safe modes.

Key concepts

- Single-axis rotation: torque τ = Iα; angular momentum H = Iω.
- A bang-bang slew accelerates for half the angle and brakes for the other half: t = 2√(θ/α).
- Reaction wheels trade momentum with the spacecraft; wheels saturate and must be unloaded with thrusters or magnetic torquers.
- Gyros measure rates and star trackers measure attitude; filters combine them.
- Safe modes put a spacecraft in a simple, power-positive attitude when something looks wrong.

Key equations

$$\tau = I\alpha$$

Rotational Newton’s law

$$t = 2\sqrt{\theta/\alpha}$$

Bang-bang slew time

$$H = I\omega$$

Angular momentum

$$\omega_{max} = \alpha t/2$$

`ω_max=α t/2`

Peak slew rate

Worked examples, step by step

Example 10.1 A spacecraft with I = 1000 kg·m² has 0.5 N·m of wheel torque (sample values). Find α and the 90° slew time.

1. $\alpha = 0.5/1000 = 5 \times 10^{-4}$ rad/s².
2. $t = 2\sqrt{(1.5708/0.0005)}$.

Answer: t = 112.1 s.

Example 10.2 Find the peak rate during that slew.

1. $\omega = 0.0005 \times 112.1/2$.

Answer: 0.0280 rad/s (1.61°/s).

Example 10.3 A wheel has I = 0.1 kg·m² spinning at 600 rad/s (sample values). Find its momentum.

1. $H = 0.1 \times 600$.

Answer: 60 N·m·s.

Practice problems

Work each on paper first, then check.

1. 45° slew time?

   79.3 s.

2. Time for 0.5 N·m to build 60 N·m·s?

   120 s.

3. What do gyros measure?

   Rotation rates.

4. Why unload wheels?

   They saturate at maximum speed.

Common misconceptions

Myth: Slew time scales with angle.

Actually: It scales with √θ for bang-bang.

Myth: Wheels add momentum forever.

Actually: They saturate.

Myth: Gyros give absolute attitude.

Actually: They give rates; trackers give attitude.

Flashcards

Reaction wheel

Momentum-exchange actuator.

Star tracker

Attitude from stars.

Gyro

Rate sensor.

Bang-bang

Full torque, then full reverse.

Saturation

Wheel at maximum speed.

Safe mode

Protective fallback state.

Self-check quiz

1. 90° slew example...

112.1 s

11 s

1000 s

56 s

112.1 s. $2\sqrt{(\theta/\alpha)}$.

2. H = ...

$I\omega$

$I\alpha$

τt^2

m v

Iω. Momentum.

3. Wheel H example...

60 N·m·s

6

600

0.1

60 N·m·s. 0.1 × 600.

4. Hubble now points with...

one gyro

six gyros

no gyros

thrusters only

one gyro. NASA 2024.

Try this quiz again

How this helps save the planet

NASA reported in June 2024 that Hubble would switch to operating with one gyro, keeping another in reserve, after a gyro began “saturating”—reading the maximum slew rate regardless of actual motion—and repeatedly sent the telescope into safe mode. NASA said most observations would be unaffected, with longer slews, less pointing flexibility and no tracking of moving objects closer than Mars.

The same attitude-control ideas point Earth-observing satellites such as GOES-19 and Landsat at the planet they measure.

On Saving Planets: GOES-19 · Orbital Mechanics

Sources: NASA Science: NASA to change how it points Hubble Space Telescope (June 4, 2024) (https://science.nasa.gov/missions/hubble/nasa-to-change-how-it-points-hubble-space-telescope/)

Free learning resources

- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT course.
- NASA Science: NASA to change how it points Hubble Space Telescope (June 4, 2024) (https://science.nasa.gov/missions/hubble/nasa-to-change-how-it-points-hubble-space-telescope/): NASA article.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Axes, Rotations and Moments
   4 problems
2. Longitudinal Static Stability
   4 problems
3. Trim and Elevator Control
   4 problems
4. Lateral-Directional Stability and Turns
   4 problems
5. Equations of Motion and Dynamic Modes
   4 problems
6. Laplace Transforms and Transfer Functions
   4 problems
7. Second-Order Time Response
   4 problems
8. Feedback and PID Control
   4 problems
9. Stability Analysis: Routh, Root Locus and Margins
   4 problems
10. Spacecraft Attitude Determination and Control
    4 problems

The lab (Flight dynamics and controls lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab runs in a free simulation or a spreadsheet, so there are no physical hazards. Do not build or fly drones or models for this course. Take screen breaks, sit with good posture, and keep cables out of walkways.

Safety reference: OSHA: Walking-working surfaces (slip, trip and fall hazards) (https://www.osha.gov/walking-working-surfaces)

Lab A: Balancing moments and finding the cg

WEEK 1

Goal: Balance masses on a plank in PhET Balancing Act, then compute an aircraft-style center of gravity in a spreadsheet.

$$x_{cg} = \frac{\sum m_i x_i}{\sum m_i}$$

`x_cg=( m_ix_i)/( m_i)`

Center of gravity

Procedure

1. In PhET Balancing Act, place 10 kg at 1 m on one side and find where 5 kg balances it.
2. Record moment = mass × distance on each side.
3. In a spreadsheet, list the Unit 1 masses and distances and compute x_cg.
4. Add 200 kg at 10 m and recompute.

Data table

Case	Σm (kg)	x_cg (m)
Base	1800	4.44
+200 kg at 10 m	2000	5.00

Computed sample values.

Sample calculation

1. 8000/1800 = 4.44 m.
2. 10,000/2000 = 5.00 m.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any browser and spreadsheet work.

PhET: Balancing Act (https://phet.colorado.edu/en/simulations/balancing-act) · NASA Glenn Beginner's Guide: Center of gravity (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/center-of-gravity/)

Planet link: NASA Glenn: in flight, airplanes and rockets rotate about their centers of gravity. NASA Glenn Beginner's Guide: Center of gravity (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/center-of-gravity/)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Static margin and trim sweep

WEEK 3

Goal: Sweep cg position and compute static margin, C_mα and the zero-elevator trim angle.

$$\alpha_{trim} = \frac{C_{m_0}}{C_{L_\alpha} SM}$$

`α_trim=_m_0C_L_αSM`

Trim angle with zero elevator

Procedure

1. Use x_np = 0.35c̄, C_Lα = 5.5 /rad, C_m0 = 0.05 (sample values).
2. Sweep x_cg from 0.20 to 0.40 c̄.
3. Compute SM, C_mα and α_trim.
4. Note what happens at and beyond the neutral point.

Data table

x_cg/c̄	SM	C_mα (/rad)	α_trim (rad)
0.20	0.15	−0.825	0.0606
0.25	0.10	−0.55	0.0909
0.30	0.05	−0.275	0.182

Computed sample values; no stable trim at or aft of the neutral point.

Sample calculation

1. C_mα = −5.5 × 0.15 = −0.825.
2. α = 0.05/0.275 = 0.182 rad.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn Beginner's Guide: Aircraft elevator interactive (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-elevator-interactive/)

Planet link: NASA reports the X-59's stability margin matched predictions in flight. NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/quesst/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

- [ ] **I completed this lab (wet or virtual)**

Lab C: Damped oscillation as a second-order mode

WEEK 6

Goal: Watch damped oscillation in PhET Masses and Springs and compare with the second-order formulas.

$$\omega_n = \sqrt{k/m},\ \zeta = \frac{c}{2\sqrt{km}}$$

ω_n=/m,\ =2

Spring-mass-damper

Procedure

1. In PhET Masses and Springs, hang a mass, add some damping and pull it down.
2. Time several cycles and estimate the period and decay.
3. In a spreadsheet, compute ω_n, ζ, period and overshoot for m = 0.5 kg, k = 20 N/m, c = 1 N·s/m (sample values).
4. Explain how this matches the short-period mode.

Data table

Quantity	Value
ω_n	6.32 rad/s
ζ	0.158
Damped period	1.006 s
Overshoot	60.5%

Computed sample values; simulation readings will differ.

Sample calculation

1. ω_n = √40 = 6.32.
2. ζ = 1/(2√10) = 0.158.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any browser and spreadsheet work.

PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs)

Planet link: NASA says the X-59 has shown no unexpected instability or excessive vibration. NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/quesst/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

☐ **I completed this lab (wet or virtual)**

Lab D: Step responses by time-stepping

WEEK 8

Goal: Simulate first- and second-order step responses with small Euler steps and compare with the formulas.

$$x_{k+1} = x_k + \Delta t\, \dot{x}_k$$

x_k+1=x_k+Δ t x_k

Euler step

Procedure

1. Simulate ẏ = −2y + 5 from y = 0 with Δt = 0.001 s.
2. Simulate ÿ + 2.8ẏ + 16y = 16 from rest.
3. Find y(0.5) for the first and the peak and peak time for the second.
4. Compare with exact formulas.

Data table

Quantity	Exact
First order y(0.5)	1.580
Second order peak	1.309
Peak time	0.838 s
Settling (2%)	2.86 s

Computed values; small steps match closely.

Sample calculation

1. 2.5(1 − e^{-1}) = 1.580.
2. 1 + 0.309 = 1.309.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet or Python works.

MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/)

Planet link: NASA's X-59 team compares flight data with a real-time digital twin. NASA: X-59 aircraft builds momentum through 25th flight (Sept. 4, 2026) (https://www.nasa.gov/blogs/quesst/2026/09/04/nasas-x-59-aircraft-builds-momentum-through-25th-flight/)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Tuning a pitch-attitude autopilot

WEEK 10

Goal: Compare P and PD control of the plant 1/(s(s + 2)) by simulation.

$$s^2 + (2 + K_d)s + K_p = 0$$

s^2+(2+K_d)s+K_p=0

Closed loop with PD

Procedure

1. Simulate θ̈ + 2θ̇ = u with u = K_p(θ_cmd − θ) − K_dθ̇.
2. Try K_p = 16, K_d = 0; then K_p = 2.04; then K_p = 16, K_d = 3.6.
3. Record overshoot and settling time.
4. Choose the best design and explain.

Data table

Design	Overshoot	Settling (s)
K_p = 16	44.4%	4.0
K_p = 2.04	4.6%	4.0
K_p = 16, K_d = 3.6	4.6%	1.43

Computed from second-order formulas.

Sample calculation

1. ζ = 0.25 ⇒ 44.4%; t_s = 4/(0.25 × 4) = 4.0 s.
2. PD: t_s = 4/(0.7 × 4) = 1.43 s.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet or Python works.

MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/)

Planet link: NASA: Dragonfly must fly autonomously at Titan without real-time help from Earth. NASA Science: Dragonfly completes first simulated flight (Oct. 23, 2026) (https://science.nasa.gov/blogs/dragonfly/2026/10/23/nasas-dragonfly-completes-first-simulated-flight/)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Reaction-wheel slew planner

WEEK 13

Goal: Plan bang-bang slews for a spacecraft and check peak rates and wheel momentum.

$$t = 2\sqrt{\theta/\alpha}$$

Bang-bang slew time

Procedure

1. Set I = 1000 kg·m² and τ = 0.5 N·m (sample values).
2. Compute slew times for 30°, 90° and 180°.
3. Compute the peak rate and the momentum the wheels must absorb.
4. Discuss how losing a sensor would change planning.

Data table

Slew	Time (s)
30°	64.7
90°	112.1
180°	158.5

Computed sample values.

Sample calculation

1. t = 2√(0.5236/0.0005) = 64.7 s.
2. t = 2√(3.1416/0.0005) = 158.5 s.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Science: NASA to change how it points Hubble Space Telescope (June 4, 2024) (https://science.nasa.gov/missions/hubble/nasa-to-change-how-it-points-hubble-space-telescope/)

Planet link: NASA: in one-gyro mode Hubble needs more time to slew and lock onto targets. NASA Science: NASA to change how it points Hubble Space Telescope (June 4, 2024) (https://science.nasa.gov/

missions/hubble/nasa-to-change-how-it-points-hubble-space-telescope/)

☐ **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. Masses 600 kg at 3 m and 400 kg at 8 m. Find the cg.

    1. (1800 + 3200)/1000.

    Answer: 5.0 m.

2. $x_{np} = 0.40\bar{c}$, $x_{cg} = 0.28\bar{c}$, $C_{L\alpha} = 5$ /rad. Find SM and $C_{m\alpha}$.

    1. SM = 0.12.
    2. −5 × 0.12.

    Answer: SM = 0.12; $C_{m\alpha} = -0.60$ /rad.

3. $C_{m0} = 0.04$, $C_{m\alpha} = -0.6$, $C_{m\delta e} = -1.0$. Find δ_e to trim at $\alpha = 0.05$ rad.

    1. $0.04 - 0.03 - \delta_e = 0$.

    Answer: $\delta_e = 0.01$ rad (0.57°).

4. Bank angle for a 1500 m radius turn at 80 m/s?

    1. $\tan\varphi = 6400/14{,}710$.

    Answer: 23.5°.

5. Find ω_n and ζ for $s^2 + 3s + 25$.

    1. $\omega_n = 5$; $\zeta = 3/10$.

    Answer: $\omega_n = 5$ rad/s; $\zeta = 0.3$.

Final practice set (Units 1–10)

1. Step response of $4/(s + 4)$ at $t = 0.25$ s?

    1. $1 - e^{-1}$.

    Answer: 0.632.

2. Overshoot and settling time for ω_n = 5, ζ = 0.3?

    1. exp(−0.3π/√0.91).
    2. 4/1.5.

    Answer: 37.2%; 2.67 s.

3. K on 1/(s(s + 4)) for ζ = 0.5?

    1. 2ζω_n = 4 ⇒ ω_n = 4.

    Answer: K = 16.

4. Max K for $s^3 + 3s^2 + 2s + K$?

    1. 3 × 2 > K.

    Answer: K < 6.

5. Slew time for 60° with α = 0.001 rad/s²?

    1. 2√(1.0472/0.001).

    Answer: 64.7 s.

6. Momentum of a 0.05 kg·m² wheel at 400 rad/s?

    1. 0.05 × 400.

    Answer: 20 N·m·s.

Formula sheet

$$x_{cg} = \sum m_i x_i / \sum m_i,\ M = Fd$$

$$SM = (x_{np} - x_{cg})/\bar{c},\ C_{m_\alpha} = -C_{L_\alpha} SM$$

$$\delta_e = -(C_{m_0} + C_{m_\alpha}\alpha)/C_{m_{\delta_e}}$$

$$\tan\phi = V^2/(gR),\ n = 1/\cos\phi,\ \tau_r = 1/|L_p|$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2,\ \omega_{ph} \approx \sqrt{2}g/V$$

$$y = K(1 - e^{-t/\tau}),\ y(\infty) = G(0)$$

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}},\ t_p = \pi/\omega_d,\ t_s \approx 4/(\zeta\omega_n)$$

$$u = K_p e + K_i \int e + K_d \dot{e},\ T = G/(1 + GH)$$

$$PM = 180^\circ + \angle G(j\omega_c),\ \zeta \approx PM/100$$

$$\tau = I\alpha,\ H = I\omega,\ t = 2\sqrt{\theta/\alpha}$$

Constants

Constant	Value
Standard gravity	$g_0 = 9.80665\ m/s^2$
1 radian	57.2958°
1 ft	0.3048 m
Settling time rule (textbook)	t_s ≈ 4/(ζω_n) for 2% settling
First-order step	63.2% of final value at t = τ
Sea-level air density (standard atmosphere)	$1.225\ kg/m^3$

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, NASA guides and simulations alongside the course.

- MIT OpenCourseWare 16.333 Aircraft Stability and Control (Fall 2004) (https://ocw.mit.edu/courses/16-333-aircraft-stability-and-control-fall-2004/): Free MIT stability and control lectures.
- MIT OpenCourseWare 16.06 Principles of Automatic Control (Fall 2012) (https://ocw.mit.edu/courses/16-06-principles-of-automatic-control-fall-2012/): Free MIT control lectures and problems.
- MIT OpenCourseWare 16.30 Feedback Control Systems (Fall 2010) (https://ocw.mit.edu/courses/16-30-feedback-control-systems-fall-2010/): Free MIT feedback control course.
- NASA Glenn Beginner's Guide: Aircraft rotations (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aircraft-rotations/): Free NASA explainer.
- NASA Glenn Beginner's Guide: Center of gravity (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/center-of-gravity/): Free NASA explainer.
- PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs): Damped oscillation interactively.

Glossary

Aileron
Roll control surface.

Center of gravity
Average location of mass.

Damping ratio
ζ, how fast oscillations decay.

Dutch roll
Coupled yaw–roll oscillation.

Elevator
Pitch control surface.

Gain margin
Gain increase to instability.

Gyroscope
Sensor that measures rotation rate.

Natural frequency
ω_n of an undamped mode.

Neutral point
cg location with neutral pitch stability.

Overshoot
Peak above the final value.

PID controller
Proportional-integral-derivative control.

Phase margin
Extra phase lag to instability.

Phugoid
Slow speed–altitude mode.

Reaction wheel
Spinning wheel for attitude control.

Root locus
Closed-loop pole paths with gain.

Rudder
Yaw control surface.

Short period
Fast pitch mode.

Static margin
Distance from cg to neutral point over chord.

Transfer function
Laplace-domain output over input.

Trim
Zero net moment.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Draws axes and block diagrams, picks the right model, checks poles, damping and margins, and explains handling and safety trades in plain words.
3 · Proficient	Solves the practice problems with occasional slips in signs or units.
2 · Developing	Follows worked examples but confuses static and dynamic stability or ω_n and ω_d.
1 · Beginning	Plugs numbers into formulas without a diagram; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Propulsion AERO 410
- Spacecraft Design AERO 430
- Heat Transfer AERO 440
- All courses