Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 2, Spring · 🧱 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester sophomore engineering dynamics course. It is modeled on the topics and depth of MIT 2.003 Engineering Dynamics and MIT 8.01 Classical Mechanics (both on MIT OpenCourseWare), Purdue ME 274 and Georgia Tech dynamics, and on the free OpenStax University Physics Volume 1, which we link throughout. None of those authors or schools endorses this page.
>
> What we add: every unit ties to real clean-energy, Earth-observation or planetary-defense work with a cited public source (electric cars, the space station, GRACE-FO, pumped storage, NASA's DART asteroid test, rockets, wind and hydro turbines, earthquake-safe buildings), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a cheap home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Dynamics is the physics of every machine that moves energy around. Electric cars turn battery energy into motion and win some of it back, hydro and wind turbines are spinning rigid bodies, pumped-storage plants bank energy by lifting water, and satellites that watch Earth's water and climate stay up because they keep falling around it.

It is also how we protect the planet: NASA's DART mission used momentum to change an asteroid's orbit, and engineers use vibration theory to keep buildings standing in earthquakes. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (ENGR 212).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Statics: free-body diagrams, moments and centroids.
- Physics I: Newton’s laws, energy and momentum.
- Calculus I and II: derivatives, integrals and simple differential equations.
- Differential Equations helps with Unit 10 (vibration) but is not required; the needed solutions are given.

How to study

- Start with kinematics: what is known about position, velocity and acceleration, and in which coordinates?
- Draw a free-body diagram and a kinetic diagram (ma and Iα) side by side.
- Choose the method that fits: F = ma for accelerations, work-energy for speeds between two positions, impulse-momentum for forces over time and impacts.
- Solve with symbols first, then put in SI numbers, and check units and limiting cases.
- Do every practice problem on paper before opening the answer.
- Retake the quizzes a few days later; spaced practice beats one long session.

What you need

- A computer or tablet for the free PhET simulations (they run in a browser).
- A smartphone with a slow-motion video camera or a free sensor app (accelerometer, stopwatch).

- A board or long book to make a ramp, a tape measure, and a ball, a can of soup, an empty can and a ring or hoop to roll.
- String, a small weight (a nut or washers) and a ruler for a pendulum.
- A few rubber bands or a soft spring and small masses (coins in a bag) for a spring-mass oscillator.
- Spreadsheet software for graphs and fits.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Kinematics of Particles in a Straight Line	Position, velocity, acceleration; constant acceleration	OpenStax UP1 3.1–3.4
2	Kinematics of Particles in a Straight Line	Variable acceleration, a(x) and a(v); Lab A	UP1 3.6
3	Curvilinear Motion: Projectiles, Normal–Tangential and Polar Coordinates	Projectiles, normal–tangential, polar, relative motion; Lab B	UP1 4.1–4.5
4	Kinetics of Particles: Newton’s Second Law	Newton’s second law, friction, connected bodies	UP1 5.1–6.2
5	Kinetics of Particles: Newton’s Second Law	Curves, banking, orbits	UP1 6.3, 13.4
6	Work, Energy and Power	Work, energy, conservation, power and efficiency	UP1 7.1–8.3
7	Impulse, Momentum and Impact	Impulse, momentum, impact, angular momentum; Lab C; midterm practice	UP1 9.1–9.4
8	Systems of Particles and Variable Mass: Jets and Rockets	Systems of particles, jets, rockets	UP1 9.5–9.7
9	Planar Kinematics of Rigid Bodies	Rigid-body rotation, gears, relative velocity	UP1 10.1–10.3
10	Planar Kinematics of Rigid Bodies	Instantaneous centers, relative acceleration, rolling	UP1 11.1
11	Planar Kinetics of Rigid Bodies: Force, Mass and Acceleration	Moments of inertia, plane motion equations, rolling; Lab D	UP1 10.4–10.7
12	Rigid Bodies: Energy and Momentum Methods	Rigid-body energy, power, angular momentum	UP1 10.8, 11.2–11.3

Week	Unit	Topics	Reading
13	Mechanical Vibrations	Free vibration, pendulums, energy method; Lab E	UP1 15.1–15.4
14	Mechanical Vibrations	Damped and forced vibration, resonance; Lab F	UP1 15.5–15.6
15	Review	Review and final practice set	All units

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Kinematics of Particles in a Straight Line

Position, velocity and acceleration as derivatives, constant acceleration, acceleration given as a function of time, speed or position, and distance versus displacement.

Learning objectives

By the end of this unit you can:

- Relate x, v and a by differentiation and integration.
- Use the constant-acceleration equations correctly.
- Solve a(x) problems with v dv = a dx.
- Tell total distance from displacement.

Key concepts

- Velocity is v = dx/dt and acceleration is a = dv/dt = d²x/dt². Integrating goes the other way.
- Only when a is constant do $v = v_0 + at$, $x = x_0 + v_0t + \frac{1}{2}at^2$ and $v^2 = v_0^2 + 2a(x - x_0)$ hold.
- If a is given as a function of position, use a dx = v dv; if a function of speed, use dt = dv/a(v).
- Distance traveled counts every leg; split the motion where v = 0 and add the lengths.
- Signs matter: pick a positive direction and keep it; slowing down means a and v have opposite signs.

Key equations

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt}$$

`v=dx/dt, a=dv/dt`

Definitions

$$v = v_0 + at, \quad x = x_0 + v_0t + \tfrac{1}{2}at^2$$

`v=v_0+at, x=x_0+v_0t+12at^2`

Constant acceleration

$$v^2 = v_0^2 + 2a(x - x_0)$$

`v^2=v_0^2+2a(x-x_0)`

Constant acceleration, no time

$$a\,dx = v\,dv$$

a dx=v dv

Acceleration as a function of position

Worked examples, step by step

Example 1.1 A particle moves with $x = t^3 - 6t^2 + 9t$ (m, s). Find when it stops, v and a at t = 2 s, and the distance traveled from 0 to 4 s.

1. $v = 3t^2 - 12t + 9 = 0$ at t = 1 s and t = 3 s; $a = 6t - 12$.
2. At t = 2 s: v = −3 m/s, a = 0.
3. x(0) = 0, x(1) = 4, x(3) = 0, x(4) = 4, so legs of 4 + 4 + 4 m.

Answer: Stops at 1 s and 3 s; v = −3 m/s, a = 0 at 2 s; 12 m traveled (displacement 4 m).

Example 1.2 A car starts from rest and accelerates at 3 m/s² for 8 s. Find its speed and distance.

1. $v = at = 3 \times 8$.
2. $x = \tfrac{1}{2}at^2 = \tfrac{1}{2} \times 3 \times 64$.

Answer: 24 m/s and 96 m.

Example 1.3 A car at 30 m/s brakes at 6 m/s². Find the stopping distance and time.

1. $x = v_0^2/(2a) = 900/12$.
2. $t = v_0/a$.

Answer: 75 m in 5 s.

Practice problems

Work each on paper first, then check.

1. A ball is thrown straight up at 15 m/s. Find its maximum height and the time to reach it.

   $h = v^2/(2g) = 11.47$ m; $t = v/g = 1.529$ s.

2. A particle starts at rest at x = 0 with $a = 2x$ (m/s²). Find v at x = 3 m.

   $v\,dv = 2x\,dx$ gives $v^2 = 2x^2$, $v = \sqrt{2} \times 3 = 4.24$ m/s.

3. Sample values: an electric car goes from 0 to 27 m/s (about 100 km/h) in 6 s. Find its average acceleration.

a = 27/6 = 4.5 m/s² (sample values).

4. A stone dropped into a well hits the water after 2.5 s. How deep is the water surface (ignore sound travel time)?

   h = ½gt² = 30.7 m.

Common misconceptions

Myth: Zero velocity means zero acceleration.

Actually: At the top of a throw v = 0 but a = −g.

Myth: Distance and displacement are the same.

Actually: Not when the particle reverses; split at v = 0.

Myth: The constant-acceleration formulas always work.

Actually: Only for constant a; otherwise integrate.

Flashcards

Velocity

v = dx/dt.

Acceleration

a = dv/dt.

Constant a

$v = v_0 + at$; $x = v_0t + \frac{1}{2}at^2$.

No-time equation

$v^2 = v_0^2 + 2a\Delta x$.

a(x)

a dx = v dv.

Distance

Split the motion where v = 0.

Self-check quiz

1. Acceleration is the derivative of...

position

velocity

distance

force

velocity. a = dv/dt.

2. At the top of a vertical throw, the acceleration is...

0

g downward

g upward

maximum speed

g downward. Gravity never switches off.

3. Doubling speed multiplies braking distance (same a) by...

2

4

√2

1

4. $x = v^2/(2a)$.

4. If a is given as a function of x, use...

v = at

a dx = v dv

x = vt

F = ma only

a dx = v dv. Chain rule.

Try this quiz again

How this helps save the planet

Every speed change costs energy, and where that energy comes from matters. FuelEconomy.gov (DOE/EPA) says electric vehicles convert over 77% of the electrical energy from the grid to power at the wheels, while conventional gasoline vehicles convert only about 12% to 30% of the energy in gasoline. The kinematics in this unit, how fast a car speeds up and how far it brakes, is the starting point for the drive-cycle models used to rate vehicle efficiency.

On Saving Planets: Physics I

Sources: FuelEconomy.gov (DOE/EPA): All-electric vehicles (https://www.fueleconomy.gov/feg/evtech.shtml) · OpenStax UP1, 3.4 Motion with constant acceleration (https://openstax.org/books/university-physics-volume-1/pages/3-4-motion-with-constant-acceleration)

Free learning resources

- OpenStax UP1, 3.1 Position, displacement and average velocity (https://openstax.org/books/university-physics-volume-1/pages/3-1-position-displacement-and-average-velocity): Free textbook section.
- OpenStax UP1, 3.4 Motion with constant acceleration (https://openstax.org/books/university-physics-volume-1/pages/3-4-motion-with-constant-acceleration): Constant-a equations.
- MIT OpenCourseWare 2.003SC Engineering Dynamics (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/): MIT lectures and problems.

Watch the lecture

Kinematics: describing the motion of particles and rigid bodies (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/kinematics-describing-the-motion-of-particles-and-rigid-bodies/) (in English)

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☐ **I finished this unit**

UNIT 2 · WEEK 3

Curvilinear Motion: Projectiles, Normal–Tangential and Polar Coordinates

Motion in a plane: rectangular components and projectiles, normal and tangential components on curved paths, polar coordinates for rotating arms, and relative motion.

Learning objectives

By the end of this unit you can:

- Solve projectile problems with independent x and y motions.
- Find tangential and normal acceleration on a curved path.
- Use polar components for motion along a rotating arm.
- Add relative velocities.

Key concepts

- A projectile has a_x = 0 and a_y = −g (no air drag); time links the two directions.
- On a curved path, a_t = dv/dt changes the speed and a_n = v²/ρ points toward the center of curvature and changes the direction.
- In polar coordinates, a_r = r̈ − rθ̇² and a_θ = rθ̈ + 2ṙθ̇; the 2ṙθ̇ term is the Coriolis part.
- Relative motion: v_A = v_B + v_A/B, and the same for accelerations (for translating frames).
- A satellite in a circular orbit has only normal acceleration: it falls toward Earth at v²/r while moving sideways fast enough to miss it.

Key equations

$$x = v_0 \cos\theta \, t, \quad y = v_0 \sin\theta \, t - \tfrac{1}{2} g t^2$$

`x=v_0θ t, y=v_0θ t-12gt^2`

Projectile

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

`R=(v_0^22θ)/(g)`

Range on level ground

$$a_t = \dot{v}, \quad a_n = \frac{v^2}{\rho}$$

`a_t= v, a_n=(v^2)/(ρ)`

Normal–tangential

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

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a_r= r-rθ^2, a_θ=rθ+2 rθ
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Polar components

Worked examples, step by step

Example 2.1 A ball is launched at 20 m/s and 30° above level ground. Find the range, maximum height and flight time.

1. v_x = 17.32 m/s, v_y = 10 m/s.
2. Time = 2v_y/g; height = v_y²/(2g); R = v_0^2 sin 60°/g.

Answer: R = 35.3 m, h = 5.10 m, t = 2.04 s.

Example 2.2 A car at 25 m/s on a curve of radius 100 m is slowing at 2 m/s². Find its total acceleration.

1. a_n = 25²/100 = 6.25 m/s²; a_t = −2 m/s².
2. |a| = √(6.25² + 2²).

Answer: 6.56 m/s².

Example 2.3 A collar slides out along an arm at 0.3 m/s (constant) while the arm turns at a constant 2 rad/s. Find its acceleration when r = 0.5 m.

1. a_r = 0 − 0.5 × 2² = −2.0 m/s².
2. a_θ = 0 + 2 × 0.3 × 2 = 1.2 m/s².

Answer: |a| = 2.33 m/s².

Practice problems

Work each on paper first, then check.

1. NASA says the station orbits Earth about every 90 minutes. Using a sample orbit radius of 6,791 km (420 km altitude), find its speed and normal acceleration.

$v = 2\pi r/T = 7{,}902$ m/s (NASA quotes about 5 miles per second, 8.0 km/s); $a_n = v^2/r = 9.19$ m/s^2.

2. A boat heads north at 4 m/s in a current flowing east at 3 m/s. Find its velocity over the ground.

   5 m/s at 36.9° east of north.

3. Which launch angle gives the longest range on level ground (no drag)?

   45°, where $\sin 2\theta = 1$.

4. Find the normal acceleration of a car at 15 m/s on a curve of radius 50 m.

   $a_n = 225/50 = 4.5$ m/s^2.

Common misconceptions

Myth: A body moving at constant speed has no acceleration.
Actually: On a curve it has $a_n = v^2/\rho$.

Myth: Horizontal speed of a projectile drops during flight.
Actually: Without drag, v_x stays constant.

Myth: Astronauts float because there is no gravity in orbit.
Actually: Gravity there is still strong: $(6371/6791)^2 = 88\%$ of surface gravity at a sample 420 km altitude. They float because they are in free fall with the station.

Flashcards

Projectile

$a_x = 0$, $a_y = -g$.

Range

$R = v_0^2 \sin 2\theta/g$.

Tangential

$a_t = dv/dt$.

Normal

$a_n = v^2/\rho$.

Polar

$a_r = \ddot{r} - r\dot{\theta}^2$, $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$.

Relative velocity

$v_A = v_B + v_{A/B}$.

Self-check quiz

1. For a projectile without drag, a_x is...

g

0

−g

v^2/r

0. Only gravity acts, vertically.

2. Normal acceleration on a curve is…

dv/dt

v^2/ρ

$r\ddot{\theta}$

0

v^2/ρ. Changes direction.

3. The term $2\dot{r}\dot{\theta}$ in polar coordinates is the…

centripetal term

Coriolis term

drag term

tangential term

Coriolis term. Coriolis.

4. Doubling speed on the same curve multiplies a_n by…

2

4

½

1

4. v^2.

Try this quiz again

How this helps save the planet

The space station shows curvilinear motion at its most extreme. NASA says it travels at five miles per second, orbiting Earth about every 90 minutes and making 16 orbits in 24 hours, and that its path takes it over 90 percent of Earth's population; astronauts have taken millions of images of the planet below. The same normal-acceleration balance keeps every Earth-observing satellite in orbit.

On Saving Planets: Intro to Aerospace Engineering

Sources: NASA: International Space Station facts and figures (https://www.nasa.gov/international-space-station/space-station-facts-and-figures/) · OpenStax UP1, 4.4 Uniform circular motion (https://openstax.org/books/university-physics-volume-1/pages/4-4-uniform-circular-motion)

Free learning resources

- OpenStax UP1, 4.3 Projectile motion (https://openstax.org/books/university-physics-volume-1/pages/4-3-projectile-motion): Free textbook section.
- OpenStax UP1, 4.4 Uniform circular motion (https://openstax.org/books/university-physics-volume-1/pages/4-4-uniform-circular-motion): Circular motion.
- PhET: Projectile Motion (https://phet.colorado.edu/en/simulations/projectile-motion): Interactive.
- PhET: Ladybug Motion 2D (https://phet.colorado.edu/en/simulations/ladybug-motion-2d): Velocity and acceleration vectors.

Watch the lecture

Definition of normal and tangential coordinates (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/definition-of-normal-and-tangential-coordinates/) (in English)

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☐ **I finished this unit**

Kinetics of Particles: Newton’s Second Law

ΣF = ma in rectangular, normal–tangential and polar coordinates, friction and connected bodies, banked curves, and orbits.

Learning objectives

By the end of this unit you can:

- Draw free-body and kinetic diagrams.
- Apply ΣF = ma with friction and connected bodies.
- Solve curve and banking problems with normal–tangential components.
- Find circular-orbit speeds.

Key concepts

- Newton’s second law: ΣF = ma, in an inertial frame. Write it in the coordinates that match the motion.
- Kinetic friction while sliding is μ_kN, opposite to the sliding direction.
- Connected bodies share accelerations through constraints (a rope of fixed length, a pulley); write one equation per body.
- On a curve, the net normal force supplies mv²/ρ; banking lets the normal force provide it without friction.
- For a circular orbit, gravity supplies the normal force: GMm/r² = mv²/r, so v = √(GM/r).

Key equations

$$\sum \vec{F} = m\vec{a}$$

`F=m a`

Newton’s second law

$$\sum F_n = \frac{mv^2}{\rho}, \quad \sum F_t = m\dot{v}$$

`F_n=(mv^2)/(ρ), F_t=m v`

Normal–tangential

$$v = \sqrt{g\rho \tan\theta}$$

Banked curve, no friction

$$v = \sqrt{GM/r}$$

Circular orbit

Worked examples, step by step

Example 3.1 A 1200 kg car skids to a stop from 25 m/s with μ_k = 0.7. Find the deceleration and stopping distance.

1. a = μ_k g = 6.87 m/s² (mass cancels).
2. x = v²/(2a) = 625/13.73.

Answer: 6.87 m/s²; 45.5 m.

Example 3.2 Masses of 5 kg and 3 kg hang on either side of a light, frictionless pulley. Find the acceleration and rope tension.

1. a = (5 − 3)g/(5 + 3).
2. T = $2m_1m_2g/(m_1 + m_2)$.

Answer: a = 2.45 m/s²; T = 36.8 N.

Example 3.3 A curve of radius 80 m is banked at 15°. At what speed is no friction needed?

1. v = √(gρ tan 15°) = √(9.81 × 80 × 0.268).

Answer: 14.5 m/s (52 km/h).

Practice problems

Work each on paper first, then check.

1. A 10 kg block is pushed with 50 N along a floor with μ_k = 0.3. Find its acceleration.

   a = (50 − 29.4)/10 = 2.06 m/s².

2. A 70 kg person stands in an elevator accelerating upward at 1.5 m/s². Find the floor force.

   N = m(g + a) = 792 N.

3. Find the circular-orbit speed at a sample radius of 6,791 km (GM = 3.986 × 10¹⁴ m³/s²).

$v = \sqrt{(GM/r)} = 7{,}661$ m/s.

4. Find the top speed on a flat curve of radius 50 m with $\mu_s = 0.8$.

   $v = \sqrt{(\mu_s g r)} = 19.8$ m/s.

Common misconceptions

Myth: A moving object needs a net force to keep moving.

Actually: Only to change its velocity.

Myth: ma is a force on the free-body diagram.

Actually: ma is the result, drawn on the kinetic diagram.

Myth: Centrifugal force pushes you out on a curve.

Actually: In an inertial frame, the net inward force makes you turn; nothing pushes outward.

Flashcards

Second law

$\Sigma F = ma$.

Kinetic diagram

Shows ma (and later $I\alpha$).

Kinetic friction

$\mu_k N$, opposite sliding.

Atwood machine

$a = (m_1 - m_2)g/(m_1 + m_2)$.

Banked curve

$v = \sqrt{(g\rho \tan \theta)}$.

Circular orbit

$v = \sqrt{(GM/r)}$.

Self-check quiz

1. Newton’s second law holds in…

any frame

an inertial frame

only on Earth

only for particles at rest

an inertial frame. Non-inertial frames need extra terms.

2. On a skidding car, μ_k g is the…

speed

deceleration

force

distance

deceleration. a = μ_kN/m.

3. A circular orbit at a larger radius is…

faster

slower

the same speed

impossible

slower. v = √(GM/r).

4. What supplies the normal acceleration on a frictionless banked curve?

gravity alone

the normal force component

engine thrust

drag

the normal force component. N sin θ.

Try this quiz again

How this helps save the planet

GRACE-FO weighs Earth's water with Newton's second law. NASA JPL explains that the twin satellites fly about 220 km apart; areas of slightly stronger gravity pull the lead satellite ahead first, and a microwave ranging system measures the tiny change in distance. An accelerometer at each satellite's center of mass measures non-gravitational accelerations such as atmospheric drag so that only gravity is counted. The result is monthly maps showing how water moves in aquifers, lakes, ice sheets and oceans.

On Saving Planets: Earth Observation

Sources: NASA JPL: GRACE-FO mission overview (https://gracefo.jpl.nasa.gov/mission/overview/) · OpenStax UP1, 6.3 Centripetal force (https://openstax.org/books/university-physics-volume-1/pages/6-3-centripetal-force)

Free learning resources

- OpenStax UP1, 6.3 Centripetal force (https://openstax.org/books/university-physics-volume-1/pages/6-3-centripetal-force): Free textbook section.
- OpenStax UP1, 6.2 Friction (https://openstax.org/books/university-physics-volume-1/pages/6-2-friction): Friction.
- MIT OpenCourseWare 8.01SC Classical Mechanics (https://ocw.mit.edu/courses/8-01sc-classical-mechanics-fall-2016/): MIT lectures.

Watch the lecture

Lecture 2: Newton's Laws and Describing the Kinematics of Particles (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/lecture-2-newtons-laws-and-describing-the-kinematics-of-particles-1/) (in English)

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☐ **I finished this unit**

Work, Energy and Power

Work of forces (gravity, springs, friction), the work–energy principle, conservative forces and potential energy, conservation of energy, power and efficiency.

Learning objectives

By the end of this unit you can:

- Compute the work of gravity, springs and friction.
- Use the work–energy principle to find speeds between two positions.
- Use conservation of mechanical energy when only conservative forces do work.
- Compute power and efficiency.

Key concepts

- Work is U = ∫F·dr. Gravity does −mgΔh, a spring does −½k(x_2^2 − x_1^2), and kinetic friction does −μ_kN × distance.
- Work–energy principle: T_1 + $\Sigma U_{1\to 2}$ = T_2, with kinetic energy T = ½mv².
- For conservative forces, work equals the loss of potential energy, so T_1 + V_1 = T_2 + V_2 when only they do work.
- Power is the rate of work: P = F·v. Efficiency is output power divided by input power.
- Energy methods give speeds directly without finding accelerations, which is why they are so useful.

Key equations

$$T_1 + \sum U_{1\to 2} = T_2, \quad T = \tfrac{1}{2}mv^2$$

`T_1+ U_12=T_2, T=12mv^2`

Work–energy principle

$$V_g = mgh, \quad V_e = \tfrac{1}{2}kx^2$$

`V_g=mgh, V_e=12kx^2`

Potential energies

$$P = \vec{F}\cdot\vec{v}, \quad \eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

`P= F· v, =_outP_in`

Power and efficiency

Worked examples, step by step

Example 4.1 A 2 kg block slides from rest down a smooth ramp, dropping 3 m. Find its speed at the bottom.

1. $mgh = \tfrac{1}{2}mv^2$.
2. $v = \sqrt{(2gh)}$.

Answer: 7.67 m/s (mass does not matter).

Example 4.2 A spring (k = 500 N/m) compressed 0.2 m launches a 0.5 kg block on a smooth floor. Find the launch speed.

1. $\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2$.
2. $v = x\sqrt{(k/m)}$.

Answer: 6.32 m/s.

Example 4.3 A 1500 kg car climbs a 5% grade at a steady 20 m/s. Ignoring drag and rolling losses, what power lifts it?

1. $\theta = \tan^{-1} 0.05 = 2.86°$.
2. $P = mg \sin\theta \times v$.

Answer: 14.7 kW.

Practice problems

Work each on paper first, then check.

1. Sample values: a pumped-storage plant raises 1 million m^3 of water by 300 m. How much energy does it store?

   $E = mgh = 10^9 \times 9.81 \times 300 = 2.94 \times 10^{12}$ J ≈ 818 MWh (sample values).

2. A motor with 85% efficiency delivers 10 kW. What power does it draw?

   P_in = 10/0.85 = 11.8 kW.

3. A 20 kg box is pushed 5 m across a floor with $\mu_k = 0.4$. How much work does friction do?

   $U = -0.4 \times 196.2 \times 5 = -392$ J.

4. A pendulum bob is released 0.5 m above its lowest point. Find its speed there.

   $v = \sqrt{(2g \times 0.5)} = 3.13$ m/s.

Common misconceptions

Myth: Work is force times distance in every case.

Actually: Only the component along the motion does work.

Myth: The normal force does work on a sliding block.

Actually: It is perpendicular to the motion, so its work is zero.

Myth: Energy is lost to friction.

Actually: It is converted to heat; total energy is conserved, mechanical energy is not.

Flashcards

Work

$U = \int F \cdot dr$.

Kinetic energy

$T = \frac{1}{2}mv^2$.

Work–energy

$T_1 + \Sigma U = T_2$.

Potential energy

mgh; $\frac{1}{2}kx^2$.

Power

$P = F \cdot v$.

Efficiency

P_out/P_in.

Self-check quiz

1. The SI unit of work is the…

watt

joule

newton

pascal

joule. N·m = J.

2. Doubling speed multiplies kinetic energy by…

2

4

8

√2

4. ½mv².

3. Mechanical energy is conserved when…

friction acts

only conservative forces do work

speed is constant

always

only conservative forces do work. Definition.

4. Power equals…

F·v

F/v

mv

mgh

F·v. Rate of work.

Try this quiz again

How this helps save the planet

Pumped storage banks clean electricity as height. The U.S. Department of Energy describes pumped storage hydropower as two reservoirs at different elevations that generate power as water flows down through a turbine and use power to pump it back up, acting like a giant battery. DOE says it accounts for 88% of all utility-scale energy storage in the United States, at 43 plants. The stored energy is simply mgh, as in the sample practice problem.

On Saving Planets: Physics II

Sources: U.S. DOE: Pumped storage hydropower (https://www.energy.gov/eere/water/pumped-storage-hydropower) · OpenStax UP1, 8.3 Conservation of energy (https://openstax.org/books/university-physics-volume-1/pages/8-3-conservation-of-energy)

Free learning resources

- OpenStax UP1, 7.3 Work-energy theorem (https://openstax.org/books/university-physics-volume-1/pages/7-3-work-energy-theorem): Free textbook section.
- OpenStax UP1, 8.3 Conservation of energy (https://openstax.org/books/university-physics-volume-1/pages/8-3-conservation-of-energy): Energy conservation.
- PhET: Energy Skate Park (https://phet.colorado.edu/en/simulations/energy-skate-park): Interactive.

Watch the lecture

Example 2b-work done equals the change in kinetic energy (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/example-2b-work-done-equals-the-change-in-kinetic-energy/) (in English)

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☐ **I finished this unit**

Impulse, Momentum and Impact

Linear impulse and momentum, conservation of momentum, direct central impact and the coefficient of restitution, and angular momentum of a particle.

Learning objectives

By the end of this unit you can:

- Use the impulse–momentum principle to find average forces.
- Apply conservation of momentum to collisions and recoil.
- Solve impacts with the coefficient of restitution.
- Use conservation of angular momentum for central-force motion.

Key concepts

- Linear momentum is L = mv. Impulse–momentum: $mv_1 + \int F\,dt = mv_2$.
- If no external impulse acts on a system, its total momentum is conserved, even when kinetic energy is not.
- The coefficient of restitution $e = (v_2' - v_1')/(v_1 - v_2)$ ranges from 0 (perfectly plastic) to 1 (perfectly elastic).
- Angular momentum about O is H_O = r × mv. If all forces pass through O (a central force like gravity), H_O is conserved.
- In orbit, conservation of angular momentum means a satellite moves fastest at periapsis: $r_1v_1 = r_2v_2$ at the apsides.

Key equations

$$m\vec{v}_1 + \int \vec{F}\,dt = m\vec{v}_2$$

```
m v_1+ F dt=m v_2
```

Impulse–momentum

$$\sum m\vec{v}_1 = \sum m\vec{v}_2$$

```
m v_1= m v_2
```

Conservation of momentum

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

```
e=(v_2'-v_1')/(v_1-v_2)
```

Coefficient of restitution

$$\vec{H}_O = \vec{r} \times m\vec{v}$$

H_0= r× m v

Angular momentum

Worked examples, step by step

Example 5.1 A 0.15 kg ball arrives at 40 m/s and leaves the bat at 30 m/s the other way after 2 ms of contact. Find the impulse and average force.

1. Impulse = m(v_2 − v_1) = 0.15 × 70.
2. F = impulse/Δt.

Answer: 10.5 N·s; 5,250 N.

Example 5.2 A 1500 kg car at 20 m/s hits a stopped 1000 kg car and they lock together. Find the common speed and kinetic energy lost.

1. 1500 × 20 = 2500v.
2. T_1 = 300 kJ; T_2 = ½ × 2500 × 12^2.

Answer: 12 m/s; 120 kJ (40%) lost.

Example 5.3 A 2 kg ball at 5 m/s hits a 3 kg ball at rest head-on with e = 0.8. Find both final velocities.

1. Momentum: 10 = 2v_1' + 3v_2'.
2. Restitution: v_2' − v_1' = 0.8 × 5 = 4.

Answer: v_1' = −0.4 m/s (bounces back), v_2' = 3.6 m/s.

Practice problems

Work each on paper first, then check.

1. NASA reports DART shortened Dimorphos's 11 h 55 min orbit to 11 h 23 min. What fraction of the orbit period is that?

32 min/715 min = 4.48%.

2. NASA gives DART's impact speed as about 22,530 km/h. Convert to m/s.

   22,530/3.6 = 6,258 m/s.

3. A satellite moves at 7.5 km/s at 7,000 km from Earth's center (periapsis). Find its speed at 8,000 km (apoapsis).

   $r_1v_1 = r_2v_2$: v_2 = 6.56 km/s.

4. A 60 kg skater at rest throws a 2 kg ball forward at 10 m/s. Find her recoil speed.

   60v = 2 × 10, v = 0.333 m/s backward.

Common misconceptions

Myth: Momentum and kinetic energy are both conserved in every collision.
Actually: Momentum is; kinetic energy only if e = 1.

Myth: A bigger force always means a bigger impulse.
Actually: Impulse is force × time; a long, gentle push can match a short, hard one.

Myth: Restitution depends only on the material.
Actually: It also depends on speed, shape and temperature.

Flashcards

Momentum

L = mv.

Impulse

∫F dt = Δ(mv).

Conservation

No external impulse: Σmv constant.

Restitution

e from 0 to 1.

Angular momentum

H_O = r × mv.

Orbit apsides

$r_1v_1 = r_2v_2$.

Self-check quiz

1. Impulse equals the change in...

energy

momentum

speed

force

momentum. Impulse–momentum.

2. For a perfectly plastic impact, e =

0

0.5

1

∞

0. They move together.

3. A central force conserves…

linear momentum

angular momentum about the center

speed

nothing

angular momentum about the center. No moment about O.

4. Air bags reduce injury by…

reducing impulse

increasing stopping time

adding mass

adding speed

increasing stopping time. Same impulse over more time.

Try this quiz again

How this helps save the planet

In 2022, NASA proved momentum can defend Earth. NASA reports that the DART spacecraft hit the asteroid Dimorphos at roughly 22,530 km/h and changed its orbit around Didymos by 32 minutes, from 11 hours 55 minutes to 11 hours 23 minutes, more than 25 times NASA's minimum success benchmark. NASA adds that the recoil from ejected debris substantially enhanced DART's push, like air streaming out of a balloon. Neither asteroid posed any hazard to Earth.

On Saving Planets: Planetary Defense

Sources: NASA: DART impact changed asteroid's motion in space (https://www.nasa.gov/news-release/nasa-confirms-dart-mission-impact-changed-asteroids-motion-in-space/) · OpenStax UP1, 9.4 Types of collisions (https://openstax.org/books/university-physics-volume-1/pages/9-4-types-of-collisions)

Free learning resources

- OpenStax UP1, 9.3 Conservation of linear momentum (https://openstax.org/books/university-physics-volume-1/pages/9-3-conservation-of-linear-momentum): Free textbook section.
- OpenStax UP1, 9.4 Types of collisions (https://openstax.org/books/university-physics-volume-1/pages/9-4-types-of-collisions): Collisions.
- PhET: Collision Lab (https://phet.colorado.edu/en/simulations/collision-lab): Interactive.

Watch the lecture

A review of the principle of linear momentum and impulse (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/a-review-of-the-principle-of-linear-momentum-and-impulse/) (in English)

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☐ **I finished this unit**

Systems of Particles and Variable Mass: Jets and Rockets

Center of mass of a system, motion of the center of mass, momentum of systems, steady fluid streams, and variable-mass systems including the rocket equation.

Learning objectives

By the end of this unit you can:

- Find the center of mass of a system and its motion.
- Use momentum for systems that push apart.
- Find the force of a steady fluid jet.
- Use the thrust and rocket equations.

Key concepts

- The center of mass moves as if all external forces acted on the total mass there: ΣF_ext = m a_G. Internal forces cancel.
- A steady stream that changes velocity needs a force: F = ṁ(v_out − v_in) for each direction.
- A rocket gains momentum by throwing mass backward. Ideal thrust is F = ṁv_e plus a pressure term (p_e − p_a)A_e.
- The ideal rocket equation, Δv = v_e ln(m$_0$/m_f), shows why rockets are mostly propellant.
- Gravity and drag reduce the real Δv, so launch vehicles use stages to drop empty mass.

Key equations

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i}, \quad \sum \vec{F}_{\text{ext}} = m\vec{a}_G$$

`x=( m_ix_i)/( m_i), F_ext=m a_G`

Center of mass

$$F = \dot{m}(v_{\text{out}} - v_{\text{in}})$$

`F= m(v_out-v_in)`

Steady stream

$$F = \dot{m}v_e + (p_e - p_a)A_e$$

`F= m v_e+(p_e-p_a)A_e`

Rocket thrust

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

`Δ v=v_e(m_0)/(m_f)`

Ideal rocket equation

Worked examples, step by step

Example 6.1 Find the center of mass of 3 kg at x = 0 and 5 kg at x = 4 m.

1. $\bar{x}$ = (3 × 0 + 5 × 4)/8.

Answer: 2.5 m.

Example 6.2 Sample values: an engine burns 250 kg/s with exhaust at 3,000 m/s. Find the ideal thrust (ignore the pressure term).

1. F = $\dot{m}$v_e = 250 × 3000.

Answer: 750 kN (sample values).

Example 6.3 A rocket stage has v_e = 3,000 m/s and burns until its mass is a quarter of its starting mass. Find the ideal Δv.

1. Δv = 3000 ln 4.

Answer: 4,159 m/s.

Practice problems

Work each on paper first, then check.

1. What mass ratio m_0/m_f gives Δv = 7,800 m/s with v_e = 3,500 m/s (ideal)?

   m_0/m_f = e^(7800/3500) = e^2.229 = 9.29.

2. A water jet of area 0.001 m² at 20 m/s hits a flat wall and stops. Find the force.

   F = ρAv² = 1000 × 0.001 × 400 = 400 N.

3. Skaters of 50 kg and 75 kg push apart from rest; the 75 kg skater moves at 2 m/s. Find the other’s speed.

50v = 75 × 2, v = 3 m/s.

4. Sample values: a 50,000 kg rocket lifts off with 750 kN of thrust. Find its initial acceleration.

   a = (750,000 − 490,500)/50,000 = 5.19 m/s² (sample values).

Common misconceptions

Myth: A rocket pushes on the air.

Actually: It pushes on its own exhaust; it works best in vacuum.

Myth: Doubling the propellant doubles Δv.

Actually: Δv grows only with the logarithm of the mass ratio.

Myth: Internal forces can move the center of mass.

Actually: Only external forces change a_G.

Flashcards

Center of mass

Σm x/Σm.

System motion

ΣF_ext = m a_G.

Steady stream

F = ṁΔv.

Thrust

ṁv_e + (p_e − p_a)A_e.

Rocket equation

Δv = v_e ln(m_0/m_f).

Staging

Drop empty tanks to improve the mass ratio.

Self-check quiz

1. The center of mass of a system is moved by...

internal forces

external forces

both

neither

external forces. $\Sigma F_{ext} = m a_G$.

2. Ideal rocket Δv depends on v_e and...

burn time only

the mass ratio

the air density

the color

the mass ratio. $\ln(m_0/m_f)$.

3. Thrust grows with...

mass flow rate and exhaust speed

rocket length

payload

altitude only

mass flow rate and exhaust speed. $F = \dot{m} v_e$ + pressure term.

4. Why stage a rocket?

to look better

to drop empty mass

to slow down

to add drag

to drop empty mass. Better mass ratio per stage.

Try this quiz again

How this helps save the planet

Every Earth-observing satellite rides a rocket built on this unit. NASA Glenn explains that thrust follows Newton's third law and depends on the mass flow rate through the engine, the exit velocity of the exhaust and the pressure at the nozzle exit. Getting more Δv from each kilogram of propellant means fewer and smaller launches for the same payloads, such as the Landsat and weather satellites that monitor forests, crops and storms.

On Saving Planets: Intro to Aerospace Engineering

Sources: NASA Glenn: Rocket thrust equation (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/rocket-thrust-equation/) · OpenStax UP1, 9.7 Rocket propulsion (https://openstax.org/books/university-physics-volume-1/pages/9-7-rocket-propulsion)

Free learning resources

- OpenStax UP1, 9.6 Center of mass (https://openstax.org/books/university-physics-volume-1/pages/9-6-center-of-mass): Free textbook section.
- OpenStax UP1, 9.7 Rocket propulsion (https://openstax.org/books/university-physics-volume-1/pages/9-7-rocket-propulsion): Rocket equation.
- NASA Glenn: Rocket thrust equation (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/rocket-thrust-equation/): Thrust explained.

Watch the lecture

Damped response of spring-mass-dashpot system to initial conditions (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/damped-response-of-spring-mass-dashpot-system-to-initial-conditions/) (in English)

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☐ **I finished this unit**

Planar Kinematics of Rigid Bodies

Rotation about a fixed axis, angular velocity and acceleration, gear trains, relative velocity and acceleration in rigid bodies, instantaneous centers, and rolling without slipping.

Learning objectives

By the end of this unit you can:

- Relate ω and α to point velocities and accelerations.
- Use gear and belt ratios.
- Use relative velocity and the instantaneous center for linkages.
- Describe rolling without slipping.

Key concepts

- For rotation about a fixed axis, a point at radius r has v = rω, a_t = rα and a_n = rω².
- Meshing gears share the contact speed: $r_1\omega_1 = r_2\omega_2$, so speed ratios follow tooth counts.
- Relative velocity in a rigid body: v_B = v_A + ω × r_B/A; relative acceleration adds α × r and −ω²r.
- The instantaneous center of zero velocity (IC) lets you treat a moving body as pure rotation for velocities at that instant.
- Rolling without slipping: v_G = rω and a_G = rα; the contact point is momentarily at rest and the top moves at 2v_G.

Key equations

$$v = r\omega, \quad a_t = r\alpha, \quad a_n = r\omega^2$$

`v=rω, a_t=rα, a_n=rω^2`

Fixed-axis rotation

$$r_1\omega_1 = r_2\omega_2$$

`r_1ω_1=r_2ω_2`

Gears and belts

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

`v_B= v_A+ω× r_B/A`

Relative velocity

$$v_G = r\omega$$

$v_G = r\omega$

Rolling without slipping

Worked examples, step by step

Example 7.1 A 0.3 m radius wheel rolls without slipping at 15 m/s. Find ω and the speed of the top point.

1. ω = v/r = 15/0.3.
2. Top: v = 2v_G.

Answer: ω = 50 rad/s; top point 30 m/s.

Example 7.2 Sample values: a wind turbine rotor turns at 15 rpm and a gearbox with ratio 100 drives the generator. Find the generator speed in rpm and rad/s.

1. n = 15 × 100.
2. ω = 1500 × 2π/60.

Answer: 1,500 rpm = 157.1 rad/s (sample values).

Example 7.3 A 1 m bar slides with end A on the floor moving right at 2 m/s and end B on a wall. Find ω and v_B when the bar makes 30° with the floor.

1. The IC is where perpendiculars to v_A and v_B meet; its distance from A is L sin 30° = 0.5 m.
2. ω = v_A/0.5; v_B = ω × L cos 30°.

Answer: ω = 4 rad/s; v_B = 3.46 m/s downward.

Practice problems

Work each on paper first, then check.

1. Sample values: a 60 m rotor turns at 15 rpm. Find the blade-tip speed and normal acceleration.

   v = 60 × 1.571 = 94.2 m/s; $a_n = v^2/r$ = 148 m/s² (sample values).

2. A flywheel spins up from rest to 3,000 rpm in 10 s at constant α. Find α.

   ω = 314.2 rad/s; α = 31.4 rad/s².

3. A 0.5 m disk has ω = 2 rad/s and α = 4 rad/s². Find the rim acceleration.

$a_t = 2$, $a_n = 2$, $|a| = 2.83$ m/s².

4. A 20-tooth gear drives an 80-tooth gear at 1,200 rpm. Find the output speed.

   1,200 × 20/80 = 300 rpm.

Common misconceptions

Myth: Every point on a rolling wheel moves at the same speed.
Actually: The contact point is at rest and the top moves at 2v.

Myth: Angular velocity differs from point to point on a rigid body.
Actually: ω is the same for the whole body.

Myth: The IC has zero acceleration.
Actually: Only zero velocity, at that instant.

Flashcards

Fixed axis

$v = r\omega$, $a_t = r\alpha$, $a_n = r\omega^2$.

Gear ratio

$\omega_2/\omega_1 = r_1/r_2 = N_1/N_2$.

Relative velocity

$v_B = v_A + \omega \times r_{B/A}$.

Instantaneous center

Point of zero velocity at an instant.

Rolling

$v_G = r\omega$.

rpm

× 2π/60 for rad/s.

Self-check quiz

1. Speed of a point at radius r on a rotating body:

ω/r

rω

rω²

r/ω

rω. $v = r\omega$.

2. Meshing gears share the same...

ω

tooth count

contact speed

radius

contact speed. $r_1\omega_1 = r_2\omega_2$.

3. The contact point of a wheel rolling without slipping moves at...

v

2v

0

$r\omega^2$

0. No slip.

4. 1 rpm in rad/s is about...

0.1047

1

6.28

60

0.1047. $2\pi/60$.

Try this quiz again

How this helps save the planet

A wind turbine is a chain of rotating rigid bodies. The U.S. Department of Energy explains that the rotor connects to the generator either directly, in a direct-drive turbine, or through a shaft and a gearbox that speeds up the rotation and allows a physically smaller generator. The gear-ratio kinematics in this unit is how engineers match a slow, huge rotor to a fast generator.

On Saving Planets: Physics II

Sources: U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work) · OpenStax UP1, 10.3 Relating angular and translational quantities (https://openstax.org/books/university-physics-volume-1/pages/10-3-relating-angular-and-translational-quantities)

Free learning resources

- OpenStax UP1, 10.3 Relating angular and translational quantities (https://openstax.org/books/university-physics-volume-1/pages/10-3-relating-angular-and-translational-quantities): Free textbook section.
- OpenStax UP1, 11.1 Rolling motion (https://openstax.org/books/university-physics-volume-1/pages/11-1-rolling-motion): Rolling.
- MIT OpenCourseWare 2.003SC Engineering Dynamics (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/): MIT rigid-body kinematics.

Watch the lecture

Kinematics: describing the motion of particles and rigid bodies (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/kinematics-describing-the-motion-of-particles-and-rigid-bodies/) (in English)

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☐ **I finished this unit**

Planar Kinetics of Rigid Bodies: Force, Mass and Acceleration

Mass moments of inertia, the parallel-axis theorem, equations of motion for translation, fixed-axis rotation and general plane motion, and rolling with friction.

Learning objectives

By the end of this unit you can:

- Compute mass moments of inertia of common shapes.
- Write ΣF = ma_G and ΣM_G = I_Gα.
- Solve fixed-axis rotation and pulley problems.
- Solve rolling problems and check for slip.

Key concepts

- Mass moment of inertia I = ∫r² dm measures resistance to angular acceleration. Solid disk ½mr², hoop mr², solid sphere ⅖mr², rod about its center mL²/12.
- Parallel-axis theorem: I = I_G + md².
- Plane motion equations: ΣF_x = m a_Gx, ΣF_y = m a_Gy, ΣM_G = I_Gα (or ΣM_P = sum of the moments of the kinetic diagram about any point P).
- A body rolling down a slope without slipping has a = g sin θ/(1 + I_G/(mr²)); friction makes the rotation happen.
- Check rolling with f ≤ μ_sN; if the needed friction is larger, the body slides and rolls at once.

Key equations

$$\sum \vec{F} = m\vec{a}_G, \quad \sum M_G = I_G\alpha$$

`F=m a_G, M_G=I_Gα`

Plane motion

$$I = I_G + md^2$$

`I=I_G+md^2`

Parallel axis

$$a = \frac{g\sin\theta}{1 + I_G/(mr^2)}$$

`a=(gθ)/(1+I_G/(mr^2))`

Rolling down a slope

Worked examples, step by step

Example 8.1 A 10 kg solid disk of radius 0.2 m on a fixed axle feels a 4 N·m torque. Find α.

1. $I = \tfrac{1}{2} \times 10 \times 0.2^2 = 0.2$ kg·m².
2. $\alpha = M/I$.

Answer: 20 rad/s².

Example 8.2 A solid cylinder rolls without slipping down a 30° slope. Find its acceleration and the smallest μ_s that prevents slip.

1. $a = g \sin 30°/(1 + \tfrac{1}{2}) = (2/3)g \sin 30°$.
2. Moments about G: $fr = \tfrac{1}{2}mr^2(a/r)$, so $f = \tfrac{1}{2}ma = (1/3)mg \sin 30°$; $\mu_s \geq f/N = (1/3) \tan 30°$.

Answer: $a = 3.27$ m/s²; $\mu_s \geq 0.192$.

Example 8.3 A 4 kg mass hangs from a rope wrapped on a 2 kg solid-disk pulley of radius 0.1 m. Find the acceleration and tension.

1. Mass: $4g - T = 4a$. Pulley: $Tr = \tfrac{1}{2}Mr^2\,(a/r)$, so $T = \tfrac{1}{2}Ma = a$.
2. $4g = 5a$.

Answer: $a = 7.85$ m/s²; $T = 7.85$ N.

Practice problems

Work each on paper first, then check.

1. Find I for a 3 kg, 2 m rod about one end.

   $I = mL^2/3 = 4$ kg·m².

2. That rod is pinned at one end and released from horizontal. Find α at release.

   $\alpha = 3g/(2L) = 7.36$ rad/s².

3. Sample values: a hydro turbine runner has $I = 5 \times 10^4$ kg·m² and a net torque of 1×10^6 N·m at start-up. Find α.

   $\alpha = 20$ rad/s² (sample values).

4. Find the acceleration of a solid sphere rolling down a 30° slope.

   $a = (5/7)\, g \sin 30° = 3.50\ m/s^2$.

Common misconceptions

Myth: Mass is all that resists angular acceleration.

Actually: Where the mass is (I) matters; a hoop accelerates slower than a disk of the same mass.

Myth: Friction always slows a rolling wheel.

Actually: Static friction in pure rolling does no work and is what makes it spin.

Myth: $\Sigma M = I\alpha$ works about any point.

Actually: Use it about G or a fixed axis; about other points include the moment of ma_G.

Flashcards

Mass moment of inertia

$I = \int r^2\, dm$.

Disk, hoop, sphere

$\frac{1}{2}mr^2$, mr^2, $\frac{2}{5}mr^2$.

Rod

$mL^2/12$ (center), $mL^2/3$ (end).

Plane motion

$\Sigma F = ma_G$, $\Sigma M_G = I_G\alpha$.

Rolling a

$g \sin\theta/(1 + I/(mr^2))$.

Slip check

$f \le \mu_s N$.

Self-check quiz

1. Which reaches the bottom of a slope first?

hoop

solid disk

solid sphere

all tie

solid sphere. Smallest $I/(mr^2)$.

2. I of a solid disk about its axis:

mr^2

$\frac{1}{2}mr^2$

$\frac{2}{5}mr^2$

$mL^2/12$

$\frac{1}{2}mr^2$. Standard result.

3. The parallel-axis theorem adds...

md^2

mr

I/d

d^2

md^2. $I = I_G + md^2$.

4. Rotational form of Newton's law:

$M = I\alpha$

$M = m\alpha$

$F = I\omega$

$P = I\alpha$

$M = I\alpha$. About G or a fixed axis.

Try this quiz again

How this helps save the planet

Hydro turbines are rigid bodies spun by water. The U.S. Department of Energy describes two main families: reaction turbines, such as Francis and Kaplan, which run in the water stream and use pressure and moving water, and impulse turbines, such as the Pelton wheel, whose buckets are struck by free jets. DOE notes Francis turbines are commonly used for heads of about 130 to 2,000 feet. Sizing the runner's moment of inertia and torque is the kinetics of this unit.

On Saving Planets: Physics II

Sources: U.S. DOE: Types of hydropower turbines (https://www.energy.gov/eere/water/types-hydropower-turbines) · OpenStax UP1, 10.7 Newton's second law for rotation (https://openstax.org/books/university-physics-volume-1/pages/10-7-newtons-second-law-for-rotation)

Free learning resources

- OpenStax UP1, 10.5 Calculating moments of inertia (https://openstax.org/books/university-physics-volume-1/pages/10-5-calculating-moments-of-inertia): Free textbook section.
- OpenStax UP1, 10.7 Newton's second law for rotation (https://openstax.org/books/university-physics-volume-1/pages/10-7-newtons-second-law-for-rotation): Rotation dynamics.
- PhET: Torque (https://phet.colorado.edu/en/simulations/torque): Interactive.

Watch the lecture

Angular momentum for rigid bodies and the mass moment of inertia matrix, [I] (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/angular-momentum-for-rigid-bodies-and-the-mass-moment-of-inertia-matrix-i/) (in English)

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- [ ] **I finished this unit**

Rigid Bodies: Energy and Momentum Methods

Kinetic energy of rigid bodies in translation, rotation and rolling, work of couples, power in rotating machines, angular impulse and momentum, and conservation of angular momentum.

Learning objectives

By the end of this unit you can:

- Write T = ½mv_G² + ½I_Gω².
- Use work–energy for rolling and pivoting bodies.
- Compute rotating-machine power P = Mω.
- Apply conservation of angular momentum.

Key concepts

- Rigid-body kinetic energy is T = ½mv_G² + ½I_Gω²; for rolling, part of the energy goes into spin.
- A couple M turning through angle θ does work Mθ; power is P = Mω.
- Angular momentum about G is H_G = I_Gω. Angular impulse–momentum: $I_G\omega_1 + \int M\,dt = I_G\omega_2$.
- If no external moment acts about an axis, I ω is conserved: pulling mass inward spins faster.
- A flywheel stores energy ½Iω²; spinning it twice as fast stores four times the energy.

Key equations

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$

`T=12mv_G^2+12I_Gω^2`

Rigid-body kinetic energy

$$U = M\theta, \quad P = M\omega$$

`U=Mθ, P=Mω`

Work and power of a couple

$$I_1 \omega_1 = I_2 \omega_2$$

`I_1ω_1=I_2ω_2`

Conservation of angular momentum

$$v = \sqrt{\frac{2gh}{1 + I_G/(mr^2)}}$$

`v=(2gh)/(1+I_G/(mr^2))`

Rolling from height h

Worked examples, step by step

Example 9.1 A flywheel with I = 10 kg·m² spins at 300 rad/s. Find its stored energy in J and kWh.

1. T = ½ × 10 × 300².
2. ÷ 3.6 × 10⁶.

Answer: 450 kJ = 0.125 kWh.

Example 9.2 A solid sphere rolls from rest down a 2 m height without slipping. Find its speed.

1. mgh = ½mv² + ½(⅖mr²)(v/r)² = (7/10)mv².
2. v = √(10gh/7).

Answer: 5.29 m/s.

Example 9.3 A skater spins at 2 rad/s with I = 4 kg·m², then pulls in her arms to I = 1.6 kg·m². Find the new spin rate.

1. 4 × 2 = 1.6ω.

Answer: 5 rad/s.

Practice problems

Work each on paper first, then check.

1. A solid cylinder rolls from rest down a 2 m height. Find its speed.

   v = √(4gh/3) = 5.11 m/s.

2. A 20 kg solid disk of radius 0.5 m spins at 100 rad/s. Find its kinetic energy.

   I = 2.5 kg·m²; T = 12.5 kJ.

3. A 2 m rod pinned at one end is released from horizontal. Find ω when it hangs vertically.

   mg(L/2) = ½(mL²/3)ω², ω = √(3g/L) = 3.84 rad/s.

4. Sample values: a turbine shaft carries 1×10^6 N·m at 150 rpm. Find the power.

   $\omega = 15.7$ rad/s; $P = M\omega = 15.7$ MW (sample values).

Common misconceptions

Myth: A rolling ball has kinetic energy $\frac{1}{2}mv^2$ only.

Actually: Add $\frac{1}{2}I\omega^2$ for the spin.

Myth: Pulling in your arms adds energy from nowhere.

Actually: Your muscles do the work; angular momentum is conserved, not kinetic energy.

Myth: Torque and power are the same thing.

Actually: $P = M\omega$; the same torque gives more power at higher speed.

Flashcards

Rigid-body T

$\frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$.

Couple work

$U = M\theta$.

Shaft power

$P = M\omega$.

Angular momentum

$H_G = I_G\omega$.

Conservation

$I_1\omega_1 = I_2\omega_2$.

Flywheel

Stores $\frac{1}{2}I\omega^2$.

Self-check quiz

1. Kinetic energy of a rolling body includes...

only $\frac{1}{2}mv^2$

only $\frac{1}{2}I\omega^2$

both terms

mgh

both terms. Translation plus rotation.

2. Power of a rotating shaft:

Mω

M/ω

Iω

mv

Mω. P = Mω.

3. Halving I with no external moment makes ω…

half

the same

double

four times

double. Iω constant.

4. Doubling a flywheel's speed multiplies its energy by…

2

4

8

√2

4. ω^2.

Try this quiz again

How this helps save the planet

Hydropower is an energy chain from height to spin to electricity. The U.S. Department of Energy explains that water flows through a pipe called a penstock, spins the blades of a turbine, and the turbine spins a generator that produces electricity for the grid. The work–energy and power equations of this unit, from mgh to P = Mω, describe each link in that chain.

On Saving Planets: Physics II

Sources: U.S. DOE: How hydropower works (https://www.energy.gov/eere/water/how-hydropower-works) · OpenStax UP1, 11.3 Conservation of angular momentum (https://openstax.org/books/university-physics-volume-1/pages/11-3-conservation-of-angular-momentum)

Free learning resources

- OpenStax UP1, 11.1 Rolling motion (https://openstax.org/books/university-physics-volume-1/pages/11-1-rolling-motion): Free textbook section.
- OpenStax UP1, 11.3 Conservation of angular momentum (https://openstax.org/books/university-physics-volume-1/pages/11-3-conservation-of-angular-momentum): Angular momentum.
- MIT OpenCourseWare 2.003SC Engineering Dynamics (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/): MIT lectures.

Watch the lecture

Angular momentum for rigid bodies and the mass moment of inertia matrix, [I] (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/angular-momentum-for-rigid-bodies-and-the-mass-moment-of-inertia-matrix-i/) (in English)

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☐ **I finished this unit**

Mechanical Vibrations

Free undamped vibration, natural frequency, energy methods, viscous damping and the damping ratio, forced vibration, resonance and magnification.

Learning objectives

By the end of this unit you can:

- Find natural frequencies of spring-mass systems and pendulums.
- Use the damping ratio to classify motion and estimate it from decay.
- Find the steady-state amplitude of forced vibration.
- Explain resonance and why low damping is dangerous.

Key concepts

- A mass on a spring obeys mẍ + kx = 0 and vibrates at ω_n = √(k/m); the period is T = 2π/ω_n. A simple pendulum has ω_n = √(g/L) for small swings.
- With viscous damping, mẍ + cẋ + kx = 0. The damping ratio ζ = c/(2√(km)) sets the behavior: ζ < 1 oscillates and decays, ζ = 1 is critical, ζ > 1 creeps back.
- The logarithmic decrement δ = ln(x_1/x_2) between successive peaks gives ζ ≈ δ/(2π) for light damping.
- A harmonic force F_0 sin ωt drives a steady response of amplitude X = (F_0/k)/√((1 − r²)² + (2ζr)²), with r = ω/ω_n.
- At resonance (r = 1) the magnification is 1/(2ζ): with light damping, a small push repeated at the right rhythm builds a large motion.

Key equations

$$\omega_n = \sqrt{k/m}, \quad T = 2\pi/\omega_n$$

ω_n=/m, T=2π/ω_n

Natural frequency

$$\zeta = \frac{c}{2\sqrt{km}}, \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

=2, ω_d=ω_n1-^2

Damping

$$X = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}, \quad r = \frac{\omega}{\omega_n}$$

X=_0/k(1-r^2)^2+(2 r)^2, r=(ω)/(ω_n)

Forced response

$$\left.\frac{X}{F_0/k}\right|_{r=1} = \frac{1}{2\zeta}$$

.(X)/(F_0/k)|_r=1=12

Resonant magnification

Worked examples, step by step

Example 10.1 A 2 kg mass hangs on a spring with k = 800 N/m. Find ω_n, f and T.

1. ω_n = √(800/2) = 20 rad/s.
2. f = ω_n/(2π); T = 1/f.

Answer: 20 rad/s, 3.18 Hz, 0.314 s.

Example 10.2 Add a damper with c = 8 N·s/m. Find ζ and the damped frequency.

1. ζ = 8/(2√(800 × 2)) = 8/80.
2. ω_d = 20√(1 − 0.01).

Answer: ζ = 0.1; ω_d = 19.9 rad/s.

Example 10.3 The undamped system is driven by F_0 = 10 N at 15 rad/s. Find the steady amplitude.

1. F_0/k = 12.5 mm; r = 0.75.
2. X = 12.5/(1 − 0.5625).

Answer: 28.6 mm.

Practice problems

Work each on paper first, then check.

1. Find the period of a 1 m simple pendulum.

   T = 2π√(1/9.81) = 2.01 s.

2. A machine sags 10 mm on its mounts. Estimate its natural frequency.

$\omega_n = \sqrt{(g/\delta)}$ gives f = 4.98 Hz.

3. Find the resonant magnification for $\zeta = 0.02$.

   $1/(2 \times 0.02) = 25$.

4. Successive peaks of a free vibration are 10 mm and 7 mm. Estimate ζ.

   $\delta = \ln(10/7) = 0.357$; $\zeta \approx \delta/(2\pi) = 0.057$.

Common misconceptions

Myth: A heavier mass vibrates faster.

Actually: $\omega_n = \sqrt{(k/m)}$: more mass, lower frequency.

Myth: More damping always means more safety.

Actually: It lowers resonant peaks but also passes more force at high frequency.

Myth: A pendulum's period depends on how heavy the bob is.

Actually: For small swings, only on L and g.

Flashcards

Natural frequency

$\omega_n = \sqrt{(k/m)}$.

Pendulum

$\omega_n = \sqrt{(g/L)}$.

Damping ratio

$\zeta = c/(2\sqrt{(km)})$.

Log decrement

$\zeta \approx \ln(x_1/x_2)/(2\pi)$.

Frequency ratio

$r = \omega/\omega_n$.

Resonance

$X/(F_0/k) = 1/(2\zeta)$ at $r = 1$.

Self-check quiz

1. Natural frequency of a spring-mass system:

$\sqrt{(m/k)}$

$\sqrt{(k/m)}$

k/m

km

√(k/m). ω_n = √(k/m).

2. Critical damping is ζ =

0

0.5

1

2

1. Fastest return without oscillation.

3. At resonance with ζ = 0.05, magnification is...

5

10

20

0.1

10. 1/(2ζ).

4. Driving well above ω_n (r ≫ 1) makes the response...

huge

small

zero frequency

critical

small. X → 0 as r grows.

Try this quiz again

How this helps save the planet

Resonance decides which buildings survive earthquakes. IRIS (now EarthScope) explains that every building has a natural period, the time it takes to sway back and forth, and that when ground motion matches it the building undergoes its largest oscillations and greatest damage. A USGS study of 41 tall buildings found their damping ratios clearly lower than 3%, and the magnification formula in this unit shows why low damping makes resonance dangerous. Engineers tune periods and add damping so homes, hospitals and infrastructure stay standing.

On Saving Planets: Solutions

Sources: IRIS/EarthScope: Building resonance (https://www.iris.edu/hq/inclass/animation/224) · USGS: Natural periods and damping of tall buildings (Çelebi, 2022) (https://www.usgs.gov/publications/preliminary-study-correlation-natural-periods-and-damping-percentages-tall-buildings)

Free learning resources

- OpenStax UP1, 15.1 Simple harmonic motion (https://openstax.org/books/university-physics-volume-1/pages/15-1-simple-harmonic-motion): Free textbook section.
- OpenStax UP1, 15.5 Damped oscillations (https://openstax.org/books/university-physics-volume-1/pages/15-5-damped-oscillations): Damping.
- OpenStax UP1, 15.6 Forced oscillations (https://openstax.org/books/university-physics-volume-1/pages/15-6-forced-oscillations): Resonance.
- PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs): Interactive.
- PhET: Pendulum Lab (https://phet.colorado.edu/en/simulations/pendulum-lab): Interactive.

Watch the lecture

Lecture 19: Introduction to Mechanical Vibration (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/resources/lecture-19-introduction-to-mechanical-vibration-1/) (in English)

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☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Kinematics of Particles in a Straight Line

4 problems

2. Curvilinear Motion: Projectiles, Normal–Tangential and Polar Coordinates
   4 problems
3. Kinetics of Particles: Newton’s Second Law
   4 problems
4. Work, Energy and Power
   4 problems
5. Impulse, Momentum and Impact
   4 problems
6. Systems of Particles and Variable Mass: Jets and Rockets
   4 problems
7. Planar Kinematics of Rigid Bodies
   4 problems
8. Planar Kinetics of Rigid Bodies: Force, Mass and Acceleration
   4 problems
9. Rigid Bodies: Energy and Momentum Methods
   4 problems
10. Mechanical Vibrations
    4 problems

The lab (Dynamics lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free PhET simulation or with household items. Roll and drop only soft, light objects, keep the floor clear, and never launch anything toward people, pets or windows.

Safety reference: OSHA: Hand and power tools (https://www.osha.gov/hand-power-tools)

Lab A: Constant acceleration on a ramp

WEEK 2

Goal: Measure position against time for a ball rolling down a gentle ramp, show that x grows as t², and extract the acceleration.

$$x = \tfrac{1}{2}at^2$$

`x=12at^2`

From rest; plot x against t² for a straight line of slope a/2

Procedure

1. Prop one end of a long board on a book to make a gentle slope; mark 0.25, 1.0, 2.25 and 4.0 m (or scaled-down marks).
2. Release a ball from rest and film it in slow motion with a phone; read the time at each mark.
3. Plot x against t² and fit a line through the origin; a = 2 × slope.
4. Optional: compare with PhET or with a = (5/7) g sin θ for a rolling solid ball.

Data table

t (s)	x (m) for a = 0.5 m/s²
1	0.25
2	1.00
3	2.25
4	4.00

Sample values for a = 0.5 m/s²; your slope sets your own a.

Sample calculation

1. x/t² = 0.25 m/s² in every row, so a = 0.50 m/s².
2. Doubling the time quadruples the distance.
3. For a solid ball, a = 0.5 m/s² needs sin θ = 0.5 × 7/(5 × 9.81), θ = 4.1°.

> **Safety**
>
> - Keep the floor clear where the ball runs out; use a soft ball.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Ladybug Motion 2D (https://phet.colorado.edu/en/simulations/ladybug-motion-2d)

Planet link: Measured acceleration data like this feeds the drive-cycle tests that rate how efficiently cars use energy. FuelEconomy.gov (DOE/EPA): All-electric vehicles (https://www.fueleconomy.gov/feg/evtech.shtml)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Projectile range and launch angle

WEEK 3

Goal: Measure how range depends on launch angle and compare with R = v_0^2 sin 2θ/g.

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

R=(v_0^22θ)/(g)

Level ground, no drag

Procedure

1. Open PhET Projectile Motion, turn off air resistance, set v_0 = 10 m/s and the launch height to 0.
2. Fire at 15°, 30°, 45°, 60° and 75° and record the range.
3. Plot R against sin 2θ; the slope should be v_0^2/g = 10.19 m.
4. Turn on air resistance and note which angle now gives the longest range.

Data table

θ (°)	R (m)
15	5.10
30	8.83
45	10.19
60	8.83
75	5.10

Values computed from the formula with g = 9.81 m/s².

Sample calculation

1. Complementary angles (15° and 75°, 30° and 60°) give the same range.
2. Maximum at 45°: R = v_0^2/g = 10.19 m.

> **Safety**
>
> - None beyond normal computer use; if you try a home version, toss only soft balls and never toward people.

Home or virtual alternative

- Optional: toss a soft ball at estimated angles in a yard and measure ranges; drag and launch height make it less exact.

PhET: Projectile Motion (https://phet.colorado.edu/en/simulations/projectile-motion)

Planet link: The same equations, with drag added, plan rocket ascent paths that place Earth-observing satellites in orbit. NASA: International Space Station facts and figures (https://www.nasa.gov/international-space-station/space-station-facts-and-figures/)

☐ **I completed this lab (wet or virtual)**

Lab C: Bounces, collisions and the coefficient of restitution

WEEK 7

Goal: Find e for a bouncing ball from bounce heights, and check momentum conservation in simulated collisions.

$$e = \sqrt{h_2/h_1}$$

e=_2/h_1

Ball dropped on a heavy floor

Procedure

1. Drop a ball from 1.0 m in front of a tape measure and film it; read the first three bounce heights.
2. Compute e for each bounce.
3. In PhET Collision Lab, collide 2 kg at 5 m/s with 3 kg at rest at elasticity 80% and check the final velocities against the Unit 5 example.
4. Repeat at 0% and 100% elasticity and compare total momentum and kinetic energy before and after.

Data table

Bounce	Height (m)	e
Drop	1.00	—
1	0.64	0.80
2	0.41	0.80
3	0.26	0.80

Sample values for a ball with e ≈ 0.8.

Sample calculation

1. e = √(0.64/1.00) = 0.80; √(0.41/0.64) = 0.80; √(0.26/0.41) = 0.80.
2. Simulation at 80%: v_1' = −0.4 m/s, v_2' = 3.6 m/s; momentum 10 kg·m/s before and after.

> **Safety**
>
> - Use a soft ball and keep breakables away.

Home or virtual alternative

- The bounce part is the home version.

PhET: Collision Lab (https://phet.colorado.edu/en/simulations/collision-lab)

Planet link: NASA's DART test was a planned collision; measuring how momentum transfers in impacts is how planetary defense is designed. NASA: DART impact changed asteroid's motion in space (https://www.nasa.gov/news-release/nasa-confirms-dart-mission-impact-changed-asteroids-motion-in-space/)

☐ **I completed this lab (wet or virtual)**

Lab D: The rolling race

WEEK 11

Goal: Race a ball, a full can and a ring down the same slope and explain the order with a = g sin θ/(1 + I/(mr²)).

$$a = \frac{g \sin\theta}{1 + I_G/(mr^2)}, \quad t = \sqrt{2L/a}$$

`a=(gθ)/(1+I_G/(mr^2)), t=2L/a`

Rolling from rest down length L

Procedure

1. Set a board at 10° (phone inclinometer) and mark a 1 m run.
2. Release pairs together: solid ball against full soup can, full can against a ring or empty can (thin-walled).
3. Film the finishes and time each run; repeat three times.
4. Compare with the predicted times.

Data table

Object	I/(mr²)	a (m/s²)	t over 1 m (s)
Solid sphere	0.4	1.22	1.28
Solid cylinder	0.5	1.14	1.33
Hoop	1.0	0.852	1.53

Values computed for θ = 10° and rolling without slipping; a full soup can behaves only roughly like a solid cylinder because the liquid can slosh.

Sample calculation

1. Sphere: a = 9.81 sin 10°/1.4 = 1.22 m/s²; t = √(2/1.22) = 1.28 s.
2. Hoop: a = 9.81 sin 10°/2 = 0.852 m/s²; t = 1.53 s.
3. Mass and radius cancel: only the shape factor matters.

> **Safety**
>
> - Catch the objects at the bottom; keep fingers out from under heavy cans.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Torque (https://phet.colorado.edu/en/simulations/torque)

Planet link: Where a rotor's mass sits changes how fast it spins up, which matters for wind and hydro turbines. U.S. DOE: Types of hydropower turbines (https://www.energy.gov/eere/water/types-hydropower-turbines)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Pendulum period and g

WEEK 13

Goal: Measure the period of a simple pendulum at several lengths and find g from a fit.

$$T = 2\pi\sqrt{L/g} \;\Rightarrow\; T^2 = \frac{4\pi^2}{g}L$$

T=2π/g\ \ T^2=(4π^2)/(g)L

Plot T^2 against L

Procedure

1. Tie a small weight to a string and hang it from a door frame. Set L (to the weight's center) at 0.25, 0.50 and 1.00 m.
2. Pull it aside about 10° and time 10 swings; divide by 10.
3. Plot T^2 against L and fit a line through the origin; g = $4\pi^2$/slope.
4. Check in PhET Pendulum Lab that mass does not change the period.

Data table

L (m)	T (s)
0.25	1.003
0.50	1.419
1.00	2.006

Values computed for g = 9.81 m/s²; small-angle formula.

Sample calculation

1. T^2/L = 4.024 s^2/m in each row, so g = $4\pi^2/4.024$ = 9.81 m/s^2.
2. Quadrupling L doubles T.

> **Safety**
>
> - Use a light weight and keep the swing away from faces.

Home or virtual alternative

- The whole lab is a home lab.

PhET: Pendulum Lab (https://phet.colorado.edu/en/simulations/pendulum-lab)

Planet link: Every building also has a natural period; matching it to ground shaking is what makes earthquakes dangerous. IRIS/EarthScope: Building resonance (https://www.iris.edu/hq/inclass/animation/224)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Spring-mass vibration and damping

WEEK 14

Goal: Measure natural frequency against mass, and estimate the damping ratio from decaying peaks.

$$f = \frac{1}{2\pi}\sqrt{k/m}, \quad \zeta \approx \frac{\ln(x_1/x_2)}{2\pi}$$

`f=(1)/(2π)/m, ≈((x_1/x_2))/(2π)`

Natural frequency and log decrement

Procedure

1. In PhET Masses and Springs, set a spring of 10 N/m and hang 100, 200 and 400 g; time 10 oscillations for each.
2. Turn on damping, release, and read three successive peak heights from the ruler.
3. Home: hang a bag of coins from a rubber band or soft spring and repeat with a phone stopwatch.
4. Plot f against 1/√m.

Data table

m (kg)	f (Hz) for k = 10 N/m
0.1	1.592
0.2	1.125
0.4	0.796

Values computed from the formula; sample damping peaks of 10, 8 and 6.4 cm are used below.

Sample calculation

1. Quadrupling m halves f: 1.592 → 0.796 Hz.
2. Peaks 10 → 8 → 6.4 cm: δ = ln 1.25 = 0.223, ζ ≈ 0.0355.
3. With ζ = 0.0355, resonant magnification would be 1/(2ζ) = 14.1.

> **Safety**
>
> - Rubber bands can snap; wear glasses and keep the mass small.

Home or virtual alternative

- The rubber-band step is the home version.

PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs)

Planet link: USGS measurements show tall buildings have little damping, so engineers add dampers to cut resonant motion. USGS: Natural periods and damping of tall buildings (Çelebi, 2022) (https://www.usgs.gov/publications/preliminary-study-correlation-natural-periods-and-damping-percentages-tall-buildings)

- [ ] **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. A car accelerates from rest at 2.5 m/s² to 30 m/s. Find the time and distance.

    1. $t = v/a$; $x = v^2/(2a)$.

    Answer: 12 s; 180 m.

2. A ball is launched at 25 m/s and 40° over level ground. Find the range.

    1. $R = v_0^2 \sin 80°/g$.

    Answer: 62.7 m.

3. A 5 kg block is pulled with 40 N along a floor with $\mu_k = 0.2$. Find its acceleration.

    1. $a = (40 - 0.2 \times 49.05)/5$.

    Answer: 6.04 m/s².

4. A 1 kg mass falls 5 m onto a spring ($k = 2000$ N/m). Find the maximum compression.

    1. $mg(5 + x) = \frac{1}{2}kx^2$, so $1000x^2 - 9.81x - 49.05 = 0$.

    Answer: 0.226 m.

5. A 0.06 kg ball at 50 m/s is stopped in 0.005 s. Find the average force.

    1. $F = m\Delta v/\Delta t$.

    Answer: 600 N.

Final practice set (all units)

1. A stage with $v_e = 2{,}800$ m/s has $m_0/m_f = 3$. Find the ideal Δv.

1. Δv = v_e ln 3.

Answer: 3,076 m/s.

2. A 0.4 m radius wheel rolls without slipping at 20 m/s. Find ω.

    1. ω = v/r.

Answer: 50 rad/s.

3. A 4 kg solid disk of radius 0.25 m feels a 2 N·m torque. Find α.

    1. I = ½mr² = 0.125 kg·m².

Answer: 16 rad/s².

4. A hoop rolls from rest down a 1.5 m height without slipping. Find its speed.

    1. mgh = mv² for a hoop, v = √(gh).

Answer: 3.84 m/s.

5. A 5 kg mass on a spring with k = 1250 N/m. Find ω_n and f.

    1. ω_n = √(k/m).

Answer: 15.8 rad/s; 2.52 Hz.

6. Find the resonant magnification for ζ = 0.05.

    1. 1/(2ζ).

Answer: 10.

Formula sheet

$$v = dx/dt,\ a = dv/dt,\ a\,dx = v\,dv$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$R = v_0^2 \sin 2\theta / g$$

$$a_n = v^2/\rho,\ a_t = \dot{v}$$

$$a_r = \ddot{r} - r\dot{\theta}^2,\ a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\sum \vec{F} = m\vec{a}$$

$$T_1 + \sum U = T_2,\ P = \vec{F}\cdot\vec{v}$$

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2,\ e = (v_2' - v_1')/(v_1 - v_2)$$

$$\Delta v = v_e \ln(m_0/m_f)$$

$$v = r\omega,\ a_t = r\alpha,\ a_n = r\omega^2$$

$$\vec{v}_B = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\sum M_G = I_G\alpha,\ I = I_G + md^2$$

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2,\ P = M\omega$$

$$\omega_n = \sqrt{k/m},\ \zeta = c/(2\sqrt{km})$$

$$X = (F_0/k)/\sqrt{(1-r^2)^2 + (2\zeta r)^2}$$

Constants

Constant	Value
Standard acceleration of gravity, g	9.80665 m/s² (we use 9.81 m/s²)
Earth's gravitational parameter, GM (rounded)	3.986 × 10¹⁴ m³/s²
Mean Earth radius (rounded)	6371 km
1 revolution per minute	2π/60 = 0.1047 rad/s
1 kilowatt-hour	3.6 × 10⁶ J
1 mile	1.609 km

Source: NIST: Standard acceleration of gravity (https://physics.nist.gov/cgi-bin/cuu/Value?gn)

Free texts and simulations

Use these free textbooks, lectures and simulations alongside the course.

- OpenStax University Physics Volume 1 (https://openstax.org/books/university-physics-volume-1/pages/3-1-position-displacement-and-average-velocity): Free textbook for the physics side of the course.
- MIT OpenCourseWare 2.003SC Engineering Dynamics (https://ocw.mit.edu/courses/2-003sc-engineering-dynamics-fall-2011/): Free MIT lectures, problem sets and exams.
- MIT OpenCourseWare 8.01SC Classical Mechanics (https://ocw.mit.edu/courses/8-01sc-classical-mechanics-fall-2016/): Free MIT mechanics lectures.
- PhET: Projectile Motion (https://phet.colorado.edu/en/simulations/projectile-motion): Trajectories.
- PhET: Collision Lab (https://phet.colorado.edu/en/simulations/collision-lab): Momentum and impacts.
- PhET: Masses and Springs (https://phet.colorado.edu/en/simulations/masses-and-springs): Vibration and damping.

Glossary

Angular momentum
H = r × mv for a particle; $I_G\omega$ for a rigid body in plane motion.

Coefficient of restitution
Ratio of separation to approach speed in an impact.

Coriolis acceleration
$2\dot{r}\dot{\theta}$ term in polar coordinates.

Damping ratio
$\zeta = c/(2\sqrt{(km)})$; 1 is critical.

Impulse
Integral of force over time.

Instantaneous center
Point of a body with zero velocity at an instant.

Kinetic diagram
Sketch showing ma_G and $I_G\alpha$.

Logarithmic decrement
ln of the ratio of successive peaks.

Mass moment of inertia
$I = \int r^2\, dm$.

Natural frequency
Frequency of free vibration, $\sqrt{(k/m)}$.

Normal acceleration
v^2/ρ, toward the center of curvature.

Power
Rate of doing work, $F\cdot v$ or $M\omega$.

Projectile
Body moving under gravity alone.

Resonance
Large response when forcing matches a natural frequency.

Rocket equation
$\Delta v = v_e \ln(m_0/m_f)$.

Rolling without slipping
$v_G = r\omega$; contact point at rest.

Tangential acceleration
dv/dt along the path.

Work–energy principle
$T_1 + \Sigma U = T_2$.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Chooses coordinates and the best method (forces, energy or momentum), draws free-body and kinetic diagrams, and checks units and limiting cases.
3 · Proficient	Solves the practice problems with occasional sign or unit slips.
2 · Developing	Follows worked examples but mixes up methods or forgets rotational terms.
1 · Beginning	Plugs numbers into formulas without diagrams; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	25%
Labs	20%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Statics ENGR 211
- Mechanics of Materials ENGR 213
- Calculus III MATH 210
- All courses