Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Spring · 🧱 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester junior aircraft performance course. It is modeled on the topics and depth of the performance parts of MIT 16.01–16.04 Unified Engineering and MIT 16.885J Aircraft Systems Engineering (free on MIT OpenCourseWare), on courses such as Purdue AAE 251 and Georgia Tech AE 3310, and on the FAA Pilot's Handbook of Aeronautical Knowledge, Chapter 11, which we link where they help. None of those authors, schools or agencies endorses this page.
>
> What we add: every unit ties to real work with a cited public source (laminar-flow wings, truss-braced wings, hybrid engines, sustainable aviation fuel and transportation energy), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation, a spreadsheet or a safe home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Performance is where aircraft design meets fuel burn. Every percent of drag removed, every kilogram saved and every better climb or cruise profile shows up directly in the Breguet range equation, and so in fuel use and emissions.

NASA lists three main ways to make aviation sustainable: efficient aircraft technology, sustainable aviation fuel, and operations and infrastructure. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (AERO 320).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Aerodynamics I: lift and drag coefficients, the drag polar and induced drag.
- Dynamics: Newton’s laws, work and energy.
- Calculus II: integrals and natural logarithms (for range and endurance).
- Thermodynamics helps with engines and the atmosphere but is not required.

How to study

- Start every problem with a free-body diagram: lift, weight, thrust and drag, and the flight-path angle.
- Write down the atmosphere first: altitude, density, temperature and speed of sound.
- Keep weight in newtons and mass in kilograms; most errors in performance come from mixing them.
- Know the key speeds by heart: stall, minimum drag, minimum power, best climb and approach.
- Check limits: zero excess thrust means zero climb; heavier aircraft need more runway, roughly as weight squared.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free NASA FoilSim and PhET Projectile Motion simulations.
- Spreadsheet software for the atmosphere, drag-polar, range and takeoff labs.

- Printer paper, a ruler or tape measure, paper clips and a stopwatch or phone timer.
- A safe open indoor space or a calm outdoor space for glider flights.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	The Standard Atmosphere, Airspeeds and Density Altitude	Standard atmosphere, airspeeds, density altitude; Lab A	FAA PHAK Ch. 11
2	Forces in Flight and the Drag Polar	Forces in flight and the drag polar; Lab C	NASA Glenn; MIT 16.01
3	Thrust and Power Required	Thrust and power required	MIT 16.01
4	Climb Performance	Climb angle and rate	FAA PHAK Ch. 11
5	Climb Performance	Specific excess power, ceilings, time to climb	MIT 16.01
6	Gliding Flight	Gliding flight; Lab B	NASA Glenn
7	Range and Endurance: the Breguet Equations	Fuel consumption and the Breguet range equation	MIT 16.885J
8	Range and Endurance: the Breguet Equations	Endurance, payload–range; Lab D; midterm practice	MIT 16.885J
9	Takeoff Performance	Stall speed, high lift, ground roll	FAA Airplane Flying Handbook
10	Takeoff Performance	Density altitude, wind, balanced field; Lab E	FAA PHAK Ch. 11
11	Landing Performance	Approach and landing distance	FAA Airplane Flying Handbook
12	Turning Flight and the V-n Diagram	Turns, load factor, turn radius	FAA Airplane Flying Handbook
13	Turning Flight and the V-n Diagram	Maneuvering speed and the V-n diagram; Lab F	MIT 16.01
14	Performance of Electric, Hybrid and Sustainable-Fuel Aircraft	Electric and hybrid aircraft performance	NASA HyTEC
15	Performance of Electric, Hybrid and Sustainable-Fuel Aircraft	Sustainable fuel, CO_2 and the full performance picture; final practice	DOE SAF; NASA

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

The Standard Atmosphere, Airspeeds and Density Altitude

Why performance depends on the air, the International Standard Atmosphere (ISA), temperature, pressure and density with altitude, the speed of sound and Mach number, indicated, calibrated, equivalent and true airspeed, and pressure and density altitude.

Learning objectives

By the end of this unit you can:

- Compute ISA temperature, pressure and density in the troposphere.
- Find the speed of sound and Mach number.
- Convert between equivalent and true airspeed.
- Explain why hot, high airports hurt performance.

Key concepts

- Lift, drag and engine thrust all depend on air density, so performance is always quoted for a stated atmosphere. The ISA is a reference: 288.15 K, 101,325 Pa and 1.225 kg/m³ at sea level.
- In the troposphere (up to 11 km) temperature falls 6.5 K per km: T = T_0 − 0.0065h. Hydrostatics plus the ideal-gas law give p/p_0 = (T/T_0)^5.26 and ρ/ρ_0 = (T/T_0)^4.26. Above 11 km temperature is constant at 216.65 K and density falls exponentially.
- The speed of sound is a = √(γRT); Mach number M = V/a. Because a depends only on temperature, it is lower at cruise altitude.
- An airspeed indicator senses dynamic pressure ½ρV². Equivalent airspeed EAS = V√σ, with density ratio σ = ρ/ρ_0; true airspeed is higher than EAS at altitude.
- Pressure altitude is the ISA altitude with the measured pressure; density altitude is the ISA altitude with the actual density. High density altitude (a high, hot or humid airfield) means thinner air, so less lift at a given speed and less engine thrust: takeoff gets longer and climb gets weaker.

Key equations

$$T = T_0 - 0.0065h$$

`T=T_0-0.0065h`

Troposphere temperature

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{4.26}$$

`(ρ)/(ρ_0)=((T)/(T_0))^4.26`

Troposphere density

$$a = \sqrt{\gamma RT},\ M = \frac{V}{a}$$

a=γ RT,\ M=V/a

Speed of sound and Mach

$$V_E = V\sqrt{\sigma}$$

V_E=Vσ

Equivalent airspeed

Worked examples, step by step

Example 1.1 Find the ISA temperature, pressure and density at 3000 m.

1. T = 288.15 − 0.0065 × 3000 = 268.65 K.
2. p = 101,325 × (268.65/288.15)^5.26 = 70.1 kPa.
3. ρ = 1.225 × (268.65/288.15)^4.26 = 0.909 kg/m³.

Answer: T = 268.65 K, p = 70.1 kPa, ρ = 0.909 kg/m³ (σ = 0.742).

Example 1.2 Find the speed of sound at sea level and at 11 km (216.65 K).

1. a_0 = √(1.4 × 287 × 288.15).
2. a = √(1.4 × 287 × 216.65).

Answer: a_0 = 340.3 m/s; a = 295.0 m/s.

Example 1.3 Find the ISA density and pressure at 11 km.

1. T/T_0 = 216.65/288.15.
2. ρ = 1.225 × (0.7519)^4.26; p = 101,325 × (0.7519)^5.26.

Answer: ρ = 0.364 kg/m³ and p = 22.6 kPa: less than a third of sea-level density.

Practice problems

Work each on paper first, then check.

1. Find ISA density at 5000 m.

0.736 kg/m³.

2. What Mach number is 250 m/s at sea level?

   250/340.3 = 0.735.

3. Find the exponent g/(aR) − 1 for density.

   9.81/(0.0065 × 287) − 1 = 4.26.

4. An airport at 1500 m on a hot day: is its density altitude above or below 1500 m?

   Above: hot air is thinner.

Common misconceptions

Myth: True airspeed equals indicated airspeed.
Actually: Only at sea-level standard conditions.

Myth: The speed of sound depends on pressure.
Actually: For an ideal gas it depends only on temperature.

Myth: Density altitude is the airport elevation.
Actually: It also depends on temperature, pressure and humidity.

Flashcards

ISA

288.15 K, 101,325 Pa, 1.225 kg/m³.

Lapse rate

0.0065 K/m.

Density ratio

$\sigma = \rho/\rho_0$.

Speed of sound

$\sqrt{(\gamma RT)}$.

EAS

$V\sqrt{\sigma}$.

Density altitude

ISA altitude with the actual density.

Self-check quiz

1. ISA sea-level density is...

1.000 kg/m³

1.225 kg/m³

0.364 kg/m³

101.3 kg/m³

1.225 kg/m³. Standard value.

2. Troposphere lapse rate is...

6.5 K/km

9.8 K/km

0

2 K/km

6.5 K/km. ISA definition.

3. The speed of sound depends on...

pressure

temperature

density only

airspeed

temperature. a = √(γRT).

4. High density altitude makes takeoff...

shorter

longer

unchanged

impossible

longer. Less lift and thrust.

Try this quiz again

How this helps save the planet

EIA reports that about 29% of total U.S. energy use in 2025 went to transporting people and goods, that petroleum products supplied about 89% of transportation energy, and that jet fuel alone was 13% of transportation energy use.

EIA also notes that jet fuel powers jet airplanes and some helicopters, and that biofuels may be blended into jet fuel. Every one of those gallons is burned in the atmosphere modeled in this unit; where an aircraft flies in that atmosphere decides how much it burns.

On Saving Planets: Aerodynamics I · Earth Observation

Sources: U.S. EIA: Use of energy for transportation (https://www.eia.gov/energyexplained/use-of-energy/transportation.php)

Free learning resources

- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): Free FAA chapter on density altitude and performance.
- NASA Glenn: Dynamic pressure (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/dynamic-pressure/): Free NASA page.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes on the atmosphere.
- FAA Pilot's Handbook of Aeronautical Knowledge (FAA-H-8083-25C) (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/phak): Full FAA handbook.

☐ **I finished this unit**

Forces in Flight and the Drag Polar

The four forces, steady level flight, lift and drag coefficients, parasite and induced drag, the parabolic drag polar C_D = C_D0 + KC_L², Oswald efficiency and aspect ratio, maximum lift-to-drag ratio, and drag in cruise.

Learning objectives

By the end of this unit you can:

- Write the equations of steady level flight.
- Use the parabolic drag polar.
- Find (L/D)max and the lift coefficient where it occurs.
- Compute cruise drag for an airliner.

Key concepts

- In steady level flight lift equals weight and thrust equals drag: L = W = ½ρV²SC_L and T = D = ½ρV²SC_D.
- Drag has two parts. Parasite (zero-lift) drag C_D0 comes from skin friction and pressure drag; induced drag KC_L² is the price of making lift, with K = 1/(πeAR) for aspect ratio AR and Oswald efficiency e.
- The drag polar C_D = C_D0 + KC_L² sets almost every performance number in this course.
- Lift-to-drag ratio L/D = C_L/C_D is the aircraft's aerodynamic efficiency. It is largest when induced drag equals parasite drag: C_L* = √(C_D0/K) and (L/D)max = 1/(2√(KC_D0)).
- At (L/D)max the drag is smallest for a given weight: D_min = W/(L/D)max. Long thin wings (high AR) lower K, which is why gliders and the most efficient airliners have them.

Key equations

$$L = W = \tfrac{1}{2}\rho V^2 S C_L$$

`L=W=12ρ V^2SC_L`

Level-flight lift

$$C_D = C_{D0} + K C_L^2$$

`C_D=C_D0+KC_L^2`

Parabolic drag polar

$$K = \frac{1}{\pi e AR}$$

K=(1)/(π eAR)

Induced-drag factor

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{KC_{D0}}}$$

(LD)_max=12_D0

Best lift-to-drag ratio

Worked examples, step by step

Example 2.1 The sample airliner has m = 60,000 kg, S = 120 m², C_D0 = 0.020 and K = 0.045. Find W, (L/D)max, C_L* and the minimum drag.

1. W = 60,000 × 9.81 = 588.6 kN.
2. (L/D)max = 1/(2√(0.045 × 0.020)) = 16.67.
3. C_L* = √(0.020/0.045) = 0.667.
4. D_min = 588,600/16.67.

Answer: (L/D)max = 16.67 at C_L = 0.667; D_min = 35.3 kN.

Example 2.2 The airliner cruises at 11 km (ρ = 0.364 kg/m³) at 230 m/s. Find q, C_L, C_D and drag.

1. q = ½ × 0.3636 × 230² = 9618 Pa.
2. C_L = 588,600/(9618 × 120) = 0.510.
3. C_D = 0.020 + 0.045 × 0.510² = 0.0317.
4. D = 9618 × 120 × 0.0317.

Answer: D = 36.6 kN; L/D = 16.1, close to the maximum.

Example 2.3 What is the cruise Mach number?

1. M = 230/295.0.

Answer: M = 0.780.

Practice problems

Work each on paper first, then check.

1. Halve C_{D0} with K unchanged. How does $(L/D)_{max}$ change?

   Rises by $\sqrt{2}$: 23.6.

2. Find K for $AR = 9$ and $e = 0.8$.

   $1/(\pi \times 0.8 \times 9) = 0.0442$.

3. At C_L^*, what fraction of drag is induced?

   Half.

4. Find the cruise L/D above.

   0.510/0.0317 = 16.1.

Common misconceptions

Myth: Drag always grows with speed.
Actually: Induced drag falls with speed; total drag has a minimum.

Myth: Bigger wings always mean less drag.
Actually: They add parasite drag.

Myth: L/D is fixed for an aircraft.
Actually: It depends on C_L, so on speed, weight and altitude.

Flashcards

Level flight

$L = W$, $T = D$.

Drag polar

$C_{D0} + KC_L^2$.

K

$1/(\pi eAR)$.

C_L^*

$\sqrt{(C_{D0}/K)}$.

$(L/D)_{max}$

$1/(2\sqrt{(KC_{D0})})$.

D_{min}

$W/(L/D)_{max}$.

Self-check quiz

1. In steady level flight...

L = W, T = D

L = T

D = W

L = 0

L = W, T = D. Force balance.

2. (L/D)max occurs when...

induced = parasite drag

$C_L = 0$

$C_L = C_{Lmax}$

V is maximum

induced = parasite drag. Minimize C_D/C_L.

3. Raising aspect ratio...

raises K

lowers K

raises C_{D0}

has no effect

lowers K. $K = 1/(\pi e AR)$.

4. D_min equals...

W(L/D)max

W/(L/D)max

T_{max}

qS

W/(L/D)max. D = W/(L/D).

Try this quiz again

How this helps save the planet

NASA describes the truss-braced wing as a long, thin wing with aerodynamically shaped structural supports that could reduce fuel and operating costs for future airliners. In December 2025, NASA and Boeing tested a semispan model of Boeing's Subsonic Ultra Green Aircraft Research (SUGAR) concept in a large pressurized wind tunnel in Farnborough, England.

A long, thin wing is a high-aspect-ratio wing: it lowers K in the drag polar and raises (L/D)max. The struts carry the loads that would make such a wing too heavy on its own.

On Saving Planets: Aerodynamics I · Mechanics of Materials

Sources: NASA: NASA, Boeing Advance Truss-Braced Wing Research in Test (Apr. 29, 2026) (https://www.nasa.gov/image-article/nasa-boeing-advance-truss-braced-wing-research-in-test/)

Free learning resources

- NASA Glenn: The drag equation (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/drag-equation/): Free NASA page.
- NASA Glenn: Induced drag coefficient (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/induced-drag-coefficient/): Free NASA page.
- NASA Glenn: Lift-to-drag ratio (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-to-drag-ratio/): Free NASA page.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): MIT aerodynamics.

☐ **I finished this unit**

Thrust and Power Required

Thrust required versus speed, the minimum-drag speed, power required and the minimum-power speed, thrust and power available from jets and propellers, maximum and minimum level speeds, and the effect of altitude and weight.

Learning objectives

By the end of this unit you can:

- Plot thrust and power required against speed.
- Find the minimum-drag and minimum-power speeds.
- Explain how altitude changes these speeds.
- Find maximum level speed from thrust available.

Key concepts

- Thrust required equals drag: T_R = ½ρV²SC_D0 + 2KW²/(ρV²S). The first term grows with V², the second falls as 1/V², so T_R has a minimum.
- The minimum-drag speed is V_md = √(2W/(ρS)) (K/C_D0)^¼. It is the speed for (L/D)max.
- Power required is P_R = T_R V. It is least at V_mp = V_md/3^¼ = 0.760 V_md, where induced drag is three times parasite drag and L/D = 0.866(L/D)max.
- At altitude, all speeds scale as 1/√σ at the same weight: the true speed for best L/D rises, but the drag is unchanged.
- Jet thrust is roughly constant with speed and falls with density; propeller power is roughly constant, so propeller thrust falls with speed. Maximum level speed is where available thrust (or power) meets required.

Key equations

$$T_R = \tfrac{1}{2}\rho V^2 S C_{D0} + \frac{2KW^2}{\rho V^2 S}$$

```
T_R=12ρ V^2SC_D0+(2KW^2)/(ρ V^2S)
```

Thrust required

$$V_{md} = \sqrt{\frac{2W}{\rho S}} \left(\frac{K}{C_{D0}}\right)^{1/4}$$

```
V_md=(2W)/(ρ S)(_D0)^1/4
```

Minimum-drag speed

$$V_{mp} = \frac{V_{md}}{3^{1/4}}$$

V_mp=_md3^1/4

Minimum-power speed

$$P_R = T_R V$$

P_R=T_RV

Power required

Worked examples, step by step

Example 3.1 Find V_md for the sample airliner at sea level.

1. √(2 × 588,600/(1.225 × 120)) = 89.5 m/s.
2. (0.045/0.020)^¼ = 1.225.
3. V_md = 89.5 × 1.225.

Answer: V_md = 109.6 m/s.

Example 3.2 Find V_md at 11 km and the speed ratio.

1. Ratio = √(1.225/0.3636) = 1.835.
2. V_md = 109.6 × 1.835.

Answer: V_md = 201.2 m/s at the same 35.3 kN drag.

Example 3.3 Find V_mp at sea level and the power required at V_md.

1. V_mp = 109.6/1.316 = 83.3 m/s.
2. P_R(V_md) = 35,316 × 109.6.

Answer: V_mp = 83.3 m/s; P_R at V_md = 3.87 MW.

Practice problems

Work each on paper first, then check.

1. Find the drag at V_mp.

W/(0.866 × 16.67) = 40.8 kN.

2. Find q at 100 m/s at sea level.

   ½ × 1.225 × 100^2 = 6125 Pa.

3. If weight rises 21%, how does V_md change?

   By √1.21 = 1.1 times.

4. Below V_md, is the aircraft speed-stable?

   No: slowing raises drag (the back side of the drag curve).

Common misconceptions

Myth: Minimum drag and minimum power happen at the same speed.
Actually: V_mp is 24% slower.

Myth: Flying higher lowers drag at best L/D.
Actually: Drag is the same; only the speed is higher.

Myth: Thrust required is the same as power required.
Actually: Power = thrust × speed.

Flashcards

Thrust required

Equals drag.

V_md

Speed for (L/D)max.

V_mp

0.760 V_md.

Power required

T_R V.

Altitude

Speeds ∝ 1/√σ.

Back side

Below V_md, drag rises as speed falls.

Self-check quiz

1. Thrust required is lowest at...

V_md

V_mp

V_stall

V_max

V_md. Minimum drag.

2. V_mp/V_md equals...

0.760

1.316

1

0.5

0.760. 3^(−¼).

3. At altitude, V_md (true) is...

higher

lower

the same

zero

higher. ∝ 1/√ρ.

4. Propeller thrust with speed...

falls

rises

is constant

is zero

falls. Constant power.

Try this quiz again

How this helps save the planet

NASA explains that during flight a thin layer of air, the boundary layer, forms very near an aircraft's surface, and that on most aircraft it becomes turbulent, increasing friction drag and fuel use. NASA's Crossflow Attenuated Natural Laminar Flow (CATNLF) design aims to keep that flow laminar on large swept wings.

A NASA computational study from 2014 to 2017 estimated that a CATNLF wing on a large, long-range aircraft like the Boeing 777 could save up to 10% of fuel a year. In this unit's terms, laminar flow lowers C_D0, which lowers thrust and power required at every speed.

On Saving Planets: Fluid Mechanics · Aerodynamics I

Sources: NASA: NASA Tests Technology Offering Potential Fuel Savings for Commercial Aviation (Jan. 21, 2026) (https://www.nasa.gov/centers-and-facilities/armstrong/nasa-tests-technology-offering-potential-fuel-savings-for-commercial-aviation/)

Free learning resources

- NASA Glenn: What is drag? (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/what-is-drag/): Free NASA page.
- NASA Glenn: Thrust-to-weight ratio (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/thrust-to-weight-ratio/): Free NASA page.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT performance notes.
- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on thrust and power curves.

☐ **I finished this unit**

Climb Performance

Forces in a steady climb, climb angle and rate of climb, excess thrust and excess power, best-angle and best-rate speeds, specific excess power, time to climb, and absolute and service ceilings.

Learning objectives

By the end of this unit you can:

- Write the force balance in a steady climb.
- Compute rate and angle of climb.
- Distinguish best-angle and best-rate speeds.
- Estimate time to climb and ceilings.

Key concepts

- In a steady climb at flight-path angle γ: T − D − W sin γ = 0 and L = W cos γ. So sin γ = (T − D)/W: excess thrust sets the climb angle.
- Rate of climb RC = V sin γ = (T − D)V/W = (P_avail − P_req)/W: excess power sets the climb rate.
- The best-angle speed (V_x) maximizes excess thrust and is used to clear obstacles; the best-rate speed (V_y) maximizes excess power and gets to altitude soonest.
- Specific excess power P_s = (T − D)V/W combines climb and acceleration: it is the rate at which energy height h + V²/(2g) can grow.
- Excess power falls with altitude. The absolute ceiling is where maximum RC is zero; the service ceiling is where it is 0.5 m/s (100 ft/min).

Key equations

$$\sin\gamma = \frac{T - D}{W}$$

`γ=(T-D)/(W)`

Climb angle

$$RC = \frac{(T - D)V}{W}$$

`RC=((T-D)V)/(W)`

Rate of climb

$$P_s = \frac{(T - D)V}{W}$$

`P_s=((T-D)V)/(W)`

Specific excess power

$$h_E = h + \frac{V^2}{2g}$$

`h_E=h+(V^2)/(2g)`

Energy height

Worked examples, step by step

Example 4.1 The sample airliner climbs at 130 m/s at sea level with 120 kN of thrust. Find C_L and drag.

1. q = ½ × 1.225 × 130² = 10,351 Pa.
2. C_L = 588,600/(10,351 × 120) = 0.474.
3. D = 10,351 × 120 × (0.020 + 0.045 × 0.474²).

Answer: C_L = 0.474; D = 37.4 kN (taking L ≈ W).

Example 4.2 Find the rate and angle of climb.

1. RC = (120,000 − 37,394) × 130/588,600.
2. sin γ = 82,606/588,600.

Answer: RC = 18.2 m/s (3590 ft/min); γ = 8.07°.

Example 4.3 At an average 10 m/s, how long does a climb to 10,000 m take?

1. t = 10,000/10 = 1000 s.

Answer: t = 16.7 min.

Practice problems

Work each on paper first, then check.

1. Convert 18.2 m/s to ft/min.

   18.2 × 196.85 = 3590 ft/min.

2. Which speed clears an obstacle best?

   V_x, best angle.

3. What is RC at the absolute ceiling?

   Zero.

4. Why does climb fall with altitude?

   Thrust and excess power fall with density.

Common misconceptions

Myth: Climb comes from extra lift.
Actually: It comes from excess thrust or power; in a climb L < W.

Myth: V_x and V_y are the same.
Actually: V_x is slower.

Myth: Climb rate is independent of weight.
Actually: RC ∝ 1/W.

Flashcards

Climb angle

sin γ = (T − D)/W.

Rate of climb

(T − D)V/W.

V_x

Best angle.

V_y

Best rate.

P_s

Specific excess power.

Ceilings

Absolute: RC = 0; service: 0.5 m/s.

Self-check quiz

1. Climb angle depends on...

excess thrust

excess lift

wing area only

Mach

excess thrust. sin γ = (T − D)/W.

2. Rate of climb depends on…

excess power

drag only

C_Lmax

K only

excess power. RC = ΔP/W.

3. Service ceiling RC is…

0

0.5 m/s

5 m/s

10 m/s

0.5 m/s. 100 ft/min.

4. In a steady climb, lift is…

W cos γ

W

W sin γ

T

W cos γ. Normal balance.

Try this quiz again

How this helps save the planet

NASA's sustainable aviation page notes that needing less energy to fly means using less fuel or, for electric aircraft, less power that must be generated elsewhere, which lowers lifecycle emissions for the whole time an aircraft is in use.

Climb is the most energy-hungry part of a short flight. Specific excess power shows why: every kilogram of weight saved and every newton of drag removed raises the climb rate and shortens the high-thrust phase.

On Saving Planets: Dynamics · Solutions

Sources: NASA: Sustainable Aviation (https://www.nasa.gov/directorates/armd/sustainable-aviation/)

Free learning resources

- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on climb performance.
- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): FAA handbook on climbs.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT climb notes.
- NASA Glenn: Thrust-to-weight ratio (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/thrust-to-weight-ratio/): Free NASA page.

☐ **I finished this unit**

Gliding Flight

Forces in an unpowered glide, glide angle and glide ratio, distance from a given height, sink rate and minimum sink, the effect of weight and wind, and engine-out planning.

Learning objectives

By the end of this unit you can:

- Relate glide angle to L/D.
- Find glide distance from a height.
- Compute sink rate.
- Explain the effects of weight and wind.

Key concepts

- In a steady glide, thrust is zero: D = W sin γ and L = W cos γ, so tan γ = D/L = 1/(L/D).
- Glide distance from height h is d = h(L/D). The flattest glide is at (L/D)max, so at V_md.
- Sink rate V_s = V sin γ ≈ V/(L/D). Minimum sink occurs at the minimum-power speed V_mp, best for staying up longest.
- Weight does not change the best glide ratio, only the speed at which it happens (heavier gliders fly faster at the same angle). Headwind shortens the distance over the ground; tailwind lengthens it.
- NASA notes that the Wright brothers measured their gliders' performance by flying parallel to a dune slope, so the glide angle equalled the slope of the hill.

Key equations

$$\tan\gamma = \frac{1}{L/D}$$

γ=(1)/(L/D)

Glide angle

$$d = h\,(L/D)$$

d=h (L/D)

Glide distance

$$V_s \approx \frac{V}{L/D}$$

Sink rate

$$D = W \sin \gamma$$

Glide force balance

Worked examples, step by step

Example 5.1 The sample airliner loses all engines at 10 km. How far can it glide at (L/D)max = 16.67 in still air?

1. d = 10,000 × 16.67.

Answer: d = 166.7 km.

Example 5.2 A sailplane with L/D = 40 is 1000 m up. Find its glide distance and angle.

1. d = 1000 × 40 = 40 km.
2. γ = arctan(1/40).

Answer: 40 km at 1.43°.

Example 5.3 Find the sailplane's sink rate at 30 m/s.

1. V_s = 30/40.

Answer: 0.75 m/s.

Practice problems

Work each on paper first, then check.

1. Find the sink rate at 25 m/s with L/D = 35.

   25/35 = 0.714 m/s.

2. Does adding water ballast change the best glide ratio?

   No, only the best-glide speed.

3. Glide for longest time or farthest distance: which speed is which?

   V_mp for time, V_md for distance.

4. Glide angle with L/D = 10?

   arctan(0.1) = 5.71°.

Common misconceptions

Myth: Heavier gliders glide less far.

Actually: Same distance in still air, just faster.

Myth: Pull the nose up to stretch a glide.

Actually: Below best-glide speed the glide gets steeper.

Myth: Minimum sink and best glide are the same speed.

Actually: Minimum sink is slower.

Flashcards

Glide angle

tan γ = 1/(L/D).

Glide distance

h(L/D).

Sink rate

V/(L/D).

Best glide

At V_md.

Minimum sink

At V_mp.

Weight

Changes speed, not ratio.

Self-check quiz

1. tan γ in a glide equals...

L/D

D/L

W/L

T/W

D/L. Force balance.

2. Best glide distance is at...

(L/D)max

C_Lmax

V_max

V_mp

(L/D)max. Flattest angle.

3. Longest time aloft is at...

V_mp

V_md

V_stall

V_max

V_mp. Minimum sink.

4. Extra weight changes...

best glide ratio

best glide speed

both equally

nothing

best glide speed. Speed ∝ √W.

Try this quiz again

How this helps save the planet

NASA's Beginner's Guide explains that the smaller the glide angle, the farther an aircraft can fly for a given change in altitude, so the glide angle is a measure of flying efficiency. The Wright brothers used gliders between 1900 and 1902 to learn aerodynamics and piloting before powered flight.

The glide ratio is just L/D, the same number that sets fuel burn in cruise. An aircraft that glides well also cruises on less fuel.

On Saving Planets: Intro to Aerospace Engineering · Aerodynamics I

Sources: NASA Glenn Beginner's Guide: Glide angle (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/glide-angle/)

Free learning resources

- NASA Glenn Beginner's Guide: Glide angle (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/glide-angle/): Free NASA page.
- NASA Glenn: Lift-to-drag ratio (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-to-drag-ratio/): Free NASA page.
- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): FAA handbook on emergency glides.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.

☐ **I finished this unit**

Range and Endurance: the Breguet Equations

Specific fuel consumption for jets and propellers, fuel weight fractions, the Breguet range and endurance equations, best-range and best-endurance speeds, cruise-climb, payload–range trade-offs and reserves.

Learning objectives

By the end of this unit you can:

- Use thrust- and power-specific fuel consumption.
- Apply the Breguet range equation for jets and propeller aircraft.
- Compute endurance.
- Identify which design changes extend range.

Key concepts

- A jet's thrust-specific fuel consumption c is fuel weight burned per unit thrust per unit time (1/s). A piston or turboprop's power-specific consumption is fuel per unit power per unit time.
- Integrating dW/dt = −cT with T = W/(L/D) at constant V and L/D gives the Breguet range for jets: R = (V/c)(L/D) ln(W_i/W_f).
- For propeller aircraft, with propeller efficiency η and power-specific consumption c_p (1/m): R = (η/c_p)(L/D) ln(W_i/W_f). Endurance for jets is E = (1/c)(L/D) ln(W_i/W_f).
- Range grows with three groups: engine efficiency (V/c or η/c_p), aerodynamics (L/D) and structures (the weight ratio). That is why the equation guides almost every aircraft improvement.
- Jets fly for best range a little faster than V_md (maximizing V·L/D) and for best endurance at V_md. Propeller aircraft fly for range at V_md and for endurance at V_mp.

Key equations

$$R = \frac{V}{c}\frac{L}{D}\ln\frac{W_i}{W_f}$$

```
R=V/cL/D(W_i)/(W_f)
```

Breguet range, jet

$$R = \frac{\eta}{c_p}\frac{L}{D}\ln\frac{W_i}{W_f}$$

```
R=()/(c_p)L/D(W_i)/(W_f)
```

Breguet range, propeller

$$E = \frac{1}{c}\frac{L}{D}\ln\frac{W_i}{W_f}$$

`E=1/cL/D(W_i)/(W_f)`

Endurance, jet

$$c_p = \frac{\mathrm{SFC}\cdot g}{3.6\times 10^6}$$

`c_p=· g3.6×10^6`

SFC in kg/kWh to 1/m

Worked examples, step by step

Example 6.1 A jet cruises at 230 m/s with c = 0.6 per hour and L/D = 16, burning fuel equal to 25% of its starting weight (W_f/W_i = 0.75). Find the range.

1. c = 0.6/3600 = 1.667 × 10⁻⁴ s⁻¹.
2. ln(1/0.75) = 0.2877.
3. R = (230/1.667 × 10⁻⁴) × 16 × 0.2877.

Answer: R = 6352 km.

Example 6.2 Find the endurance for the same aircraft.

1. E = (1/1.667 × 10⁻⁴) × 16 × 0.2877 = 27,618 s.

Answer: E = 7.67 h.

Example 6.3 A turboprop has η = 0.8, SFC = 0.3 kg/kWh, L/D = 15 and burns 15% of its weight (W_f/W_i = 0.85). Find the range.

1. c_p = 0.3 × 9.81/3.6 × 10⁶ = 8.175 × 10⁻⁷ m⁻¹.
2. ln(1/0.85) = 0.1625.
3. R = (0.8/8.175 × 10⁻⁷) × 15 × 0.1625.

Answer: R = 2386 km.

Practice problems

Work each on paper first, then check.

1. Raise L/D from 16 to 18 in the jet example. New range?

   7146 km (12.5% more).

2. Burn 30% of the weight instead of 25%. Range?

   ln(1/0.7) = 0.3567: 7875 km.

3. Which term does a laminar-flow wing improve?

   L/D.

4. Which term does a lighter structure improve?

   The weight ratio.

Common misconceptions

Myth: Range is proportional to fuel carried.
Actually: It grows with the log of the weight ratio.

Myth: Best range and best endurance are the same speed.
Actually: Endurance is flown slower.

Myth: Breguet needs constant altitude.
Actually: It assumes constant V and L/D, which means a slow cruise-climb.

Flashcards

TSFC

Fuel weight per thrust per time.

Breguet jet

(V/c)(L/D) ln(W_i/W_f).

Breguet prop

(η/c_p)(L/D) ln(W_i/W_f).

Endurance

(1/c)(L/D) ln(W_i/W_f).

Three levers

Engine, aerodynamics, structures.

Cruise-climb

Constant V and L/D as fuel burns.

Self-check quiz

1. Jet range is proportional to...

V(L/D)/c

c/V

L/D only

W_f

V(L/D)/c. Breguet.

2. The weight term in Breguet is...

W_i − W_f

ln(W_i/W_f)

W_i/W_f

W_f/W_i

ln(W_i/W_f). Integration of dW/W.

3. Jet endurance is best at...

V_md

V_mp

V_max

V_stall

V_md. Minimum drag = minimum fuel flow.

4. Propeller range is best at...

V_md

V_mp

V_max

Mach 1

V_md. (L/D)max.

Try this quiz again

How this helps save the planet

NASA and GE Aerospace tested an integrated hybrid-electric jet engine, a modified GE Passport, in December 2025 at GE Aerospace's Peebles Test Operation site in Ohio. NASA's Hybrid Thermally Efficient Core (HyTEC) project aims for a hybrid engine that burns up to 10% less fuel than today's best-in-class engines.

In Breguet terms, a 10% lower fuel burn means a smaller c: the same flight on less fuel, or a longer flight on the same fuel. The engine term, the aerodynamics term and the structures term each give engineers a lever.

On Saving Planets: Thermodynamics · Solutions

Sources: NASA: NASA, GE Aerospace Hybrid Engine System Marks Successful Test (Jan. 26, 2026) (https://www.nasa.gov/aeronautics/hybrid-engine-tested/)

Free learning resources

- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes on the Breguet equation.
- MIT OpenCourseWare 16.885J Aircraft Systems Engineering (Fall 2005) (https://ocw.mit.edu/courses/16-885j-aircraft-systems-engineering-fall-2005/): MIT aircraft systems.
- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on range and endurance.
- NASA Glenn: Lift-to-drag ratio (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-to-drag-ratio/): Free NASA page.

☐ **I finished this unit**

Takeoff Performance

Stall speed and C_Lmax, high-lift devices, the takeoff ground roll, liftoff speed, the effect of weight, density altitude, wind and runway slope, the airborne segment over an obstacle, and balanced field length.

Learning objectives

By the end of this unit you can:

- Find stall and liftoff speeds.
- Estimate takeoff ground roll.
- Show how weight and density altitude lengthen takeoff.
- Explain balanced field length.

Key concepts

- Stall speed is the slowest steady flight speed: V_s = √(2W/(ρSC_Lmax)). Flaps and slats raise C_Lmax for takeoff and landing.
- Liftoff speed is typically V_LOF ≈ 1.2V_s, giving a margin above stall.
- With thrust much larger than drag and friction, the ground roll is about s_g ≈ 1.44W²/(gρSC_LmaxT).
- Takeoff distance therefore grows roughly with W², and with 1/ρ twice over for jets (through V_s and through lower thrust). A 10 kt headwind shortens the roll; a tailwind lengthens it a lot.
- Balanced field length is where, if an engine fails at the decision speed V_1, stopping and continuing need the same distance.

Key equations

$$V_s = \sqrt{\frac{2W}{\rho S C_{L,max}}}$$

```
V_s=2Wρ SC_L,max
```

Stall speed

$$V_{LOF} \approx 1.2 V_s$$

Liftoff speed

$$s_g \approx \frac{1.44 W^2}{g \rho S C_{L,max} T}$$

Ground roll

$$\frac{s_2}{s_1} = \left(\frac{W_2}{W_1}\right)^2$$

`(s_2)/(s_1)=((W_2)/(W_1))^2`

Weight effect

Worked examples, step by step

Example 7.1 The sample airliner has C_Lmax = 2.4 with takeoff flaps at sea level. Find V_s and V_LOF.

1. V_s = √(2 × 588,600/(1.225 × 120 × 2.4)).
2. V_LOF = 1.2 × 57.8.

Answer: V_s = 57.8 m/s; V_LOF = 69.3 m/s.

Example 7.2 Estimate the ground roll with 180 kN thrust.

1. s_g = 1.44 × 588,600²/(9.81 × 1.225 × 120 × 2.4 × 180,000).

Answer: s_g = 801 m.

Example 7.3 Repeat at a 1500 m airport (ISA ρ = 1.058 kg/m³), same thrust.

1. s_g = 801 × 1.225/1.058.

Answer: s_g = 927 m, 1.158 times longer (more with the real thrust loss).

Practice problems

Work each on paper first, then check.

1. The aircraft is loaded to 70,000 kg. Ratio of ground rolls?

   (70/60)² = 1.361.

2. Stall speed ratio at 70,000 kg?

   √(70/60) = 1.080.

3. Why do flaps shorten takeoff?

   Higher C_Lmax, lower V_s.

4. What is V_1?

   The decision speed for a rejected or continued takeoff.

Common misconceptions

Myth: Takeoff distance is proportional to weight.
Actually: Closer to weight squared.

Myth: A hot day only affects engines.
Actually: It also lowers lift at a given speed.

Myth: Liftoff happens at stall speed.
Actually: A margin above stall, about 1.2V_s.

Flashcards

Stall speed

$\sqrt{(2W/(\rho S C_{Lmax}))}$.

V_LOF

$\approx 1.2V_s$.

Ground roll

$1.44W^2/(g\rho S C_{LmaxT})$.

Weight effect

$\propto W^2$.

Density altitude

Longer roll.

V_1

Decision speed.

Self-check quiz

1. Stall speed scales with...

$\sqrt{W}$

W

W^2

1/W

√W. $V_s \propto \sqrt{(W/\rho)}$.

2. Ground roll scales roughly with…

W

W^2

√W

1/W

W^2. $s_g \propto W^2/(\rho T)$.

3. Flaps raise…

C_{Lmax}

C_{D0} only

Mach

K only

C_{Lmax}. High-lift devices.

4. Balanced field length equalizes…

stop and go distances

V_1 and V_2

thrust and drag

L and W

stop and go distances. Definition.

Try this quiz again

How this helps save the planet

NASA and Boeing’s truss-braced wing model included slats, flaps and other moving surfaces so the team could set it to the low-speed, high-lift settings of takeoff and landing. NASA notes that the pressurized QinetiQ tunnel is used to predict airplane behavior in takeoff and landing conditions.

A very efficient cruise wing still has to meet this unit’s stall and runway numbers, which is why high-lift performance is tested as early as cruise drag.

On Saving Planets: Aerodynamics I · Statics

Sources: NASA: NASA, Boeing Advance Truss-Braced Wing Research in Test (Apr. 29, 2026) (https://www.nasa.gov/image-article/nasa-boeing-advance-truss-braced-wing-research-in-test/)

Free learning resources

- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): FAA handbook on takeoffs.
- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on takeoff performance charts.
- FAA PHAK Chapter 10: Weight and Balance (PDF) (https://www.faa.gov/sites/faa.gov/files/12_phak_ch10.pdf): FAA chapter on weight and balance.
- NASA Glenn: The lift equation (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/lift-equation/): Free NASA page.

☐ **I finished this unit**

Landing Performance

The approach, flare and ground roll, approach speed, braking, reverse thrust and spoilers, runway condition, landing distance estimates and the energy that must be removed.

Learning objectives

By the end of this unit you can:

- Find approach speed from stall speed.
- Estimate landing ground roll from deceleration.
- Explain how runway condition and spoilers matter.
- Compute the kinetic energy at touchdown.

Key concepts

- Landing has three segments: the approach over a 15 m (50 ft) obstacle, the flare and the ground roll.
- Approach speed is typically V_app ≈ 1.3V_s in the landing configuration.
- With an average deceleration a, the ground roll is s = V²/(2a). Brakes, spoilers (which dump lift so the wheels can brake) and reverse thrust all raise a.
- Wet or icy runways lower friction and can greatly lengthen the roll; performance charts give corrections.
- The kinetic energy at touchdown ½mV² must be absorbed, mostly as heat in the brakes.

Key equations

$$V_{app} \approx 1.3 V_s$$

Approach speed

$$s = \frac{V^2}{2a}$$

`s=(V^2)/(2a)`

Ground roll

$$E_k = \tfrac{1}{2} m V^2$$

`E_k=12mV^2`

Kinetic energy

$$a = \mu g$$

a=μ g

Ideal braking deceleration

Worked examples, step by step

Example 8.1 With landing flaps, C_Lmax = 2.8. Find V_s and V_app for the sample airliner at sea level.

1. V_s = √(2 × 588,600/(1.225 × 120 × 2.8)) = 53.5 m/s.
2. V_app = 1.3 × 53.5.

Answer: V_s = 53.5 m/s; V_app = 69.5 m/s.

Example 8.2 Estimate the ground roll from 69.5 m/s at an average 0.4g.

1. s = 69.5²/(2 × 0.4 × 9.81).

Answer: s = 616 m.

Example 8.3 Find the kinetic energy at 69.5 m/s.

1. E = ½ × 60,000 × 69.5².

Answer: E = 145 MJ.

Practice problems

Work each on paper first, then check.

1. Halve the deceleration. New ground roll?

   Doubles: 1232 m.

2. Why deploy spoilers at touchdown?

   They dump lift so the brakes work.

3. Approach 10% fast: change in roll?

$1.1^2 = 1.21$, 21% longer.

4. What is the approach obstacle height?

   15 m (50 ft).

Common misconceptions

Myth: Landing distance is just the ground roll.
Actually: It includes the approach and flare.

Myth: A few knots fast doesn't matter.
Actually: Roll grows with V^2.

Myth: Brakes work fully at touchdown.
Actually: Not until lift is dumped.

Flashcards

V_app

≈ 1.3V_s.

Ground roll

$V^2/(2a)$.

Spoilers

Dump lift.

Reverse thrust

Extra deceleration.

Wet runway

Longer roll.

Brake energy

$\frac{1}{2}mV^2$.

Self-check quiz

1. V_app is about...

1.3V_s

V_s

2V_s

0.8V_s

1.3V_s. Margin above stall.

2. Ground roll with deceleration a is...

$V^2/(2a)$

Va

V/a

2aV

$V^2/(2a)$. Kinematics.

3. Spoilers help braking by...

dumping lift

adding thrust

raising C_Lmax

cooling brakes

dumping lift. More weight on wheels.

4. 10% faster approach lengthens roll by...

10%

21%

5%

0%

21%. V^2.

Try this quiz again

How this helps save the planet

NASA lists operations and infrastructure as one of the three main ways to make aviation sustainable: using less energy on the ground or by choice of flight path reduces fuel burned and saves money.

Approach and landing are part of that. A well-planned continuous descent and a landing at the right speed use less fuel and less braking energy than a rushed, fast approach.

On Saving Planets: Dynamics · Solutions

Sources: NASA: Sustainable Aviation (https://www.nasa.gov/directorates/armd/sustainable-aviation/)

Free learning resources

- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): FAA handbook on approaches and landings.
- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on landing performance charts.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- PhET: Projectile Motion (https://phet.colorado.edu/en/simulations/projectile-motion): Kinematics interactively.

- [ ] **I finished this unit**

Turning Flight and the V-n Diagram

Level coordinated turns, bank angle and load factor, turn radius and rate, stall speed in a turn, maneuvering speed, the V-n diagram with limit load factors and gust loads, and pull-ups.

Learning objectives

By the end of this unit you can:

- Relate bank angle to load factor.
- Compute turn radius and rate.
- Find stall speed in a turn and maneuvering speed.
- Read a V-n diagram.

Key concepts

- In a level coordinated turn, lift tilts with the bank angle φ: L cos φ = W and L sin φ = mV²/R. The load factor n = L/W = 1/cos φ.
- Turn radius R = V²/(g√(n² − 1)) = V²/(g tan φ); turn rate ω = V/R.
- Stall speed rises with load factor: V_s,turn = V_s√n. At 60° bank, n = 2 and stall speed is 41% higher.
- Maneuvering speed V_A = V_s√n_limit is the speed at which full control deflection stalls the wing before the limit load is reached.
- The V-n diagram plots allowed load factor against speed: stall curves bound the left, limit load factors the top and bottom, and dive speed the right. Gust lines add the loads from turbulence.

Key equations

$$n = \frac{1}{\cos \phi}$$

Load factor in a level turn

$$R = \frac{V^2}{g \tan \phi}$$

R=(V^2)/(g)

Turn radius

$$\omega = \frac{V}{R}$$

Turn rate

$$V_{s,n} = V_s\sqrt{n}$$

`V_s,n=V_s n`

Stall speed under load

Worked examples, step by step

Example 9.1 Find n, turn radius and turn rate at 100 m/s for 30°, 45° and 60° bank.

1. n = 1/cos φ = 1.155, 1.414, 2.00.
2. R = 100²/(9.81 tan φ) = 1766 m, 1019 m, 589 m.
3. ω = V/R.

Answer: Turn rates 3.25°/s, 5.62°/s and 9.74°/s.

Example 9.2 How much does stall speed rise at 30°, 45° and 60° bank?

1. √n = 1.075, 1.189, 1.414.

Answer: 7.5%, 18.9% and 41.4%.

Example 9.3 A light aircraft has V_s = 50 m/s and n_limit = 2.5. Find V_A.

1. V_A = 50 × √2.5.

Answer: V_A = 79.1 m/s.

Practice problems

Work each on paper first, then check.

1. Find the turn radius at 60 m/s and 30° bank.

   60²/(9.81 × tan 30°) = 636 m.

2. Bank angle for n = 2?

   60°.

3. Why do airliners use about 25–30° bank?

Comfortable load factor, small stall-speed rise.

4. What bounds the left of the V-n diagram?

   The stall curve $n = (V/V_s)^2$.

Common misconceptions

Myth: A steeper bank makes a tighter turn at no cost.
Actually: It raises load factor and stall speed.

Myth: Maneuvering speed is a maximum cruise speed.
Actually: It protects against full control inputs.

Myth: Load factor depends only on speed.
Actually: It depends on bank angle in a level turn.

Flashcards

Load factor

$n = L/W = 1/\cos \varphi$.

Turn radius

$V^2/(g \tan \varphi)$.

Turn rate

V/R.

Stall in turn

$V_s\sqrt{n}$.

V_A

$V_s\sqrt{n_{limit}}$.

V-n diagram

Flight envelope.

Self-check quiz

1. Load factor at 60° bank is...

1.15

1.41

2

3

2. 1/cos 60°.

2. Turn radius is…

$V^2/(g \tan \varphi)$

gV

V/g

V tan φ

$V^2/(g \tan \varphi)$. Centripetal balance.

3. Stall speed in a turn is…

$V_s\sqrt{n}$

V_s n

V_s/n

V_s

$V_s\sqrt{n}$. More lift needed.

4. The V-n diagram shows…

allowed loads vs speed

drag vs speed

range vs fuel

altitude vs time

allowed loads vs speed. Flight envelope.

Try this quiz again

How this helps save the planet

NASA's 2026 CATNLF flight tests mounted a roughly 3-foot (40-inch) scale wing model vertically under an F-15B research jet. The first flight, on January 29, 2026, lasted 75 minutes and confirmed the aircraft could maneuver safely with the test article attached.

NASA says the first flight focused on envelope expansion, with turns, steady holds and gentle pitch changes between about 20,000 and nearly 34,000 feet, and that it was the first of up to 15 planned flights. Expanding an envelope step by step is the V-n diagram of this unit put into practice.

On Saving Planets: Dynamics · Mechanics of Materials

Sources: NASA: NASA Completes First Flight of Laminar Flow Scaled Wing Design (Feb. 11, 2026) (https://www.nasa.gov/image-article/nasa-completes-first-flight-of-laminar-flow-scaled-wing-design/)

Free learning resources

- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): FAA handbook on turns and maneuvers.
- FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf): FAA chapter on load factors.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- MIT OpenCourseWare 16.885J Aircraft Systems Engineering (Fall 2005) (https://ocw.mit.edu/courses/16-885j-aircraft-systems-engineering-fall-2005/): MIT aircraft systems.

☐ **I finished this unit**

Performance of Electric, Hybrid and Sustainable-Fuel Aircraft

Range of battery-electric aircraft, why weight does not fall as batteries discharge, specific energy of batteries versus jet fuel, hybrid-electric engines, sustainable aviation fuel, CO_2 from fuel burn, and putting all the performance levers together.

Learning objectives

By the end of this unit you can:

- Derive the range equation for a battery-electric aircraft.
- Compare the specific energy of batteries and jet fuel.
- Compute CO_2 from fuel burned.
- Rank technologies by which Breguet term they improve.

Key concepts

- A battery-electric aircraft does not get lighter as it flies, so there is no logarithm: R = (e_b η/g)(L/D)(m_b/m), with battery specific energy e_b (J/kg), total efficiency η and battery mass fraction m_b/m.
- Jet fuel holds about 43 MJ/kg (about 11,900 Wh/kg, a textbook value). Batteries of about 250 Wh/kg (a sample value) hold roughly 48 times less per kilogram, which is why electric aircraft today are small and short-range.
- Hybrid-electric engines use electric motors to help a jet engine run more efficiently, lowering the specific fuel consumption c.
- Burning a kilogram of jet fuel makes about 3.16 kg of CO_2 (a textbook value). Fuel saved by better L/D, lighter structures or better engines cuts CO_2 in direct proportion.
- Sustainable aviation fuel (SAF) has properties similar to conventional jet fuel, so it does not change the performance equations; it changes where the carbon comes from.

Key equations

$$R = \frac{e_b \eta}{g} \frac{L}{D} \frac{m_b}{m}$$

R=(e_b)/(g)L/D(m_b)/(m)

Battery-electric range

$$m_{CO_2} \approx 3.16\, m_{fuel}$$

CO_2 from jet fuel

$$\frac{e_{fuel}}{e_b} = \frac{43\ \mathrm{MJ/kg}}{e_b}$$

`_fuele_b=43\ MJ/kge_b`

Specific-energy ratio

$$R = \frac{V}{c}\frac{L}{D}\ln\frac{W_i}{W_f}$$

`R=V/cL/D(W_i)/(W_f)`

Breguet, for comparison

Worked examples, step by step

Example 10.1 An electric aircraft has 250 Wh/kg batteries (sample value), η = 0.8, L/D = 15 and a 30% battery fraction. Find its range.

1. e_b = 250 × 3600 = 900,000 J/kg.
2. R = (900,000 × 0.8/9.81) × 15 × 0.3.

Answer: R = 330 km.

Example 10.2 How much more energy per kilogram does jet fuel hold?

1. 43 × 10⁶/3600 = 11,944 Wh/kg.
2. 11,944/250.

Answer: About 47.8 times.

Example 10.3 How much CO_2 comes from burning 1000 kg of jet fuel?

1. 3.16 × 1000.

Answer: 3160 kg.

Practice problems

Work each on paper first, then check.

1. Repeat the electric range with 400 Wh/kg cells.

528 km.

2. An aircraft burns 2500 kg of fuel per hour. CO_2 per hour?

   7900 kg.

3. Which Breguet term does a hybrid engine improve?

   c, the fuel consumption.

4. Why is there no logarithm in the battery range equation?

   Mass stays constant.

Common misconceptions

Myth: Electric aircraft get lighter as they fly.
Actually: Battery mass stays the same.

Myth: SAF needs new engines.
Actually: It is designed to work in existing aircraft.

Myth: Only engines matter for emissions.
Actually: Aerodynamics, weight and operations all count.

Flashcards

Electric range

(e_bη/g)(L/D)(m_b/m).

Jet fuel

≈ 43 MJ/kg.

Battery

≈ 250 Wh/kg (sample).

CO_2

≈ 3.16 kg per kg fuel.

Hybrid engine

Lowers c.

SAF

Drop-in fuel from biomass and wastes.

Self-check quiz

1. Battery-electric range is proportional to...

e_b(L/D)(m_b/m)

ln(W_i/W_f)

c

V/c

e_b(L/D)(m_b/m). Constant mass.

2. Jet fuel vs 250 Wh/kg battery holds about...

2×

10×

48×

500×

48×. 11,944/250.

3. CO_2 per kg of jet fuel is about...

1 kg

3.16 kg

10 kg

0.5 kg

3.16 kg. Carbon plus oxygen.

4. SAF changes...

the carbon source

L/D

stall speed

K

the carbon source. Similar fuel properties.

Try this quiz again

How this helps save the planet

DOE says sustainable aviation fuel made from biomass and waste resources can deliver performance similar to petroleum jet fuel. DOE's 2023 Billion-Ton Report concluded that the United States could triple its biomass production to more than 1 billion tons a year, producing an estimated 60 billion gallons of liquid biofuels, enough feedstock to meet the projected fuel demand of U.S. aviation.

NASA notes that SAF does not release new CO_2 into the environment and has been shown to reduce contrails that can trap heat in Earth's atmosphere. NASA also estimates that with today's fleet and efficiency, worldwide air traffic in 2050 would need more than 620 megatons of fuel and produce close to 2,000 megatons of CO_2; the United States has committed to net-zero greenhouse gas emissions from U.S. aviation by 2050.

On Saving Planets: Solutions · Thermodynamics

Sources: U.S. DOE: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels) · NASA: Sustainable Aviation (https://www.nasa.gov/directorates/armd/sustainable-aviation/)

Free learning resources

- U.S. DOE: Sustainable Aviation Fuels (https://www.energy.gov/eere/bioenergy/sustainable-aviation-fuels): DOE overview of SAF.
- NASA: Sustainable Aviation (https://www.nasa.gov/directorates/armd/sustainable-aviation/): NASA overview of sustainable aviation.
- NASA: NASA, GE Aerospace Hybrid Engine System Marks Successful Test (Jan. 26, 2026) (https://www.nasa.gov/aeronautics/hybrid-engine-tested/): NASA hybrid engine test.
- MIT OpenCourseWare 16.885J Aircraft Systems Engineering (Fall 2005) (https://ocw.mit.edu/courses/16-885j-aircraft-systems-engineering-fall-2005/): MIT aircraft systems.

Watch the lecture

Lecture 12: Aircraft Performance (https://ocw.mit.edu/courses/16-687-private-pilot-ground-school-january-iap-2019/resources/lecture-12-aircraft-performance/) (in English)

MIT 16.687 Private Pilot Ground School (IAP 2019), MIT OpenCourseWare. License: CC BY-NC-SA 4.0 (https://creativecommons.org/licenses/by-nc-sa/4.0/) · Terms of use (https://ocw.mit.edu/pages/privacy-and-terms-of-use/). Linked for free study only; Saving Planets is not affiliated with or endorsed by this university.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. The Standard Atmosphere, Airspeeds and Density Altitude
   4 problems
2. Forces in Flight and the Drag Polar
   4 problems
3. Thrust and Power Required
   4 problems
4. Climb Performance
   4 problems
5. Gliding Flight
   4 problems
6. Range and Endurance: the Breguet Equations
   4 problems
7. Takeoff Performance
   4 problems
8. Landing Performance
   4 problems
9. Turning Flight and the V-n Diagram
   4 problems
10. Performance of Electric, Hybrid and Sustainable-Fuel Aircraft
    4 problems

The lab (Flight performance lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation, a spreadsheet, or with paper and a tape measure. Fly paper gliders only in a clear space away from people’s faces, stairs and traffic, and keep walkways free of trip hazards.

Safety reference: OSHA: Walking-working surfaces (slip, trip and fall hazards) (https://www.osha.gov/walking-working-surfaces)

Lab A: Build a standard-atmosphere calculator

WEEK 1

Goal: Build a spreadsheet of ISA temperature, density and speed of sound from sea level to 11 km.

$$T = T_0 - 0.0065h,\ \ \rho = \rho_0 (T/T_0)^{4.26},\ \ a = \sqrt{\gamma R T}$$

`T=T_0-0.0065h,\ \ ρ=ρ_0(T/T_0)^4.26,\ \ a=γ RT`

Troposphere model

Procedure

1. Make a column of altitudes every 500 m from 0 to 11,000 m.
2. Compute T, then ρ with the exponent 9.81/(0.0065 × 287) − 1.
3. Compute a = √(1.4 × 287 × T).
4. Plot ρ and a against altitude and compare with the values in Unit 1.

Data table

Altitude (m)	T (K)	ρ (kg/m³)	a (m/s)
0	288.15	1.225	340.3
3000	268.65	0.909	328.5
5000	255.65	0.736	320.5
11,000	216.65	0.364	295.0

Computed ISA values.

Sample calculation

1. ρ(3000) = 1.225 × (268.65/288.15)^4.26 = 0.909 kg/m³.
2. a(5000) = √(1.4 × 287 × 255.65) = 320.5 m/s.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn: Dynamic pressure (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/dynamic-pressure/) · MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unifie

d-engineering-i-ii-iii-iv-fall-2005-spring-2006/)

Planet link: EIA reports jet fuel was 13% of U.S. transportation energy use in 2025. U.S. EIA: Use of energy for transportation (https://www.eia.gov/energyexplained/use-of-energy/transportation.php)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Paper glider glide ratio

WEEK 6

Goal: Measure the glide ratio of a paper glider from its launch height and flight distance, and find its glide angle and sink rate.

$$\frac{L}{D} = \frac{d}{h}, \quad \gamma = \arctan\frac{h}{d}$$

LD= dh,\ \ γ= hd

Glide ratio and angle

Procedure

1. Fold a simple glider from printer paper; add a paper clip at the nose to trim it.
2. In a clear space, release it gently (no throw) from a measured height, about 1.5 m.
3. Measure where it lands and time the flight; repeat five times and average.
4. Move the paper clip and see how the glide ratio changes.

Data table

Quantity	Value
Release height	1.5 m
Distance	6.0 m
Glide ratio	4.0
Glide angle	14.0°
Flight time	2.0 s
Average speed	3.09 m/s

Sample values; real paper gliders have far lower L/D than sailplanes.

Sample calculation

1. L/D = 6.0/1.5 = 4.0; γ = arctan(1.5/6.0) = 14.0°.
2. Speed along the path = √(6² + 1.5²)/2.0 = 3.09 m/s; sink rate = 1.5/2.0 = 0.75 m/s.

> **Safety**
>
> - Fly only in a clear space, never toward people’s faces; keep walkways clear.
> - Do not stand on chairs or stairs to launch; use your standing height.

Home or virtual alternative

- This is a home lab.

NASA Glenn Beginner's Guide: Glide angle (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/glide-angle/) · PhET: Projectile Motion (https://phet.colorado.edu/en/simulations/projectile-motion)

Planet link: NASA notes the Wright brothers studied gliders from 1900 to 1902 and used glide angle to measure performance. NASA Glenn Beginner's Guide: Glide angle (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/glide-angle/)

☐ **I completed this lab (wet or virtual)**

Lab C: Measuring a drag polar in FoilSim

WEEK 2

Goal: Collect lift and drag coefficients at several angles of attack, fit a parabolic drag polar and find (L/D)max.

$$C_D = C_{D0} + KC_L^2$$

C_D=C_D0+KC_L^2

Parabolic drag polar

Procedure

1. Open NASA FoilSim and choose a wing shape and speed.
2. Record C_L and C_D at five angles of attack.
3. Plot C_D against C_L² and fit a straight line: the intercept is C_D0 and the slope is K.
4. Compute (L/D)max = 1/(2√(KC_D0)).

Data table

C_L	C_D	L/D
0.2	0.0218	9.17
0.6	0.0362	16.6
1.0	0.0650	15.4

Sample values generated from the course polar (C_D0 = 0.020, K = 0.045); your FoilSim numbers will differ.

Sample calculation

1. C_D(0.6) = 0.020 + 0.045 × 0.36 = 0.0362.
2. (L/D)max = 1/(2√(0.045 × 0.020)) = 16.67 at C_L = 0.667.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any browser works.

NASA Glenn: FoilSim (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/foilsim/) · NASA Glenn: Induced drag coefficient (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/induced-drag-coefficient/)

Planet link: NASA's CATNLF wing aims to keep laminar flow on swept wings, lowering C_D0. NASA: NASA Tests Technology Offering Potential Fuel Savings for Commercial Aviation (Jan. 21, 2026) (https://www.nasa.gov/centers-and-facilities/armstrong/nasa-tests-technology-offering-potential-fuel-savings-for-commercial-aviation/)

- [ ] **I completed this lab (wet or virtual)**

Lab D: Breguet range trade study

WEEK 8

Goal: Use a spreadsheet to see how L/D, fuel consumption and fuel fraction change a jet's range.

$$R = \frac{V}{c}\frac{L}{D}\ln\frac{W_i}{W_f}$$

`R=V/cL/D(W_i)/(W_f)`

Breguet range

Procedure

1. Enter V = 230 m/s, c = 0.6 per hour and W_f/W_i = 0.75.
2. Compute range for L/D = 14, 16 and 18.
3. Then cut c by 10% and compare.
4. Rank the changes by kilometres gained.

Data table

L/D	Range (km)
14	5558
16	6352
18	7146

Computed values for a sample jet.

Sample calculation

1. R(14) = (230/1.667 × 10^{-4}) × 14 × 0.2877 = 5558 km.
2. Cutting c by 10% multiplies range by 1/0.9 = 1.111.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/)

Planet link: NASA's HyTEC project targets a hybrid engine burning up to 10% less fuel than today's best engines. NASA: NASA, GE Aerospace Hybrid Engine System Marks Successful Test (Jan. 26, 2026) (https://www.nasa.gov/aeronautics/hybrid-engine-tested/)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Takeoff distance and density altitude

WEEK 10

Goal: Compute how takeoff ground roll grows with airport elevation and aircraft weight.

$$s_g \approx \frac{1.44 W^2}{g \rho S C_{L,max} T}$$

Ground roll

Procedure

1. Use the sample airliner: W = 588.6 kN, S = 120 m², C_Lmax = 2.4, T = 180 kN.
2. Compute s_g at 0, 1500 and 3000 m ISA density, holding thrust constant.
3. Repeat at 70,000 kg.
4. Discuss why real jets lose even more because thrust also falls.

Data table

Elevation (m)	ρ (kg/m³)	Ground roll (m)
0	1.225	801
1500	1.058	927
3000	0.909	1079

Computed values with constant thrust (a simplification).

Sample calculation

1. s_g(0) = 1.44 × 588,600²/(9.81 × 1.225 × 120 × 2.4 × 180,000) = 801 m.
2. s_g(3000) = 801 × 1.225/0.909 = 1079 m.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

FAA PHAK Chapter 11: Aircraft Performance (PDF) (https://www.faa.gov/sites/faa.gov/files/13_phak_ch11.pdf) · FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook)

Planet link: NASA notes truss-braced wing models are tested in takeoff and landing high-lift settings. NASA: NASA, Boeing Advance Truss-Braced Wing Research in Test (Apr. 29, 2026) (https://www.nasa.gov/image-article/nasa-boeing-advance-truss-braced-wing-research-in-test/)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Turn performance table

WEEK 13

Goal: Build a table of load factor, turn radius and stall-speed increase against bank angle.

$$n = \frac{1}{\cos\phi}, \quad R = \frac{V^2}{g\tan\phi}$$

`n=(1)/(),\ \ R=(V^2)/(g)`

Level-turn relations

Procedure

1. Set V = 100 m/s.
2. Compute n and R for bank angles from 15° to 60°.
3. Add √n for the stall-speed increase.
4. Mark the bank angle where stall speed rises 10%.

Data table

Bank	n	R (m)
15°	1.035	3804
30°	1.155	1766
45°	1.414	1019
60°	2.000	589

Computed values.

Sample calculation

1. R(15°) = 100²/(9.81 × tan 15°) = 3804 m.
2. n(60°) = 1/cos 60° = 2.000.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook)

Planet link: NASA's first CATNLF flight included turns and holds from about 20,000 to nearly 34,000 feet. NASA: NASA Completes First Flight of Laminar Flow Scaled Wing Design (Feb. 11, 2026) (https://www.nasa.gov/image-article/nasa-completes-first-flight-of-laminar-flow-scaled-wing-design/)

- [ ] **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. Find the ISA temperature at 2000 m.
    1. 288.15 − 0.0065 × 2000.

    Answer: 275.15 K.

2. Find the ISA density at 2000 m.
    1. 1.225 × (275.15/288.15)^4.26.

    Answer: 1.006 kg/m³.

3. Find (L/D)max for C_D0 = 0.025 and K = 0.040.
    1. 1/(2√(0.04 × 0.025)).

    Answer: 15.81.

4. An aircraft weighing 500 kN has 50 kN excess thrust at 120 m/s. Find the rate of climb.
    1. 50,000 × 120/500,000.

    Answer: 12.0 m/s.

5. How far can an aircraft with L/D = 12 glide from 2000 m?
    1. 2000 × 12.

    Answer: 24 km.

Final practice set (all units)

1. Find the range of a jet with V = 240 m/s, c = 0.55 per hour, L/D = 17 and W_f/W_i = 0.8.

1. (240/(0.55/3600)) × 17 × ln 1.25.

Answer: 5959 km.

2. Find the sea-level stall speed for W = 100 kN, S = 30 m², C_Lmax = 1.8.

    1. √(2 × 100,000/(1.225 × 30 × 1.8)).

Answer: 55.0 m/s.

3. Takeoff weight rises 20%. Ratio of ground rolls?

    1. 1.2².

Answer: 1.44.

4. Find the landing roll from 60 m/s at 3 m/s².

    1. 60²/(2 × 3).

Answer: 600 m.

5. Find the turn radius at 80 m/s and 45° bank.

    1. 80²/(9.81 × tan 45°).

Answer: 652 m.

6. Find the range of an electric aircraft with 300 Wh/kg, η = 0.85, L/D = 18 and a 35% battery fraction.

    1. (300 × 3600 × 0.85/9.81) × 18 × 0.35.

Answer: 590 km.

Formula sheet

$$T = T_0 - 0.0065h,\ \rho/\rho_0 = (T/T_0)^{4.26},\ a = \sqrt{\gamma RT}$$

$$L = W = \tfrac{1}{2}\rho V^2 S C_L,\ C_D = C_{D0} + K C_L^2,\ K = 1/($$

$$(L/D)_{max} = 1/(2\sqrt{K C_{D0}}),\ C_L^* = \sqrt{C_{D0}/K}$$

$$V_{md} = \sqrt{2W/(\rho S)}(K/C_{D0})^{1/4},\ V_{mp} = V_{md}/3^{1/4}$$

$$\sin\gamma = (T - D)/W,\ RC = (T - D)V/W$$

$$\tan\gamma_{glide} = 1/(L/D),\ d = h(L/D)$$

$$R = (V/c)(L/D)\ln(W_i/W_f),\ E = (1/c)(L/D)\ln($$

$$V_s = \sqrt{2W/(\rho S C_{L,max})},\ s_g \approx 1.44W^2/(g\rho S C_{L,ma}$$

$$s = V^2/(2a),\ V_{app} \approx 1.3V_s \qquad n = 1/\cos\phi,\ R = V^2/(g\tan\phi),\ V_A = V_s\sqrt{n_{lim}}$$

$$R_{elec} = (e_b\eta/g)(L/D)(m_b/m)$$

Constants

Constant	Value
International Standard Atmosphere, sea level (textbook values)	ρ_0 = 1.225 kg/m³, T_0 = 288.15 K, p_0 = 101,325 Pa, lapse rate 0.0065 K/m to 11 km
Air	R = 287 J/(kg·K), γ = 1.4
Gravity	g = 9.81 m/s²
Sample airliner used throughout (sample values)	m = 60,000 kg, W = 588.6 kN, S = 120 m², C_D0 = 0.020, K = 0.045
Jet fuel (textbook values)	about 43 MJ/kg; about 3.16 kg CO_2 per kg burned
1 m/s	196.85 ft/min

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, handbooks and simulations alongside the course.

- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Free MIT lecture notes, including aircraft performance.
- MIT OpenCourseWare 16.885J Aircraft Systems Engineering (Fall 2005) (https://ocw.mit.edu/courses/16-885j-aircraft-systems-engineering-fall-2005/): Free MIT course on aircraft systems.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Free MIT aerodynamics course.
- FAA Pilot's Handbook of Aeronautical Knowledge (FAA-H-8083-25C) (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/phak): Free FAA handbook; Chapter 11 covers performance.
- FAA Airplane Flying Handbook (https://www.faa.gov/regulations_policies/handbooks_manuals/aviation/airplane_handbook): Free FAA handbook on takeoff, landing and maneuvers.
- NASA Glenn: Beginner's Guide to Aeronautics (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/): Free NASA explanations of lift, drag and gliding.
- NASA Glenn: FoilSim (https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/foilsim/): Free wing simulation.

Glossary

Absolute ceiling
Altitude where the maximum rate of climb is zero.

Balanced field length
Runway length where stopping and continuing after an engine failure need the same distance.

Breguet equation
Range or endurance from engine efficiency, L/D and weight ratio.

Density altitude
ISA altitude with the same air density as the actual air.

Drag polar
Relation between drag and lift coefficients.

Equivalent airspeed
True airspeed times $\sqrt{\sigma}$.

Glide ratio
Horizontal distance per unit of height lost; equal to L/D.

Induced drag
Drag caused by producing lift.

Load factor
Lift divided by weight.

Maneuvering speed
Speed at which full control input stalls the wing before limit load.

Parasite drag
Drag not due to lift.

Rate of climb
Vertical speed in a climb.

Service ceiling
Altitude where maximum climb rate is 0.5 m/s.

Specific excess power
Excess power per unit weight.

Specific fuel consumption
Fuel used per unit thrust or power per unit time.

Stall speed
Slowest steady flight speed, at C_{Lmax}.

Standard atmosphere
Reference model of temperature, pressure and density with altitude.

Sustainable aviation fuel
Jet fuel made from biomass and wastes.

V-n diagram
Allowed load factor against airspeed.

Wing loading
Weight divided by wing area.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Starts from the atmosphere and the force balance, picks the right speed (V_md, V_mp, V_x, V_y, V_s), keeps weight and mass straight, and explains which design lever changes the answer.
3 · Proficient	Solves the practice problems with occasional unit slips.
2 · Developing	Follows worked examples but mixes up range and endurance speeds or forgets density.
1 · Beginning	Plugs numbers into formulas without a diagram; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Fluid Mechanics ENGR 330
- Aerospace Structures AERO 330
- Aerodynamics I AERO 310
- All courses