Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Spring

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester junior aerospace structures course. It is modeled on the topics and depth of MIT 16.20 Structural Mechanics and the structures parts of MIT 16.01–16.04 Unified Engineering (free on MIT OpenCourseWare), and on courses such as Purdue AAE 352 and Georgia Tech AE 3140, using the FAA Pilot's Handbook chapter on aircraft construction and PhET simulations, which we link where they help. None of those authors, schools or agencies endorses this page.
>
> What we add: every unit ties to real work with a cited public source (composite truss-braced wings, high-rate composite manufacturing and lightweight materials), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation, a spreadsheet or a safe home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

An aircraft carries its own structure on every flight, so every kilogram of structure costs fuel for the aircraft's whole life. Lighter wings and fuselages, made possible by better analysis and better materials, are one of the main levers for cleaner flight.

NASA lists efficient aircraft technology as one of the three main ways to make aviation sustainable. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample or textbook values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (AERO 330).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Mechanics of Materials: stress, strain, bending, torsion and buckling basics.
- Statics: free-body diagrams, shear and moment diagrams.
- Calculus II and Linear Algebra: integrals and matrices.
- Aircraft Performance helps with load factors but is not required.

How to study

- Start every problem with a free-body diagram and the load path: where does the load enter, and how does it reach the supports?
- Keep limit and ultimate loads separate, and write the safety factor explicitly.
- For thin walls, think in shear flow q (N/m) first, then divide by thickness.
- Check stiffness and buckling, not just strength: light structures usually fail by buckling.
- Compare materials per kilogram (specific stiffness and strength), not per cubic metre.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for the free PhET Hooke’s Law simulation.
- A wooden or plastic ruler, a table edge, coins or small weights, tape and a second ruler for measuring.
- Dry spaghetti, a kitchen scale, paper clips and safety glasses.

- Spreadsheet software for the materials, fuselage and composites labs.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Flight Loads, Safety Factors and Materials	Load paths, load factors, safety factors, materials; Lab A	FAA PHAK Ch. 3
2	Wing Shear and Bending Loads	Wing shear and bending diagrams	MIT 16.01
3	Bending of Thin-Walled Beams	Bending stress and idealized sections; Lab B	MIT 16.20
4	Bending of Thin-Walled Beams	Spar cap sizing, unsymmetric bending	MIT 16.20
5	Torsion and Shear Flow in Closed Sections	Torsion and shear flow in closed sections	MIT 16.20
6	Shear in Open Sections and the Shear Center	Open sections and the shear center	MIT 16.20
7	Buckling of Columns, Stringers and Plates	Column buckling; midterm practice	MIT 16.20
8	Buckling of Columns, Stringers and Plates	Plate buckling and stiffened panels; Lab C	MIT 16.20
9	Pressurized Fuselages and Thin Shells	Pressurized fuselages; Lab D	FAA PHAK Ch. 3
10	Composite Materials and Laminates	Fibers, matrices, rule of mixtures	MIT 3.032
11	Composite Materials and Laminates	Laminates and manufacturing; Lab E	NASA HiCAM
12	Fatigue, Fracture and Damage Tolerance	Fatigue and Miner’s rule	MIT 3.032
13	Fatigue, Fracture and Damage Tolerance	Fracture and damage tolerance; Lab F	MIT 3.032
14	Lightweight Design and Sustainable Structures	Weight, fuel and material indices	DOE lightweight materials

Week	Unit	Topics	Reading
15	Lightweight Design and Sustainable Structures	Lifecycle and sustainable structures; final practice	NASA; DOE

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Flight Loads, Safety Factors and Materials

The structural parts of an aircraft and their load paths, load factors, limit and ultimate loads, the 1.5 safety factor, margin of safety, and choosing materials by specific stiffness and specific strength.

Learning objectives

By the end of this unit you can:

- Trace the load path through wing, fuselage and tail.
- Compute limit and ultimate loads from a load factor.
- Compute a margin of safety.
- Compare materials by specific stiffness and strength.

Key concepts

- Lift on the wing skins goes into stringers and ribs, then into spars, then through the wing box into the fuselage. Every part of an aircraft is sized by following that load path.
- Loads scale with load factor n = L/W. The limit load is the largest load expected in service (for example n = 2.5 for many transports); the structure must carry it without permanent deformation.
- The ultimate load is the limit load times a safety factor, traditionally 1.5 in aviation; the structure must carry it without failing, if only for a few seconds.
- Margin of safety MS = allowable/actual − 1. A positive margin passes; aircraft designers aim for small positive margins because extra margin means extra weight.
- Because weight matters, materials are compared per kilogram: specific stiffness E/ρ and specific strength σ/ρ. Aluminum, titanium and steel have almost the same E/ρ; carbon composites beat all three.

Key equations

$$n = \frac{L}{W}$$

`n=L/W`

Load factor

$$P_{ult} = 1.5\, P_{limit}$$

`P_ult=1.5 P_limit`

Ultimate load

$$MS = \frac{\sigma_{allow}}{\sigma} - 1$$

MS=σ_allowσ-1

Margin of safety

$$\frac{E}{\rho}, \frac{\sigma}{\rho}$$

Specific stiffness and strength

Worked examples, step by step

Example 1.1 The sample airliner weighs 588.6 kN and has a limit load factor of 2.5. Find the limit and ultimate wing loads.

1. Limit = 2.5 × 588.6 = 1471.5 kN.
2. Ultimate = 1.5 × 1471.5.

Answer: Limit 1471.5 kN; ultimate 2207 kN.

Example 1.2 A 7075 part (σ_y = 503 MPa) sees 400 MPa at limit load. Find the margin of safety on yield.

1. MS = 503/400 − 1.

Answer: MS = 0.258 (positive, so it passes).

Example 1.3 Compare the specific stiffness of 7075 aluminum and the sample carbon laminate.

1. Al: 71.7 × 10^9/2810 = 25.5 MJ/kg.
2. CFRP: 70 × 10^9/1600 = 43.75 MJ/kg.

Answer: The laminate is 1.71 times stiffer per kilogram.

Practice problems

Work each on paper first, then check.

1. Find the specific strength of 7075 aluminum.

503 × 10^6/2810 = 179 kN·m/kg.

2. Find the specific strength of Ti-6Al-4V.

   880 × 10^6/4430 = 199 kN·m/kg.

3. Find E/ρ for 4130 steel.

   205 × 10^9/7850 = 26.1 MJ/kg, almost the same as aluminum.

4. What does MS = −0.1 mean?

   The part fails the requirement; redesign.

Common misconceptions

Myth: A bigger safety factor is always better.
Actually: It adds weight and fuel burn for life.

Myth: Steel is stiffer, so it makes stiffer aircraft.
Actually: Per kilogram it is no stiffer than aluminum.

Myth: Ultimate load is the expected load.
Actually: Limit load is the expected maximum; ultimate is 1.5 times it.

Flashcards

Load factor

n = L/W.

Limit load

Largest expected in service.

Ultimate load

1.5 × limit.

Margin of safety

σ_allow/σ − 1.

Specific stiffness

E/ρ.

Specific strength

σ/ρ.

Self-check quiz

1. Ultimate load equals...

1.5 × limit

limit

2 × limit

limit/1.5

1.5 × limit. Aviation safety factor.

2. Specific stiffness is...

E/ρ

Eρ

σ/E

ρ/E

E/ρ. Per unit mass.

3. A positive margin of safety means...

the part passes

it fails

it is too light

nothing

the part passes. Allowable exceeds actual.

4. Lift reaches the fuselage mainly through...

the spars and wing box

the windows

the engines

the landing gear

the spars and wing box. Load path.

Try this quiz again

How this helps save the planet

DOE's lightweight-materials program reports that for cars and trucks a 10% cut in vehicle weight can improve fuel economy by 6% to 8%, because it takes less energy to accelerate a lighter object. DOE lists typical mass reductions of 50–70% for carbon fiber composites, 30–60% for aluminum and 40–55% for titanium when they replace conventional steel parts.

Aircraft gain even more than cars from weight savings, because they must also hold their weight up for the whole flight. Choosing materials by specific stiffness and strength is where that saving starts.

On Saving Planets: Mechanics of Materials · Solutions

Sources: U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks)

Free learning resources

- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on airframe parts.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT structures notes.
- PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law): Stiffness interactively.

I finished this unit

Wing Shear and Bending Loads

The wing as a cantilever beam, spanwise lift distributions, shear force and bending moment diagrams, root loads, relief from wing weight and fuel, and how load factor scales everything.

Learning objectives

By the end of this unit you can:

- Model a wing as a cantilever.
- Draw shear and bending moment diagrams for a distributed lift.
- Compute root shear and root bending moment.
- Explain bending relief from fuel and engines on the wing.

Key concepts

- Each wing half carries half the lift and acts as a cantilever clamped at the fuselage. Shear and moment are largest at the root.
- For a uniform lift w (N/m) on a semispan s, the root shear is V = ws and the root moment is M = ws²/2.
- An elliptical lift distribution puts the half-lift's centroid at 4s/(3π) from the root, a little inboard of the uniform case's s/2, so the root moment is lower.
- Wing structure and fuel weigh down on the wing, opposite to lift. This bending relief is why airliners carry fuel in the wings and why wing-mounted engines help.
- All loads scale with load factor: at n = 2.5 the root moment is 2.5 times its 1-g value.

Key equations

$$V(y) = \int_{y}^{s} w \, dy$$

`V(y)=_y^s w dy`

Shear from distributed load

$$M(y) = \int_{y}^{s} V \, dy$$

`M(y)=_y^s V dy`

Moment from shear

$$M_{root} = \tfrac{1}{2} w s^2$$

`M_root=12ws^2`

Uniform lift

$$M_{root} = \frac{L}{2} \cdot \frac{4s}{3\pi}$$

`M_root=2·4s3π`

Elliptical lift

Worked examples, step by step

Example 2.1 The sample airliner (588.6 kN) has a 17 m semispan. Assuming uniform lift at 1 g, find w, root shear and root moment for one wing.

1. Half-lift = 294.3 kN; w = 294,300/17 = 17.31 kN/m.
2. V = 294.3 kN.
3. M = 17,312 × 17^2/2.

Answer: w = 17.31 kN/m; V = 294.3 kN; M = 2.50 MN·m.

Example 2.2 Repeat the moment for the elliptical distribution.

1. M = 294,300 × 4 × 17/(3π).

Answer: M = 2.12 MN·m, 15.1% lower.

Example 2.3 Find the uniform-lift root moment at limit (n = 2.5) and ultimate (×1.5).

1. Limit: 2.5 × 2.50 = 6.25 MN·m.
2. Ultimate: 1.5 × 6.25.

Answer: Limit 6.25 MN·m; ultimate 9.38 MN·m.

Practice problems

Work each on paper first, then check.

1. Halve the semispan at the same lift. Root moment?

   Halves (M = Ls/4 for uniform lift).

2. Find the moment ratio, elliptical to uniform.

$8/(3\pi) = 0.849$.

3. Why do airliners use wing fuel first or last?

   Fuel in the wing relieves bending; keeping it longer helps.

4. Root shear at n = 2.5?

   2.5 × 294.3 = 735.8 kN.

Common misconceptions

Myth: The wing tip carries the biggest loads.
Actually: The root does.

Myth: Wing weight adds to the bending moment.
Actually: It relieves it.

Myth: Moment scales with the semispan.
Actually: At fixed w it scales with s^2.

Flashcards

Cantilever wing

Clamped at the root.

Root shear

Half-lift.

Uniform root moment

$ws^2/2$.

Elliptical centroid

$4s/(3\pi)$.

Bending relief

Wing weight and fuel.

Load factor

Scales all loads.

Self-check quiz

1. Root moment for uniform lift w on span s is...

$ws^2/2$

ws

ws^3

w/s

$ws^2/2$. Cantilever.

2. Elliptical lift centroid is at...

$4s/(3\pi)$

s/2

s

s/3

$4s/(3\pi)$. From the root.

3. Fuel in the wing...

relieves bending

adds bending

does nothing

adds torsion only

relieves bending. Acts against lift.

4. Doubling n doubles...

all loads

only shear

only weight

nothing

all loads. Linear.

Try this quiz again

How this helps save the planet

NASA's SWEET-15 test article is a 15-foot composite wing supported by an aerodynamic strut, based on NASA's Transonic Truss-Braced Wing concept. A strut takes part of the bending load near mid-span, so the wing can be long and thin without a huge root moment.

NASA engineers bent the test wing for several months in the Flight Loads Laboratory at Armstrong, using numerous strain and load sensors including fiber-optic strain sensors. The data confirmed the computer models' predictions and showed the wing withstood the anticipated in-flight forces.

On Saving Planets: Statics · Aircraft Performance

Sources: NASA: NASA Pushes New Wing Design to Find Structural Limits (Jul. 17, 2026) (https://www.nasa.gov/centers-and-facilities/armstrong/nasa-pushes-new-wing-design-to-find-structural-limits/)

Free learning resources

- OpenStax UP1, 12.2 Examples of static equilibrium (https://openstax.org/books/university-physics-volume-1/pages/12-2-examples-of-static-equilibrium): Free textbook section.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT beam notes.
- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT structures.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on wings.

☐ **I finished this unit**

Bending of Thin-Walled Beams

Bending stress σ = My/I, idealizing a wing box as flanges and skins, second moment of area, flange forces, unsymmetric bending, and sizing spar caps for ultimate load.

Learning objectives

By the end of this unit you can:

- Compute bending stress from M, y and I.
- Idealize a box beam as concentrated flange areas.
- Size spar caps for an ultimate moment.
- Recognize unsymmetric bending.

Key concepts

- In bending, σ = My/I: stress grows linearly from the neutral axis. Material far from the neutral axis does most of the work.
- Wing boxes are idealized as concentrated flange (boom) areas carrying direct stress, joined by thin skins carrying shear.
- For two flanges of area A a depth h apart, I = 2A(h/2)² = Ah²/2 and the flange force is F = M/h.
- Deeper wing boxes need smaller flanges for the same moment. That is why thick wing roots save weight, and why thin, high-aspect-ratio wings need help such as struts.
- If the section is not symmetric, or the moment is not about a principal axis, bending about one axis causes deflection about the other (unsymmetric bending).

Key equations

$$\sigma = \frac{My}{I}$$

σ=My/I

Bending stress

$$I = \sum A_i y_i^2$$

I= A_iy_i^2

Idealized boom section

$$F = \frac{M}{h}$$

F=M/h

Flange force

$$A = \frac{M}{\sigma_{allow} h}$$

`A=σ_allowh`

Required flange area

Worked examples, step by step

Example 3.1 The ultimate root moment is 9.38 MN·m and the wing box is 0.6 m deep. Find the flange force.

1. F = 9.38 × 10⁶/0.6.

Answer: F = 15.6 MN.

Example 3.2 Size each flange for 400 MPa allowable stress.

1. A = 9.38 × 10⁶/(400 × 10⁶ × 0.6).

Answer: A = 0.0391 m² (391 cm²) per flange.

Example 3.3 Find I for two 0.0391 m² flanges 0.6 m apart, and check σ.

1. I = 0.0391 × 0.6²/2 = 0.00704 m⁴.
2. σ = 9.38 × 10⁶ × 0.3/0.00704.

Answer: σ = 400 MPa, as designed.

Practice problems

Work each on paper first, then check.

1. Double the box depth. Required flange area?

   Halves.

2. Where is bending stress zero?

   At the neutral axis.

3. Find I for flanges of 0.01 m^2 at ±0.25 m.

   $2 \times 0.01 \times 0.25^2 = 0.00125\ m^4$.

4. Why are spar caps placed at the top and bottom?

   Largest y gives the most I per kilogram.

Common misconceptions

Myth: Thin skins carry most of the bending.

Actually: In the idealization, flanges and stringers carry direct stress; skins carry shear.

Myth: Stress is uniform across a beam.

Actually: It varies linearly with y.

Myth: A deeper beam is heavier.

Actually: It usually needs less flange material.

Flashcards

Flexure

$\sigma = My/I$.

Boom idealization

Flanges carry σ, skins carry q.

Two-flange I

$Ah^2/2$.

Flange force

M/h.

Depth

Deeper is lighter.

Unsymmetric bending

Coupled deflections.

Self-check quiz

1. Bending stress is...

My/I

M/A

VQ/It

Tr/J

My/I. Flexure formula.

2. Flange force for depth h is...

M/h

Mh

M/h^2

Vh

M/h. Couple.

3. Deeper boxes need...

smaller flanges

larger flanges

the same

no flanges

smaller flanges. $A \propto 1/h$.

4. In idealized boxes, skins carry...

shear

direct stress only

nothing

torsion only

shear. Shear panels.

Try this quiz again

How this helps save the planet

NASA describes the truss-braced wing as a long, thin wing with aerodynamically shaped structural supports that could cut fuel and operating costs for future airliners. NASA and Boeing tested a semispan model of the concept in a large pressurized wind tunnel in December 2025.

A thin wing has a shallow wing box, so by F = M/h its flanges would need to be heavy. The strut lowers M, which lets the wing stay thin without growing heavy.

On Saving Planets: Mechanics of Materials · Aerodynamics I

Sources: NASA: NASA, Boeing Advance Truss-Braced Wing Research in Test (Apr. 29, 2026) (https://www.nasa.gov/image-article/nasa-boeing-advance-truss-braced-wing-research-in-test/)

Free learning resources

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on beam bending.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on spars.

☐ **I finished this unit**

Torsion and Shear Flow in Closed Sections

Shear flow q, Bredt–Batho torsion of thin-walled closed sections, skin shear stress, the rate of twist, multi-cell boxes, and why wings are built as closed boxes.

Learning objectives

By the end of this unit you can:

- Use shear flow q = τt.
- Apply Bredt–Batho: T = 2Aq.
- Compute the rate of twist of a closed section.
- Explain why closed sections resist torsion so well.

Key concepts

- In a thin wall, shear stress times thickness is the shear flow q = τt (N/m). Around a closed section carrying pure torque, q is constant.
- Bredt–Batho: T = 2A_e q, where A_e is the area enclosed by the wall's midline.
- The rate of twist is dθ/dz = (T/(4A_e²G)) ∮ds/t.
- Closed boxes are far stiffer in torsion than open sections of the same material, which is why wings are built as torsion boxes from spars and skins.
- Wing torsion comes from lift acting ahead of or behind the shear center, and from ailerons and engines. Too little torsional stiffness can cause aileron reversal or flutter.

Key equations

$$q = \tau t$$

`q=τ t`

Shear flow

$$T = 2A_e q$$

`T=2A_eq`

Bredt–Batho

$$\frac{d\theta}{dz} = \frac{T}{4A_e^2 G} \oint \frac{ds}{t}$$

`(dθ)/(dz)=(T)/(4A_e^2G)ds/t`

Rate of twist

$$\tau = \frac{T}{2A_e t}$$

`τ=(T)/(2A_et)`

Skin shear stress

Worked examples, step by step

Example 4.1 A 2.0 m × 0.6 m wing box carries a 500 kN·m torque. Find the shear flow.

1. A_e = 1.2 m².
2. q = 500,000/(2 × 1.2).

Answer: q = 208.3 kN/m.

Example 4.2 Find the skin shear stress for 4 mm skins.

1. τ = 208,333/0.004.

Answer: τ = 52.1 MPa.

Example 4.3 Find the rate of twist (G = 27 GPa, uniform 4 mm wall).

1. ∮ds/t = 5.2/0.004 = 1300.
2. dθ/dz = 500,000 × 1300/(4 × 1.2² × 27 × 10⁹).

Answer: dθ/dz = 4.18 × 10⁻³ rad/m.

Practice problems

Work each on paper first, then check.

1. A thin tube of radius 0.1 m and wall 2 mm carries 10 kN·m. Find τ.

   T/(2πr²t) = 79.6 MPa.

2. Find J for that tube.

   2πr³t = 1.257 × 10⁻⁵ m⁴.

3. Double the enclosed area at the same torque. Shear flow?

Halves.

4. Why does cutting a slit in a tube ruin its torsional stiffness?

   It becomes an open section.

Common misconceptions

Myth: Shear flow varies around a closed section in pure torsion.
Actually: It is constant.

Myth: Thicker skins always help torsion most.
Actually: Enclosed area matters more.

Myth: Open and closed sections are similar in torsion.
Actually: Closed ones are many times stiffer.

Flashcards

Shear flow

$q = \tau t$.

Bredt–Batho

$T = 2A_e q$.

Twist rate

$T\oint ds/t/(4A_e^2G)$.

Torsion box

Closed spars and skins.

Thin tube

$\tau = T/(2\pi r^2 t)$.

Flutter

Needs torsional stiffness.

Self-check quiz

1. Bredt–Batho gives T =...

2Aq

Aq

qt

q/A

2Aq. Closed section.

2. Shear flow is...

τt

τ/t

σt

T/A

τt. Definition.

3. Wings are built as closed boxes for...

torsional stiffness

looks

fuel only

lift

torsional stiffness. Torsion box.

4. In pure torsion q is...

constant

zero

largest at corners

random

constant. Equilibrium.

Try this quiz again

How this helps save the planet

NASA reports that SWEET-15 combined five advanced composite manufacturing and assembly technologies, and that the manufacturing approach, developed at Langley with the Integrated Structural Assembly of Advanced Composites robot, aims to produce lighter and stronger composite structures for aerospace vehicles.

A composite wing box still has to satisfy Bredt–Batho: the joints between skins and spars carry the shear flow that keeps the box from twisting.

On Saving Planets: Mechanics of Materials · Dynamics

Sources: NASA: NASA Pushes New Wing Design to Find Structural Limits (Jul. 17, 2026) (https://www.nasa.gov/centers-and-facilities/armstrong/nasa-pushes-new-wing-design-to-find-structural-limits/)

Free learning resources

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on torsion.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on wing structure.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.

☐ **I finished this unit**

UNIT 5 · WEEK 6

Shear in Open Sections and the Shear Center

Shear stress in beams τ = VQ/(It), shear flow in open thin-walled sections, the shear center, why channel sections twist under load, and shear lag.

Learning objectives

By the end of this unit you can:

- Use τ = VQ/(It).
- Find shear flow in an open thin-walled section.
- Locate the shear center qualitatively.
- Explain why loads off the shear center cause twist.

Key concepts

- Transverse shear produces shear stress τ = VQ/(It), where Q is the first moment of area beyond the cut.
- In open thin-walled sections, shear flow starts at zero at a free edge and builds up along the wall: q = VQ/I.
- The shear center is the point where a transverse load causes bending without twist. For symmetric I-beams and closed boxes it is on the symmetry axis; for a channel it lies outside the web.
- Loads applied away from the shear center twist the section. Designers place spars and stringers so the wing's shear center sits where aerodynamic loads act, or provide enough torsional stiffness.
- Shear lag: in wide flanges, direct stress is highest near the webs and lower in between, so not all of the flange is fully effective.

Key equations

$$\tau = \frac{VQ}{It}$$

`τ=VQ/It`

Beam shear stress

$$q = \frac{VQ}{I}$$

`q=VQ/I`

Shear flow

$$Q = \int y \, dA$$

`Q= y dA`

First moment of area

$$\tau_{max} = \frac{3V}{2A}$$

`τ_max=3V/2A`

Rectangle, maximum

Worked examples, step by step

Example 5.1 A solid rectangular beam 20 mm wide and 60 mm deep carries 6 kN of shear. Find the maximum shear stress.

1. A = 1200 mm².
2. τ_max = 1.5 × 6000/1200.

Answer: τ_max = 7.5 MPa at the neutral axis.

Example 5.2 An I-beam web is 4 mm thick; at the neutral axis Q = 50 × 10³ mm³ and I = 20 × 10⁶ mm⁴. Find τ for V = 40 kN.

1. τ = 40,000 × 50,000/(20 × 10⁶ × 4).

Answer: τ = 25 MPa.

Example 5.3 Find the shear flow there.

1. q = τt = 25 × 4.

Answer: q = 100 N/mm (100 kN/m).

Practice problems

Work each on paper first, then check.

1. Where is shear flow zero in an open section?

   At free edges.

2. Shear center of a closed symmetric box under symmetric load?

On the symmetry axis.

3. Why does a channel twist when loaded through its web?

   The shear center lies outside the web.

4. Double V on the rectangle. τ_max?

   15 MPa.

Common misconceptions

Myth: Shear stress is largest at the outer fibres.
Actually: It is largest at the neutral axis.

Myth: The shear center is the centroid.
Actually: Only for doubly symmetric sections.

Myth: Wide flanges are fully effective.
Actually: Shear lag reduces effectiveness.

Flashcards

Shear formula

τ = VQ/(It).

Open section

q starts at zero at free edges.

Shear center

No-twist load point.

Channel

Shear center outside web.

Rectangle

τ_max = 1.5V/A.

Shear lag

Uneven flange stress.

Self-check quiz

1. Beam shear stress is...

VQ/(It)

My/I

Tr/J

P/A

VQ/(It). Shear formula.

2. At a free edge, open-section shear flow is...

zero

maximum

constant

negative

zero. Boundary.

3. Loading through the shear center causes...

no twist

twist

buckling

yield

no twist. Definition.

4. τ_max in a rectangle is...

1.5V/A

V/A

2V/A

3V/A

1.5V/A. Parabolic.

Try this quiz again

How this helps save the planet

NASA's SWEET-15 test ended with a deliberate test to failure. The wing failed at roughly 127% of its design limit load, with visible damage near the back edge of the wing and in the upper wing cover, giving insight into how the joints to the main strut and the jury strut behave beyond the flight envelope.

NASA notes this was the first time a representative composite truss-braced wing configuration had undergone this type of structural evaluation. Tests like this check exactly the shear and torsion predictions in Units 4 and 5.

On Saving Planets: Mechanics of Materials · Statics

Sources: NASA: NASA Pushes New Wing Design to Find Structural Limits (Jul. 17, 2026) (https://www.nasa.gov/centers-and-facilities/armstrong/nasa-pushes-new-wing-design-to-find-structural-limits/)

Free learning resources

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on shear flow.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- OpenStax UP1, 12.2 Examples of static equilibrium (https://openstax.org/books/university-physics-volume-1/pages/12-2-examples-of-static-equilibrium): Free textbook section.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.

☐ I finished this unit

Buckling of Columns, Stringers and Plates

Euler column buckling and end conditions, slenderness, stringers as columns, plate buckling of skin panels, post-buckling strength of stiffened panels, and why light structures are usually sized by stability.

Learning objectives

By the end of this unit you can:

- Compute Euler buckling loads with end-fixity factors.
- Compute the buckling stress of a skin panel.
- Explain why stiffened panels carry load after skin buckling.
- Recognize when buckling, not yield, governs.

Key concepts

- A slender column buckles at P_cr = π²EI/(KL)², where K = 1 for pinned ends, 0.5 for fixed ends and 2 for a cantilever.
- Buckling depends on stiffness E and geometry, not strength, so high-strength materials do not help a slender column.
- A thin flat plate in compression buckles at σ_cr = kπ²E/(12(1 − ν²)) (t/b)², with k = 4 for a long simply supported panel.
- Aircraft skins are allowed to buckle in shear or compression at loads well below ultimate; the stringers and spars then carry the load (tension-field and post-buckling behavior).
- Because light structures are thin, buckling usually sets the size of compression skins, stringers and spar webs.

Key equations

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

`P_cr=(π^2EI)/((KL)^2)`

Euler buckling

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

`σ_cr=(kπ^2E)/(12(1-ν^2))( tb)^2`

Plate buckling

$$\lambda = \frac{KL}{r}$$

=KL/r

Slenderness

$$r = \sqrt{I/A}$$

Radius of gyration

Worked examples, step by step

Example 6.1 An aluminum stringer (E = 71.7 GPa, I = 2 × 10^{-8} m^4) spans 0.5 m between ribs, pinned. Find P_cr.

1. P_cr = π^2 × 71.7 × 10^9 × 2 × 10^{-8}/0.5^2.

Answer: P_cr = 56.6 kN.

Example 6.2 Repeat with fixed ends (K = 0.5).

1. Four times the pinned value.

Answer: P_cr = 226 kN.

Example 6.3 Find the buckling stress of a 2 mm skin panel 150 mm wide (k = 4, ν = 0.33).

1. σ_cr = $4\pi^2$ × 71.7 × 10^9/(12 × 0.891) × $(0.002/0.15)^2$.

Answer: σ_cr = 47.1 MPa, far below the 503 MPa yield.

Practice problems

Work each on paper first, then check.

1. Halve the rib spacing. Stringer buckling load?

   Four times.

2. Double the skin thickness. σ_cr?

Four times: 188 MPa.

3. Does a stronger alloy with the same E raise P_cr?

   No.

4. K for a flagpole (cantilever)?

   2.

Common misconceptions

Myth: Buckling is a strength failure.
Actually: It is a stiffness (stability) failure.

Myth: A buckled skin means a failed wing.
Actually: Stiffened panels carry load after skin buckling.

Myth: Longer columns buckle at proportionally lower load.
Actually: At $1/L^2$, much lower.

Flashcards

Euler

$\pi^2 EI/(KL)^2$.

K

Pinned 1, fixed 0.5, cantilever 2.

Plate

$k\pi^2 E(t/b)^2/(12(1-\nu^2))$.

k

4 for a long supported plate.

Post-buckling

Stiffeners carry load.

Slenderness

KL/r.

Self-check quiz

1. Euler load scales with...

$1/L^2$

$1/L$

L

L^2

$1/L^2$. π^2EI/L^2.

2. Fixed ends multiply P_cr by...

4

2

0.5

1

4. K = 0.5.

3. Plate buckling stress scales with...

$(t/b)^2$

t/b

b/t

t

$(t/b)^2$. Plate formula.

4. Buckling depends on...

E and geometry

yield strength

density

color

E and geometry. Stability.

Try this quiz again

How this helps save the planet

NASA's HiCAM project lists resin infusion among the technologies it is evaluating: liquid molding that can make large, integrated structures with fewer fasteners, and thermoplastics that can be melted and welded for rapid joining.

Integrated stiffened panels, where stringers and skin are made as one piece, resist buckling without the weight of fasteners and splices. That is one route to the lighter composite airframes HiCAM aims for.

On Saving Planets: Mechanics of Materials · Linear Algebra

Sources: NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/)

Free learning resources

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on buckling.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on stringers and skins.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.

☐ **I finished this unit**

Pressurized Fuselages and Thin Shells

Cabin pressurization, hoop and axial stresses in thin cylinders, spherical bulkheads, combined pressure and bending, cutouts and stress concentrations around windows and doors, and fatigue from pressure cycles.

Learning objectives

By the end of this unit you can:

- Compute hoop and axial stress in a pressurized cylinder.
- Find the skin thickness needed for a given stress.
- Explain why windows are rounded.
- Relate pressure cycles to fatigue.

Key concepts

- A thin cylinder of radius r and wall t under pressure difference Δp has hoop stress σ_h = Δp r/t and axial stress σ_a = Δp r/(2t): the hoop stress is twice the axial.
- A spherical shell has σ = Δp r/(2t), so domed pressure bulkheads can be thinner than the cylinder.
- The fuselage also bends under its own weight and tail loads, adding to the axial stress on top or bottom.
- Cutouts for windows and doors concentrate stress; rounded corners and doublers keep the concentration low. Sharp corners can concentrate stress by a factor of three or more.
- Each flight is one pressure cycle. Over tens of thousands of flights, these cycles drive fatigue cracking, the subject of Unit 9.

Key equations

$$\sigma_h = \frac{\Delta p\, r}{t}$$

`σ_h=(Δ p r)/(t)`

Hoop stress

$$\sigma_a = \frac{\Delta p\, r}{2t}$$

`σ_a=(Δ p r)/(2t)`

Axial stress

$$\sigma_{sphere} = \frac{\Delta p\, r}{2t}$$

`σ_sphere=(Δ p r)/(2t)`

Spherical shell

$$t = \frac{\Delta p\, r}{\sigma_{allow}}$$

`t=Δ p rσ_allow`

Required thickness

Worked examples, step by step

Example 7.1 A fuselage of radius 2 m with 1.6 mm skin has a 55 kPa cabin pressure difference (sample value). Find the hoop and axial stresses.

1. σ_h = 55,000 × 2/0.0016.
2. σ_a = σ_h/2.

Answer: σ_h = 68.75 MPa; σ_a = 34.4 MPa.

Example 7.2 What thickness keeps the hoop stress at 100 MPa?

1. t = 55,000 × 2/100 × 10^6.

Answer: t = 1.1 mm (before fatigue and damage-tolerance allowances).

Example 7.3 Find the stress in a hemispherical bulkhead of the same radius and 1.6 mm thickness.

1. σ = 55,000 × 2/(2 × 0.0016).

Answer: σ = 34.4 MPa.

Practice problems

Work each on paper first, then check.

1. Double the fuselage radius at the same skin. Hoop stress?

   Doubles.

2. Why do fuselage cracks often run lengthwise?

Hoop stress, which opens them, is twice the axial.

3. Why are airliner windows rounded?

   To lower stress concentration.

4. Lower the cabin pressure difference by 20%. Stress?

   20% lower.

Common misconceptions

Myth: Axial and hoop stresses are equal.
Actually: Hoop is twice axial.

Myth: A bigger fuselage can use the same skin.
Actually: Stress grows with radius.

Myth: Pressure only matters at cruise.
Actually: Each cycle adds fatigue damage.

Flashcards

Hoop stress

Δp r/t.

Axial stress

Δp r/(2t).

Sphere

Δp r/(2t).

Cutouts

Round corners, doublers.

Pressure cycle

One per flight.

Thickness

Δp r/σ.

Self-check quiz

1. Hoop stress is...

Δp r/t

Δp r/(2t)

Δp t/r

Δp/t

Δp r/t. Thin cylinder.

2. Hoop/axial ratio is...

2

1

0.5

4

2. Cylinder.

3. Rounded windows reduce...

stress concentration

weight only

pressure

drag

stress concentration. Geometry.

4. Each flight is one...

pressure cycle

buckling event

yield

nothing

pressure cycle. Fatigue.

Try this quiz again

How this helps save the planet

NASA's HiCAM project is working toward large-scale manufacturing demonstrations of a composite fuselage barrel and wing box in 2028 and 2029. NASA notes that composite materials are already used in major portions of modern aircraft, including fuselage and wing sections of aircraft such as the Boeing 787.

A composite fuselage barrel carries the same hoop and axial stresses as a metal one; the gain is that composites carry them with less mass and do not corrode in the same way.

On Saving Planets: Mechanics of Materials · Thermodynamics

Sources: NASA: NASA Hosts 2026 Review on Advanced Composite Manufacturing (Jun. 4, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/nasa-hosts-2026-review-on-advanced-composite-manufacturing/)

Free learning resources

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on shells.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on fuselages.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.

☐ **I finished this unit**

Composite Materials and Laminates

Fibers and matrices, fiber volume fraction, the rule of mixtures for stiffness and density, transverse stiffness, ply orientation and laminates, quasi-isotropic layups, and composite manufacturing.

Learning objectives

By the end of this unit you can:

- Use the rule of mixtures for longitudinal stiffness and density.
- Estimate transverse stiffness.
- Explain why laminates use several ply angles.
- Compare composite and metal panels at equal stiffness.

Key concepts

- A composite ply is stiff fibers (carbon, glass) in a softer matrix (epoxy or a thermoplastic). Fibers carry the load; the matrix holds them together and transfers load.
- Along the fibers the rule of mixtures holds: E_1 = E_fV_f + E_mV_m, and density ρ = ρ_fV_f + ρ_mV_m.
- Across the fibers the inverse rule applies: 1/E_2 = V_f/E_f + V_m/E_m, so a ply is many times less stiff transversely.
- Laminates stack plies at 0°, ±45° and 90° so the part is stiff in every direction it is loaded. A quasi-isotropic layup behaves almost like a metal sheet in-plane, but lighter.
- For a panel in bending at equal stiffness, mass scales as ρ/E^(1/3), which is why low-density materials win for skins.

Key equations

$$E_1 = E_f V_f + E_m V_m$$

`E_1=E_fV_f+E_mV_m`

Rule of mixtures

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

`(1)/(E_2)=(V_f)/(E_f)+(V_m)/(E_m)`

Inverse rule

$$\rho = \rho_f V_f + \rho_m V_m$$

`ρ=ρ_fV_f+ρ_mV_m`

Density

$$\frac{m_2}{m_1} = \frac{\rho_2}{\rho_1}\left(\frac{E_1}{E_2}\right)^{1/3}$$

(m_2)/(m_1)=(ρ_2)/(ρ_1)((E_1)/(E_2))^1/3

Equal-stiffness panel mass

Worked examples, step by step

Example 8.1 Carbon fibers (E = 230 GPa, ρ = 1800 kg/m³) in epoxy (E = 3.5 GPa, ρ = 1200 kg/m³) at V_f = 0.6 (textbook values). Find E_1 and ρ.

1. E_1 = 230 × 0.6 + 3.5 × 0.4.
2. ρ = 1800 × 0.6 + 1200 × 0.4.

Answer: E_1 = 139.4 GPa; ρ = 1560 kg/m³.

Example 8.2 Find E_2.

1. $1/E_2$ = 0.6/230 + 0.4/3.5.

Answer: E_2 = 8.55 GPa, 16.3 times lower than E_1.

Example 8.3 A 7075 skin panel is replaced by the sample laminate (E = 70 GPa, ρ = 1600) at equal bending stiffness. Mass ratio?

1. m_2/m_1 = (1600/2810) × (71.7/70)^(1/3).

Answer: m_2/m_1 = 0.574: 42.6% lighter.

Practice problems

Work each on paper first, then check.

1. Find E_1 at V_f = 0.5.

   230 × 0.5 + 3.5 × 0.5 = 116.8 GPa.

2. Why not make every ply 0°?

It would be weak and floppy in other directions.

3. Find the specific stiffness of the unidirectional ply above.

   $139.4 \times 10^9 / 1560 = 89.4$ MJ/kg.

4. What does the matrix do?

   Holds fibers and transfers load.

Common misconceptions

Myth: Composites are stiff in every direction.
Actually: Only along the fibers; layups fix this.

Myth: More fiber is always better.
Actually: Too little matrix leaves dry spots.

Myth: A composite is just light metal.
Actually: It is anisotropic and fails differently.

Flashcards

Fiber

Carries load.

Matrix

Holds and transfers.

Rule of mixtures

$E_f V_f + E_m V_m$.

Inverse rule

$1/E_2 = V_f/E_f + V_m/E_m$.

Quasi-isotropic

0/±45/90 layup.

Panel index

ρ/E^(1/3).

Self-check quiz

1. Longitudinal modulus follows...

the rule of mixtures

the inverse rule

Euler

Hooke only

the rule of mixtures. Parallel springs.

2. Transverse modulus follows...

the inverse rule

the rule of mixtures

Bredt

Paris

the inverse rule. Series springs.

3. ±45° plies mainly carry...

shear

axial load only

pressure

heat

shear. Shear directions.

4. Panel mass at equal stiffness scales as...

ρ/E^(1/3)

ρE

E/ρ

ρ

ρ/E^(1/3). Bending.

Try this quiz again

How this helps save the planet

NASA says HiCAM aims to meet the demand to replace older, heavier transports with lightweight composite aircraft that reduce fuel consumption and carbon emissions. With its partners in the Advanced Composites Consortium, NASA is working to raise the production rate of composite aircraft.

NASA's 2026 HiCAM spring review, held May 5–7 at Langley, brought together about 150 people from the consortium, a 22-member public-private partnership, and selected the technologies with the greatest impact on manufacturing rate for the next airplane program.

On Saving Planets: General Chemistry I · Solutions

Sources: NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/) · NASA: NASA Hosts 2026 Review on Advanced Composite Manufacturing (Jun. 4, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/nasa-hosts-2026-review-on-advanced-composite-manufacturing/)

Free learning resources

- MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/): MIT materials course.
- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes on composites.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on composite construction.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.

☐ I finished this unit

Fatigue, Fracture and Damage Tolerance

S-N curves and endurance, Miner’s rule for cumulative damage, stress concentrations, the stress intensity factor, fracture toughness and critical crack length, crack growth and the Paris law, and damage-tolerant design and inspection.

Learning objectives

By the end of this unit you can:

- Use Miner’s rule.
- Compute the stress intensity factor for a crack.
- Find a critical crack length.
- Explain safe-life, fail-safe and damage-tolerant design.

Key concepts

- Repeated loads well below yield can grow cracks until a part fails: fatigue. S-N curves give cycles to failure at each stress amplitude.
- Miner’s rule adds damage from different stress levels: D = Σ n_i/N_i, with failure predicted at D = 1.
- A crack of length a under stress σ has stress intensity K = σ√(πa) (times a geometry factor). Fracture happens when K reaches the material’s fracture toughness K_IC.
- So the critical crack length is a_c = (K_IC/σ)²/π. Cracks grow per cycle as da/dN = C(ΔK)^m (Paris law), so inspection intervals are set to find cracks long before a_c.
- Modern aircraft are damage tolerant: they are designed to carry limit load with a crack or damage present until the next inspection finds it.

Key equations

$$D = \sum \frac{n_i}{N_i}$$

`D=(n_i)/(N_i)`

Miner’s rule

$$K = \sigma \sqrt{\pi a}$$

Stress intensity

$$a_c = \frac{1}{\pi} \left(\frac{K_{IC}}{\sigma} \right)^2$$

`a_c=(1)/(π)(_ICσ)^2`

Critical crack length

$$\frac{da}{dN} = C(\Delta K)^m$$

da/dN=C(Δ K)^m

Paris law

Worked examples, step by step

Example 9.1 A part sees 10^4 cycles at a level with N = 10^5, and 2×10^5 cycles at a level with N = 10^6. Find the Miner damage.

1. D = $10^4/10^5 + 2 \times 10^5/10^6$.

Answer: D = 0.30: 30% of life used.

Example 9.2 Find K for a 10 mm crack under 100 MPa.

1. K = 100 × √(π × 0.01).

Answer: K = 17.7 MPa√m.

Example 9.3 Find the critical crack length at 100 MPa for an alloy with K_IC = 37 MPa√m (sample value).

1. a_c = $(37/100)^2/\pi$.

Answer: a_c = 43.6 mm.

Practice problems

Work each on paper first, then check.

1. Double the stress. Critical crack length?

   Quarter: 10.9 mm.

2. What D does Miner predict at failure?

   1.

3. Why do aircraft get scheduled inspections?

   To find cracks before they reach a_c.

4. Find K for a 40 mm crack at 50 MPa.

   $50\sqrt{(\pi \times 0.04)} = 17.7$ MPa√m.

Common misconceptions

Myth: Fatigue needs stress above yield.
Actually: It happens well below yield.

Myth: A crack means the part must fail.
Actually: Damage-tolerant parts carry load until inspection.

Myth: Miner's rule is exact.
Actually: It is an estimate; load order matters.

Flashcards

Fatigue

Failure under repeated loads.

S-N curve

Cycles to failure vs stress.

Miner

$\Sigma\, n/N = 1$.

Stress intensity

$\sigma\sqrt{(\pi a)}$.

Critical crack

$(K_{IC}/\sigma)^2/\pi$.

Damage tolerance

Safe until found.

Self-check quiz

1. Miner failure at...

D = 1

D = 0

D = 0.5

D = 2

D = 1. Cumulative damage.

2. K scales with...

σ√a

σa

σ/a

a^2

σ√a. σ√(πa).

3. a_c scales with...

$(K_{IC}/\sigma)^2$

K_{IC}/σ

σ

K_{IC}

$(K_{IC}/\sigma)^2$. Fracture.

4. Paris law describes...

crack growth

buckling

yield

creep

crack growth. da/dN.

Try this quiz again

How this helps save the planet

NASA's HiCAM portfolio includes automated inspection: robotic systems that find defects during the build, reducing time-consuming quality checks and repairs after production, and digital engineering models that predict structural performance and manufacturing defects before assembly.

Finding flaws early and predicting how they grow is the heart of damage tolerance. It lets designers use thinner, lighter structure with confidence.

On Saving Planets: Mechanics of Materials · Statics

Sources: NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/)

Free learning resources

- MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/): MIT course on fatigue and fracture.
- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT notes.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): FAA chapter on construction.
- OpenStax UP1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.

☐ **I finished this unit**

Lightweight Design and Sustainable Structures

Weight breakdowns, the snowball effect of weight growth, structural weight and fuel via the Breguet equation, material indices for lightweight design, multi-material and integrated structures, recycling and life-cycle thinking.

Learning objectives

By the end of this unit you can:

- Estimate fuel saved by removing structural weight.
- Explain the weight snowball.
- Use material indices to choose materials.
- Discuss recycling and life-cycle trade-offs.

Key concepts

- Structure is typically about a quarter to a third of an airliner's takeoff weight (a textbook range), so structural weight is a major lever on fuel.
- Weight snowballs: extra structure needs more lift, more fuel and bigger engines, which add more structure. Savings snowball the same way in reverse.
- From the Breguet equation at fixed range, fuel burned = W_landing (e^(Rc/(V·L/D)) − 1), so fuel scales with landing weight; every kilogram saved cuts fuel in proportion.
- Material indices pick the lightest material for each job: E/ρ for ties, E^(1/2)/ρ for beams and E^(1/3)/ρ for panels in bending.
- Lifecycle matters too: manufacturing energy, repairability and recycling of composites and alloys all count alongside in-service fuel.

Key equations

$$m_{fuel} = m_{land}\left(e^{Rc/(V\,L/D)} - 1\right)$$

`m_fuel=m_land(e^Rc/(V L/D)-1)`

Fuel at fixed range

$$\Delta m_{fuel} = \Delta m\left(e^{Rc/(V\,L/D)} - 1\right)$$

`Δ m_fuel=Δ m(e^Rc/(V L/D)-1)`

Fuel saved per kilogram

$$\frac{E^{1/2}}{\rho}$$

Beam index

$$\frac{E^{1/3}}{\rho}$$

Panel index

Worked examples, step by step

Example 10.1 A jet lands at 45,000 kg after a flight whose Breguet exponent is 0.2877 (e^0.2877 = 1.333; sample values). How much fuel did it burn?

1. m_fuel = 45,000 × (1.3333 − 1).

Answer: m_fuel = 15,000 kg.

Example 10.2 Composite parts save 600 kg of structure. How much fuel is saved per flight?

1. Δm_fuel = 600 × 0.3333.

Answer: About 200 kg per flight.

Example 10.3 Rank aluminum, titanium, steel and the sample laminate by panel index E^(1/3)/ρ (GPa-based).

1. Al: 71.7^(1/3)/2810 × 1000 = 1.48.
2. Ti: 1.09; steel: 0.75; laminate: 2.58.

Answer: Laminate first, then aluminum, titanium, steel.

Practice problems

Work each on paper first, then check.

1. At 1000 flights a year, how much fuel does 200 kg per flight save?

   200 tonnes a year.

2. Using 3.16 kg CO_2 per kg of fuel (a textbook value), CO_2 saved per year?

   632 tonnes.

3. Which index applies to a wing spar in bending?

   E^(1/2)/ρ (beam).

4. Why can saving weight on a new design save more than its own mass?

   The snowball effect.

Common misconceptions

Myth: Only engines decide fuel burn.
Actually: Weight and drag matter as much.

Myth: Composites are always greener.
Actually: Manufacturing and recycling also count.

Myth: A kilogram saved saves a kilogram of fuel.
Actually: It saves a fraction per flight, over thousands of flights.

Flashcards

Snowball

Weight changes compound.

Fuel and weight

Fuel ∝ landing weight.

Tie index

E/ρ.

Beam index

E^(1/2)/ρ.

Panel index

E^(1/3)/ρ.

Lifecycle

Make, use, recycle.

Self-check quiz

1. Fuel at fixed range scales with...

landing weight

wing color

altitude only

nothing

landing weight. Breguet.

2. Panel index is...

E^(1/3)/ρ

E/ρ

Eρ

σ/E

E^(1/3)/ρ. Bending panel.

3. Weight snowball means...

savings compound

nothing

weight is fixed

only fuel changes

savings compound. Resizing.

4. Beam index is...

E^(1/2)/ρ

E/ρ

E^(1/3)/ρ

ρ/E

E^(1/2)/ρ. Bending beam.

Try this quiz again

How this helps save the planet

DOE estimates that using lightweight components and high-efficiency engines in one quarter of the U.S. vehicle fleet could save more than 5 billion gallons of fuel a year by 2030, and notes that lightweight materials are especially important for electric vehicles, where they offset battery weight and extend range.

NASA notes that needing less energy to fly means less fuel or, for electric aircraft, less power generated elsewhere, lowering lifecycle emissions for the whole time an aircraft is in use. Lightweight structure is how designers make that happen.

On Saving Planets: Aircraft Performance · Solutions

Sources: U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks) · NASA: Sustainable Aviation (https://www.nasa.gov/directorates/armd/sustainable-aviation/)

Free learning resources

- MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/): MIT materials course.
- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): MIT structures.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): MIT notes.
- U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks): DOE lightweight materials.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Flight Loads, Safety Factors and Materials
   4 problems
2. Wing Shear and Bending Loads
   4 problems
3. Bending of Thin-Walled Beams
   4 problems
4. Torsion and Shear Flow in Closed Sections
   4 problems
5. Shear in Open Sections and the Shear Center
   4 problems
6. Buckling of Columns, Stringers and Plates
   4 problems

7. Pressurized Fuselages and Thin Shells
   4 problems
8. Composite Materials and Laminates
   4 problems
9. Fatigue, Fracture and Damage Tolerance
   4 problems
10. Lightweight Design and Sustainable Structures
    4 problems

The lab (Structures lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation, a spreadsheet, or with household items. Wear safety glasses when bending or breaking anything that can snap, keep loads small and close to the floor, and keep the work area clear of trip hazards.

Safety reference: OSHA: Walking-working surfaces (slip, trip and fall hazards) (https://www.osha.gov/walking-working-surfaces)

Lab A: Material selection by specific properties

WEEK 1

Goal: Build a spreadsheet of specific stiffness and strength for aerospace materials and rank them for ties, beams and panels.

$$\frac{E}{\rho}, \quad \frac{\sigma}{\rho}, \quad \frac{E^{1/3}}{\rho}$$

Material indices

Procedure

1. Enter E, σ and ρ for 7075 aluminum, Ti-6Al-4V, 4130 steel and the sample carbon laminate.
2. Compute E/ρ and σ/ρ.
3. Add the beam and panel indices.
4. Rank the materials for each job.

Data table

Material	E/ρ (MJ/kg)	σ/ρ (kN·m/kg)
7075 Al	25.5	179
Ti-6Al-4V	25.7	199
4130 steel	26.1	55.4
Carbon laminate	43.75	375

Computed from textbook and sample values.

Sample calculation

1. Al: $71.7 \times 10^9 / 2810 = 25.5$ MJ/kg.
2. Laminate: $600 \times 10^6 / 1600 = 375$ kN·m/kg.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

PhET: Hooke’s Law (https://phet.colorado.edu/en/simulations/hookes-law) · MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials

-fall-2007/)

Planet link: DOE lists 50–70% mass reductions for carbon fiber composites replacing steel parts. U.S. DOE: Lightweight materials for cars and trucks (https://www.energy.gov/eere/vehicles/lightweight-materials-cars-and-trucks)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Ruler cantilever deflection

WEEK 3

Goal: Measure how a ruler clamped at a table edge deflects under a small weight and compare with beam theory.

$$\delta = \frac{PL^3}{3EI}, \quad I = \frac{bt^3}{12}$$

```
=(PL^3)/(3EI),\ \ I=(bt^3)/(12)
```

Cantilever tip deflection

Procedure

1. Explore stiffness in PhET Hooke's Law.
2. Tape a ruler flat to a table so 0.25 m overhangs.
3. Hang a 100 g bag of coins (about 1 N) at the tip; measure the tip drop against a second ruler.
4. Repeat at 0.20 m overhang and compare with $\delta \propto L^3$.

Data table

Overhang	Load	Predicted deflection
0.25 m	1 N	7.72 mm
0.20 m	1 N	3.95 mm

Predicted for a wooden ruler 30 mm × 3 mm with E = 10 GPa (sample values); your ruler will differ.

Sample calculation

1. $I = 0.03 \times 0.003^3/12 = 6.75 \times 10^{-11}$ m⁴.
2. $\delta = 1 \times 0.25^3/(3 \times 10 \times 10^9 \times 6.75 \times 10^{-11}) = 7.72$ mm.

> **Safety**
>
> - Keep loads small; a snapping ruler can fling coins. Wear safety glasses.

Home or virtual alternative

- This is a home lab.

PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law)

Planet link: NASA bent the SWEET-15 wing for months while fiber-optic sensors tracked strain. NASA: NASA Pushes New Wing Design to Find Structural Limits (Jul. 17, 2026) (https://www.nasa.gov/cent

ers-and-facilities/armstrong/nasa-pushes-new-wing-design-to-find-structural-limits/)

- [ ] **I completed this lab (wet or virtual)**

Lab C: Spaghetti column buckling

WEEK 8

Goal: Load single strands of dry spaghetti end-on on a kitchen scale and check Euler's 1/L² law.

$$P_{cr} = \frac{\pi^2 EI}{L^2}, \quad I = \frac{\pi d^4}{64}$$

`P_cr=(π^2EI)/(L^2),\ \ I=(π d^4)/(64)`

Euler buckling, pinned ends

Procedure

1. Cut strands to 0.20 m and 0.10 m.
2. Stand one strand on a tared kitchen scale and press down gently with a flat object until it bows; note the peak reading.
3. Repeat five times for each length and average.
4. Compare the ratio with the predicted factor of 4.

Data table

Length	Predicted P_cr	Scale reading
0.20 m	0.381 N	38.9 g
0.10 m	1.53 N	155.5 g

Predicted for 1.8 mm strands with E = 3 GPa (sample values); real pasta varies.

Sample calculation

1. $I = \pi \times 0.0018^4/64 = 5.15 \times 10^{-13}$ m⁴.
2. $P_{cr}(0.20) = \pi^2 \times 3 \times 10^9 \times 5.15 \times 10^{-13}/0.04 = 0.381$ N.

> **Safety**
>
> - Wear safety glasses; snapped pasta flies.
> - Press gently with a book, not your palm.

Home or virtual alternative

- This is a home lab.

MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/)

Planet link: NASA HiCAM evaluates resin infusion for large integrated structures with fewer fasteners. NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/)

- [ ] **I completed this lab (wet or virtual)**

Lab D: Fuselage pressure stress spreadsheet

WEEK 9

Goal: Compute hoop stress for fuselages of different sizes and find the skin needed for a target stress.

$$\sigma_h = \frac{\Delta p\, r}{t}$$

```
σ_h=(Δ p r)/(t)
```

Hoop stress

Procedure

1. Set Δp = 55 kPa and t = 1.6 mm.
2. Compute σ_h for radii of 1.5, 2.0 and 3.0 m.
3. Find the skin thickness for 100 MPa at each radius.
4. Discuss why wide-body skins are thicker.

Data table

Radius	Hoop stress
1.5 m	51.6 MPa
2.0 m	68.75 MPa
3.0 m	103.1 MPa

Computed values with a sample pressure difference.

Sample calculation

1. σ_h(3.0) = 55,000 × 3/0.0016 = 103.1 MPa.
2. t for 100 MPa at 3.0 m = 1.65 mm.

> **Safety**
>
> - Computer only. Never pressurize bottles or cans to test this.

Home or virtual alternative

- Any spreadsheet works.

FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf)

Planet link: NASA HiCAM plans a composite fuselage barrel demonstration in 2028–2029. NASA: NASA Hosts 2026 Review on Advanced Composite Manufacturing (Jun. 4, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/nasa-hosts-2026-review-on-advanced-composite-manufacturing/)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Composite ply calculator

WEEK 11

Goal: Compute longitudinal and transverse stiffness of a carbon/epoxy ply against fiber volume fraction.

$$E_1 = E_f V_f + E_m V_m, \quad \frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

`E_1=E_fV_f+E_mV_m,\ \ (1)/(E_2)=(V_f)/(E_f)+(V_m)/(E_m)`

Micromechanics

Procedure

1. Enter E_f = 230 GPa and E_m = 3.5 GPa.
2. Compute E_1 and E_2 for V_f = 0.4, 0.5 and 0.6.
3. Plot both against V_f.
4. Explain why E_2 barely changes.

Data table

V_f	E_1 (GPa)	E_2 (GPa)
0.4	94.1	5.77
0.5	116.8	6.90
0.6	139.4	8.55

Computed from textbook fiber and matrix values.

Sample calculation

1. $E_1(0.5) = 230 \times 0.5 + 3.5 \times 0.5 = 116.8$ GPa.
2. $E_2(0.4) = 1/(0.4/230 + 0.6/3.5) = 5.77$ GPa.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/)

Planet link: NASA says HiCAM aims to replace older, heavier transports with lightweight composite aircraft. NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Paper clip fatigue and Miner’s rule

WEEK 13

Goal: Bend paper clips back and forth to failure at two bend angles and test Miner’s rule.

$$D = \sum \frac{n_i}{N_i}$$

`D=(n_i)/(N_i)`

Miner’s rule

Procedure

1. Straighten several identical paper clips.
2. Bend one through ±90° until it breaks; count cycles. Repeat for ±45°.
3. On a fresh clip, do half the ±90° life, then bend at ±45° to failure.
4. Compare the second-stage cycles with Miner’s prediction.

Data table

Bend	Cycles to failure
±90°	12
±45°	30
6 at ±90°, then ±45°	15 predicted

Sample values; your clips will differ, and load order changes the result.

Sample calculation

1. 6/12 = 0.5 of the life used at ±90°.
2. Remaining 0.5 × 30 = 15 cycles at ±45°.

> **Safety**
>
> - Wear safety glasses; broken ends are sharp and the wire gets hot.
> - Dispose of pieces safely.

Home or virtual alternative

- This is a home lab.

MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/)

Planet link: NASA HiCAM develops automated inspection to catch defects during the build. NASA: HiCAM Project Overview (Feb. 15, 2026) (https://www.nasa.gov/directorates/armd/aavp/hicam/about-hicam/)

☐ **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. A 10 kN aircraft has a limit load factor of 3.8. Find the ultimate load.

    1. 1.5 × 3.8 × 10.

    Answer: 57 kN.

2. Find the root moment for uniform lift of 2 kN/m on a 5 m semispan.

    1. $2 \times 5^2/2$.

    Answer: 25 kN·m.

3. Find the flange force for 25 kN·m on a 0.1 m deep box.

    1. 25/0.1.

    Answer: 250 kN.

4. A closed section encloses 0.05 m^2 and carries 20 kN·m. Find q.

    1. 20,000/(2 × 0.05).

    Answer: 200 kN/m.

5. Find τ_max in a rectangular section of 600 mm^2 under 3 kN.

    1. 1.5 × 3000/600.

    Answer: 7.5 MPa.

Final practice set (all units)

1. Find the Euler load of a 1 m pinned column with E = 70 GPa and I = 10^{-8} m^4.

    1. $\pi^2 \times 70 \times 10^9 \times 10^{-8}/1$.

    Answer: 6909 N.

2. Find the hoop stress for r = 1.8 m, Δp = 60 kPa, t = 1.5 mm.

    1. 60,000 × 1.8/0.0015.

    Answer: 72 MPa.

3. Find E_1 for V_f = 0.55 (E_f = 230, E_m = 3.5 GPa).

    1. 230 × 0.55 + 3.5 × 0.45.

    Answer: 128.1 GPa.

4. Find a_c for K_IC = 30 MPa√m at 150 MPa.

    1. (30/150)²/π.

    Answer: 12.7 mm.

5. Find Miner damage for 2 × 10^4 of 10^5 and 5 × 10^5 of 10^6 cycles.

    1. 0.2 + 0.5.

    Answer: 0.7.

6. With a fuel factor of 0.25 per kilogram, how much fuel does 300 kg of saved structure save per flight?

    1. 300 × 0.25.

    Answer: 75 kg.

Formula sheet

$$n = L/W,\ P_{ult} = 1.5 P_{lim},\ MS = \sigma_{allow}/\sigma - 1$$

$$M_{root} = w s^2/2,\ M_{ell} = (L/2)(4s/3\pi)$$

$$\sigma = My/I,\ F = M/h$$

$$q = \tau t,\ T = 2 A_e q,\ d\theta/dz = T \oint ds/t/(4A_e^2 G)$$

$$\tau = VQ/(It),\ \tau_{max} = 3V/(2A)$$

$$P_{cr} = \pi^2 EI/(KL)^2,\ \sigma_{cr} = k\pi^2 E(t/b)^2/(12(1-\nu^2$$

$$\sigma_h = \Delta p r/t,\ \sigma_a = \Delta p r/(2t)$$

$$E_1 = E_f V_f + E_m V_m,\ 1/E_2 = V_f/E_f + V_m/E_m$$

$$D = \sum n_i/N_i,\ K = \sigma\sqrt{\pi a},\ a_c = (K_{IC}/\sigma)^2/\pi$$

$$E/\rho,\ E^{1/2}/\rho,\ E^{1/3}/\rho$$

Constants

Constant	Value
Aluminum 7075-T6 (textbook values)	E = 71.7 GPa, σ_y = 503 MPa, ρ = 2810 kg/m³, G = 27 GPa, ν = 0.33
Titanium Ti-6Al-4V (textbook values)	E = 114 GPa, σ_y = 880 MPa, ρ = 4430 kg/m³
Steel 4130 (textbook values)	E = 205 GPa, σ_y = 435 MPa, ρ = 7850 kg/m³
Quasi-isotropic carbon/epoxy laminate (sample values)	E = 70 GPa, strength 600 MPa, ρ = 1600 kg/m³
Sample airliner (sample values)	W = 588.6 kN, semispan 17 m, limit load factor 2.5, safety factor 1.5
Gravity	g = 9.81 m/s²

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, handbooks and simulations alongside the course.

- MIT OpenCourseWare 16.20 Structural Mechanics (Fall 2002) (https://ocw.mit.edu/courses/16-20-structural-mechanics-fall-2002/): Free MIT structures lectures and problem sets.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Free MIT notes on beams and materials.
- MIT OpenCourseWare 3.032 Mechanical Behavior of Materials (Fall 2007) (https://ocw.mit.edu/courses/3-032-mechanical-behavior-of-materials-fall-2007/): Free MIT course on fatigue and fracture.
- FAA PHAK Chapter 3: Aircraft Construction (PDF) (https://www.faa.gov/sites/faa.gov/files/05_phak_ch3.pdf): Free FAA chapter on aircraft structures.
- OpenStax University Physics Volume 1, 12.3 Stress, strain and elastic modulus (https://openstax.org/books/university-physics-volume-1/pages/12-3-stress-strain-and-elastic-modulus): Free textbook section.
- PhET: Hooke's Law (https://phet.colorado.edu/en/simulations/hookes-law): Stiffness interactively.

Glossary

Buckling
Sudden sideways instability of a compressed member.

Damage tolerance
Ability to carry load safely with cracks until inspection finds them.

Fatigue
Failure under repeated loading below yield.

Fracture toughness
Stress intensity at which a crack runs.

Hoop stress
Circumferential stress in a pressurized cylinder.

Laminate
Stack of composite plies at chosen angles.

Limit load
Largest load expected in service.

Load factor
Lift divided by weight.

Margin of safety
Allowable over actual, minus one.

Miner's rule
Cumulative fatigue damage sum.

Rib
Chordwise member that shapes the wing and supports stringers.

Shear center
Point where a transverse load causes no twist.

Shear flow
Shear force per unit length along a thin wall.

Spar
Main spanwise beam of a wing.

Specific stiffness
Elastic modulus divided by density.

Stress intensity factor
Measure of stress near a crack tip.

Stringer
Spanwise stiffener attached to the skin.

Torsion box
Closed wing box resisting twist.

Ultimate load
Limit load times the safety factor.

Wing box
Spars and skins forming the main wing structure.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Follows the load path, separates limit and ultimate loads, checks strength, stiffness, buckling and fatigue, and chooses materials by specific properties.
3 · Proficient	Solves the practice problems with occasional unit slips.
2 · Developing	Follows worked examples but checks only strength, not buckling.
1 · Beginning	Plugs numbers into formulas without a load path; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Aircraft Performance AERO 320
- Orbital Mechanics AERO 340
- Aerodynamics I AERO 310
- All courses