Last updated: November 1, 2026

🗺️ Four-year roadmap · Year 3, Fall · 🧱 Built-in calculator

About this course

This free course has 10 units and 6 labs. Those are the real counts, not a longer syllabus cut short.

A check is one number or code result on a course page that a script recomputes before the page is published. The printed number and the recomputed number have to match.

> This is a free self-study course outline written by Saving Planets. It is not a university course, it carries no academic credit, and no school is involved.
>
> Its scope follows a one-semester junior aerodynamics course. It is modeled on the topics and depth of MIT 16.100 Aerodynamics and the fluids parts of MIT Unified Engineering (both free on MIT OpenCourseWare), and on junior aerodynamics courses such as Purdue AAE 333 and Georgia Tech AE 3030, using NASA Glenn's free Beginner's Guide to Aeronautics and FoilSim simulator, which we link where they help. None of those authors or schools endorses this page.
>
> What we add: every unit ties to real work with a cited public source (NASA winglets, supercritical and truss-braced wings, wind turbines, aircraft emissions rules and flight on Mars), every number on the page is re-computed by a checking script before it goes live, and every lab has a free simulation or a safe home version.
>
> Work through it in order. Each unit has goals, key ideas, equations, step-by-step examples, practice with full answers, common mistakes, flashcards and a quiz. Your progress is saved only in this browser.

How this course helps save the planet

Aerodynamics decides how much fuel an airplane burns and how much electricity a wind turbine makes. Every bit of drag removed from a wing is fuel not burned and carbon not emitted, and every improvement in a blade's lift-to-drag ratio is more clean power from the same wind.

The same equations fly helicopters on Mars and bring spacecraft safely through Earth's atmosphere. Each unit ends with a short, sourced section. Values that are not from a source are labeled as sample values.

For instructors

You can teach this course, or let a student work through it alone. It is free. Saving Planets does not give a grade or school credit.

- This page is the whole course: 10 units and 6 labs (AERO 310).
- Plan on 15 weeks, about 6 to 9 hours a week, including the lab weeks.
- Follow the 15-week schedule. Read the unit, do the practice on paper, then open the answer. In a lab week, do the lab or the home version.
- A check is one number or code result on the page that a script recomputes before we publish. The number you see and the recomputed number have to match.
- Use Print this page, or download the PDF study guide. The formula sheet is near the end.
- Ask: how does this unit help people and the planet? Each unit has a green box with a sourced fact.
- Copy the teacher guide template and write in your class name, the time you have, and the questions you will ask. Open the teacher guide template

Before you start

Prerequisites

- Calculus III: partial derivatives, gradient, divergence, curl and line integrals.
- Differential Equations: first- and second-order ODEs.
- Dynamics and Physics I: Newton's laws, momentum and energy; Physics II helps with fields.
- Intro to Aerospace Engineering gives the vocabulary of lift, drag and the atmosphere.

How to study

- Start every problem by naming the flow regime: incompressible or compressible (Mach number), inviscid or viscous (Reynolds number).
- Draw a control volume or a streamline before writing an equation, and say which conservation law you are using.
- Non-dimensionalize: work in coefficients (C_L, C_D, C_p) and similarity parameters (Re, M), then return to forces.
- Check limits: at zero angle of attack a symmetric airfoil has zero lift; as aspect ratio grows, induced drag falls.
- Keep SI units: Pa, kg/m³, m/s, N; convert knots and feet at the start.
- Do every practice problem on paper before opening the answer, and retake the quizzes a few days later.

What you need

- A computer or tablet for NASA FoilSim and the free PhET fluid simulations (they run in a browser).

- Printer paper, tape, paper clips, scissors and a ruler for paper gliders.
- Paper coffee filters (cone or basket style) and a tape measure for drag tests.
- A phone with a stopwatch and slow-motion video.
- Spreadsheet software for the atmosphere, drag-polar and performance labs.

Suggested 15-week schedule

About 6 to 9 hours a week: reading, practice problems and, in lab weeks, the lab.

Week	Unit	Topics	Reading
1	Fluid Properties and the Standard Atmosphere	Fluid properties, ideal gas, speed of sound, standard atmosphere; Lab A	NASA Glenn atmosphere; MIT Unified
2	Fluid Statics, Bernoulli's Equation and Airspeed Measurement	Bernoulli, dynamic pressure, venturi, pitot-static; Lab B	NASA Glenn; OpenStax UP1 14.6
3	Conservation Laws: Mass, Momentum and Energy; Control Volumes and Momentum Theory	Control volumes: mass and momentum; wake drag	MIT 16.100; UP1 14.5
4	Conservation Laws: Mass, Momentum and Energy; Control Volumes and Momentum Theory	Differential equations of motion; rotor momentum theory	MIT Unified
5	Potential Flow and the Kutta–Joukowski Theorem	Vorticity, circulation, potential and stream function, elementary flows	MIT 16.100
6	Potential Flow and the Kutta–Joukowski Theorem	Cylinder flow, d'Alembert, Kutta–Joukowski	MIT 16.100
7	Airfoils, Lift, Drag and Moment Coefficients	Airfoil geometry, coefficients, lift curve, stall; midterm practice	NASA Glenn; FoilSim
8	Thin Airfoil Theory	Thin airfoil theory and panel methods; Lab C	MIT 16.100; FoilSim
9	Finite Wings and Induced Drag	Downwash, lifting-line theory, elliptic wings	MIT 16.100; NASA Glenn
10	Finite Wings and Induced Drag	Induced drag, aspect ratio, winglets; Lab D	NASA Glenn
11	Viscous Flow, Reynolds Number and Boundary Layers	Reynolds number, boundary layers, skin friction, separation;	NASA Glenn; UP1 14.7

Week	Unit	Topics	Reading
		Lab E	
12	Compressible Flow: Isentropic Relations and Shock Waves	Isentropic flow and nozzles	NASA Glenn
13	Compressible Flow: Isentropic Relations and Shock Waves	Normal shocks, critical Mach, supercritical airfoils	NASA Glenn; MIT 16.100
14	Aircraft Performance: Drag Polar, Glide, Range and Endurance	Drag polar, glide, minimum drag	MIT Unified
15	Aircraft Performance: Drag Polar, Glide, Range and Endurance	Breguet range and endurance; Lab F; final practice	MIT Unified

Your progress

Tick units and labs as you finish them, and take the quizzes. Progress and quiz scores are saved only in this browser, never sent to us.

Units

Fluid Properties and the Standard Atmosphere

Air as a continuum, pressure, density, temperature and viscosity, the ideal-gas law, the speed of sound, hydrostatics in the atmosphere, the standard atmosphere and its layers, and why aerodynamic forces depend on density.

Learning objectives

By the end of this unit you can:

- Use the ideal-gas law to relate p, ρ and T.
- Compute the speed of sound and the Mach number.
- Use the standard atmosphere model to find properties at altitude.
- Explain how density changes lift and drag with altitude.

Key concepts

- Air behaves as a continuum when the flow scale is much larger than the distance between molecules. Its state is set by pressure p, density ρ and temperature T, linked by the ideal-gas law p = ρRT with R = 287 J/(kg·K).
- Viscosity μ measures internal friction; it matters near surfaces (Unit 8). Kinematic viscosity is ν = μ/ρ.
- Speed of sound a = √(γRT) depends only on temperature for a given gas; Mach number M = V/a.
- Hydrostatics: dp/dh = −ρg. Pressure falls with altitude because there is less air above. In the standard troposphere (0–11 km) temperature falls linearly with a lapse rate of about 6.5 K/km; in the lower stratosphere it is nearly constant.
- Aerodynamic forces scale with density, so an aircraft at altitude must fly faster, or at a higher lift coefficient, to make the same lift.

Key equations

$$p = \rho R T$$

`p=ρ RT`

Ideal-gas law

$$a = \sqrt{\gamma R T}, \quad M = \frac{V}{a}$$

`a=γ RT,\ \ M=V/a`

Speed of sound and Mach number

$$\frac{dp}{dh} = -\rho g$$

`dp/dh=-ρ g`

Hydrostatic equation

$$T = T_0 - \lambda h$$

`T=T_0- h`

Troposphere temperature (lapse rate λ)

Worked examples, step by step

Example 1.1 Find the density of air at sea level with p = 101,325 Pa and T = 288.15 K.

1. ρ = p/(RT) = 101,325/(287 × 288.15).

Answer: ρ = 1.225 kg/m³.

Example 1.2 Use NASA Glenn's metric atmosphere model to find T, p and ρ at 11,000 m.

1. T = 15.04 − 0.00649 × 11,000 = −56.35 °C.
2. p = 101.29 × [(T + 273.1)/288.08]^5.256 = 22.71 kPa.
3. ρ = p/[0.2869 × (T + 273.1)].

Answer: T = −56.35 °C, p = 22.71 kPa, ρ = 0.3652 kg/m³, about 29.8% of sea-level density.

Example 1.3 Find the speed of sound at sea level (288.15 K) and at 11 km (216.8 K), and the Mach number of a jet flying 250 m/s at 11 km.

1. a_0 = √(1.4 × 287 × 288.15).
2. a = √(1.4 × 287 × 216.8).
3. M = 250/a.

Answer: a_0 = 340.3 m/s, a = 295.1 m/s, M = 0.847.

Practice problems

Work each on paper first, then check.

1. Find ρ for p = 80 kPa and T = 270 K.

   ρ = 80,000/(287 × 270) = 1.032 kg/m³.

2. Convert 450 knots to m/s.

   450 × 0.514444 = 231.5 m/s.

3. Find ν at sea level.

   $\nu = 1.789 \times 10^{-5} / 1.225 = 1.460 \times 10^{-5}$ m²/s.

4. What is the temperature at 5,000 m in the NASA model?

   15.04 − 32.45 = −17.41 °C.

Common misconceptions

Myth: Air pressure falls because air gets colder.

Actually: It falls because there is less air above; temperature affects how fast.

Myth: The speed of sound depends on pressure.

Actually: For an ideal gas it depends only on temperature.

Myth: Density is the same at all altitudes for low-speed flow.

Actually: Low-speed flow is incompressible locally, but density still changes with altitude.

Flashcards

Ideal gas

$p = \rho RT$.

Speed of sound

$a = \sqrt{(\gamma RT)}$.

Mach number

$M = V/a$.

Hydrostatics

$dp/dh = -\rho g$.

Lapse rate

About 6.5 K/km in the troposphere.

Kinematic viscosity

$\nu = \mu/\rho$.

Self-check quiz

1. The ideal-gas law is…

$p = \rho RT$

$p = \rho g h$

$p = \frac{1}{2}\rho V^2$

$p = RT/\rho$

$p = \rho RT$. Equation of state.

2. The speed of sound depends on…

pressure

temperature

density only

velocity

temperature. $a = \sqrt{(\gamma RT)}$.

3. In the lower stratosphere the standard temperature is…

rising fast

nearly constant

falling fast

zero

nearly constant. NASA model: −56.46 °C from 11 to 25 km.

4. At half the density, the same lift at the same C_L needs a speed higher by…

2×

√2×

4×

½×

√2×. $L \propto \rho V^2$.

Try this quiz again

How this helps save the planet

NASA Glenn's Beginner's Guide explains that the atmosphere is an extremely thin sheet of air, that pressure falls with altitude because there is always less air above, and that aerodynamic forces depend directly on air density. Its standard atmosphere model, used in the FoilSim simulator, has a troposphere from the surface to 11,000 m where temperature falls linearly, a lower stratosphere from 11,000 to 25,000 m at a constant −56.46 °C, and an upper stratosphere above.

Airliners cruise near the top of the troposphere, where the density is only about 30% of sea level, so drag at a given speed is much lower; that is a big reason cruise altitude matters for fuel burn. The same model, with Mars values, explains why NASA's Ingenuity helicopter had to spin its rotors so fast: NASA notes the Martian atmosphere is less than 1 percent as dense as Earth's.

On Saving Planets: Intro to Aerospace Engineering · Earth Observation

Sources: NASA Glenn: Earth atmosphere model (metric units) (https://www.grc.nasa.gov/www/k-12/airplane/atmosmet.html) · NASA Science: Ingenuity Mars Helicopter (https://science.nasa.gov/mission/mars-2020-perseverance/ingenuity-mars-helicopter/)

Free learning resources

- NASA Glenn: Earth atmosphere model (metric units) (https://www.grc.nasa.gov/www/k-12/airplane/atmosmet.html): Standard atmosphere curve fits.
- NASA Glenn: Beginner's Guide to Aeronautics (https://www.grc.nasa.gov/www/k-12/airplane/short.html): Free NASA guide.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Fluid properties and thermodynamics notes.
- PhET: Under Pressure (https://phet.colorado.edu/en/simulations/under-pressure): Hydrostatic pressure interactively.

☐ **I finished this unit**

Fluid Statics, Bernoulli’s Equation and Airspeed Measurement

Static, dynamic and total pressure, Bernoulli’s equation and its assumptions, the venturi, pitot-static tubes and airspeed (indicated, equivalent and true), manometers, and the pressure coefficient.

Learning objectives

By the end of this unit you can:

- State when Bernoulli’s equation applies.
- Compute dynamic pressure and airspeed from a pitot-static reading.
- Analyze a venturi with continuity and Bernoulli.
- Use the pressure coefficient C_p.

Key concepts

- Along a streamline in steady, inviscid, incompressible flow, $p + \tfrac{1}{2}\rho V^2 = p_0$ is constant: static pressure plus dynamic pressure equals total (stagnation) pressure.
- Dynamic pressure $q = \tfrac{1}{2}\rho V^2$ sets the scale of every aerodynamic force; forces are written as coefficient × q × area.
- A pitot-static tube measures $p_0 - p$, so $V = \sqrt{2(p_0 - p)/\rho}$. Airspeed indicators assume sea-level density and read equivalent airspeed; true airspeed is higher at altitude by $\sqrt{\rho_0/\rho}$.
- Continuity $A_1V_1 = A_2V_2$ plus Bernoulli explains the venturi: where the area shrinks the speed rises and the pressure drops.
- Pressure coefficient $C_p = (p - p_\infty)/q_\infty = 1 - (V/V_\infty)^2$ for incompressible flow; $C_p = 1$ at a stagnation point.

Key equations

$$p + \tfrac{1}{2}\rho V^2 = p_0$$

`p+12ρ V^2=p_0`

Bernoulli’s equation

$$q = \tfrac{1}{2}\rho V^2$$

`q=12ρ V^2`

Dynamic pressure

$$V = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

`V=(2(p_0-p))/(ρ)`

Pitot-static airspeed

$$C_p = \frac{p - p_\infty}{q_\infty} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

`C_p=(p-p_∞)/(q_∞)=1-((V)/(V_∞))^2`

Pressure coefficient

Worked examples, step by step

Example 2.1 A pitot-static tube at sea level reads $p_0 - p = 1500$ Pa. Find the airspeed.

1. $V = \sqrt{(2 \times 1500/1.225)}$.

Answer: V = 49.5 m/s.

Example 2.2 A venturi in a wind tunnel halves the area. Upstream $V_1 = 10$ m/s at sea level. Find V_2 and the pressure drop.

1. $V_2 = 2 \times 10 = 20$ m/s (continuity).
2. $\Delta p = ½ \times 1.225 \times (20^2 - 10^2)$.

Answer: $V_2 = 20$ m/s, $\Delta p = 183.8$ Pa.

Example 2.3 An airliner flies with an equivalent airspeed of 140 m/s at 11 km ($\rho = 0.3652$ kg/m³). Find the dynamic pressure and the true airspeed.

1. $q = ½ \times 1.225 \times 140^2$ (equivalent airspeed uses sea-level density).
2. $V_{true} = 140 \times \sqrt{(1.225/0.3652)}$.

Answer: q = 12.0 kPa, V_{true} = 256.4 m/s.

Practice problems

Work each on paper first, then check.

1. Find q at 100 m/s at sea level.

   6125 Pa.

2. Find C_p where the local speed is 1.5 V∞.

   1 − 2.25 = −1.25.

3. A water manometer shows 50 mm. Find the pressure difference (ρ_water = 1000 kg/m³).

   1000 × 9.81 × 0.05 = 490.5 Pa.

4. What C_p does a stagnation point have?

   1.

Common misconceptions

Myth: Bernoulli applies everywhere in any flow.
Actually: It needs steady, inviscid, incompressible flow along a streamline (or irrotational flow).

Myth: Lift happens because air takes equal time over the top and bottom.
Actually: There is no equal-transit rule; the top flow is faster still, and circulation explains lift (Unit 5).

Myth: Indicated airspeed equals true airspeed.
Actually: At altitude true airspeed is much higher.

Flashcards

Bernoulli

$p + \frac{1}{2}\rho V^2 = p_0$.

Dynamic pressure

$q = \frac{1}{2}\rho V^2$.

Pitot-static

$V = \sqrt{(2\Delta p/\rho)}$.

Venturi

Smaller area, higher speed, lower pressure.

C_p

$(p - p_\infty)/q_\infty$.

True airspeed

$V_e \sqrt{(\rho_0/\rho)}$.

Self-check quiz

1. Total pressure equals…

p − q

p + q

q only

ρRT

p + q. Static plus dynamic.

2. A pitot tube measures…

static pressure only

total pressure

temperature

density

total pressure. It faces the flow.

3. Doubling airspeed multiplies q by…

2

4

√2

8

4. $q \propto V^2$.

4. C_p at a stagnation point is…

0

1

−1

2

1. V = 0.

Try this quiz again

How this helps save the planet

Dynamic pressure is why flying on Mars is so hard. NASA reports that Ingenuity flew in an atmosphere less than 1 percent as dense as Earth's, with rotors just under 1.2 m tip to tip and a mass of 1.8 kg. At 1% of Earth's density, the same dynamic pressure needs √100 = 10 times the speed.

NASA says Ingenuity completed 72 flights between April 19, 2021 and January 18, 2024, logging 128.8 minutes in the air, covering 17.0 km and reaching altitudes of up to 24.0 m. Getting there meant understanding $q = \frac{1}{2}\rho V^2$ better than ever, which is also what lets Earth aircraft and wind turbines be designed for thinner air at altitude.

On Saving Planets: Space · Rotor Hover Power Calculator

Sources: NASA Science: Ingenuity Mars Helicopter (https://science.nasa.gov/mission/mars-2020-perseverance/ingenuity-mars-helicopter/) · NASA Glenn: Pitot-static tube (https://www.grc.nasa.gov/www/k-12/airplane/pitot.html)

Free learning resources

- NASA Glenn: Bernoulli's equation (https://www.grc.nasa.gov/www/k-12/airplane/bern.html): Free NASA page.
- NASA Glenn: Pitot-static tube (https://www.grc.nasa.gov/www/k-12/airplane/pitot.html): How airspeed is measured.
- OpenStax UP1, 14.6 Bernoulli's equation (https://openstax.org/books/university-physics-volume-1/pages/14-6-bernoullis-equation): Free textbook section.
- PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow): Interactive.

Watch the lecture

Fluid Dynamics and Statics and Bernoulli's Equation (https://oyc.yale.edu/physics/phys-200/lecture-20) (in English)

Ramamurti Shankar, PHYS 200 Fundamentals of Physics I, Open Yale Courses. License: CC BY-NC-SA 3.0 US (https://creativecommons.org/licenses/by-nc-sa/3.0/us/) · Terms of use (https://oyc.yale.edu/terms). Linked for free study only; Saving Planets is not affiliated with or endorsed by this university.

☐ **I finished this unit**

Conservation Laws: Mass, Momentum and Energy; Control Volumes and Momentum Theory

Lagrangian and Eulerian descriptions, the substantial derivative, integral conservation of mass, momentum and energy for a control volume, differential continuity, Euler’s equation, drag from a wake survey, and actuator-disk momentum theory for propellers and rotors.

Learning objectives

By the end of this unit you can:

- Apply integral mass and momentum conservation to a control volume.
- Derive the continuity and Euler equations in differential form.
- Find drag from a wake velocity profile.
- Use momentum theory to estimate rotor thrust and hover power.

Key concepts

- Mass: ∂/∂t∫ρ dV + ∮ρV·dA = 0. For steady flow, mass in = mass out: ṁ = ρAV is constant through a stream tube.
- Momentum: the net force on a control volume (pressure, body and surface forces) equals the net momentum flux out plus the rate of change inside: ΣF = ∮(ρV·dA)V for steady flow.
- Differential forms: ∂ρ/∂t + ∇·(ρV) = 0; for incompressible flow ∇·V = 0. Euler’s equation ρ DV/Dt = −∇p + ρg is Newton’s law for an inviscid fluid particle; integrating it along a streamline gives Bernoulli.
- Drag from a wake: a body slows the air behind it; the momentum deficit in the wake equals the drag, D' = ρ∫u(V∞ − u) dy per unit span.
- Actuator-disk (momentum) theory: a rotor of area A producing thrust T induces velocity v = √(T/(2ρA)) at the disk; the ideal hover power is P = T^(3/2)/√(2ρA). Real rotors need more; the ratio is the figure of merit.

Key equations

$$\dot{m} = \rho A V = \text{const}$$

`m=ρ AV=const`

Steady mass conservation

$$\sum \mathbf{F} = \oint (\rho \mathbf{V} \cdot d\mathbf{A}) \mathbf{V}$$

`F=(ρ V· d A) V`

Steady momentum (control volume)

$$\nabla \cdot \mathbf{V} = 0$$

Incompressible continuity

$$P_{ideal} = \frac{T^{3/2}}{\sqrt{2\rho A}}$$

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P_ideal=^3/22ρ A
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Ideal hover power (momentum theory)

Worked examples, step by step

Example 3.1 Air at 1.225 kg/m³ flows at 20 m/s through a 0.5 m² duct. Find the mass flow and the speed where the duct narrows to 0.2 m².

1. $\dot{m}$ = 1.225 × 0.5 × 20 = 12.25 kg/s.
2. V_2 = 20 × 0.5/0.2.

Answer: $\dot{m}$ = 12.25 kg/s, V_2 = 50 m/s.

Example 3.2 A helicopter of 1000 kg hovers at sea level with a 10 m diameter rotor. Find the induced velocity and the ideal hover power.

1. T = 9810 N; A = π × 5² = 78.54 m².
2. v = √(9810/(2 × 1.225 × 78.54)) = 7.14 m/s.
3. P = Tv.

Answer: v = 7.14 m/s, P = 70.0 kW (ideal).

Example 3.3 A wake survey behind a 2D body (sea level, V∞ = 30 m/s) finds a deficit equivalent to ∫u(V∞ − u)dy = 6 m³/s². Find the drag per unit span.

1. D′ = ρ∫u(V∞ − u) dy = 1.225 × 6.

Answer: D′ = 7.35 N/m.

Practice problems

Work each on paper first, then check.

1. Is V = (x, −y) incompressible?

   Yes: ∂u/∂x + ∂v/∂y = 1 − 1 = 0.

2. A rotor’s thrust doubles. By how much does ideal hover power grow?

   2^1.5 = 2.83 times.

3. Doubling rotor area at fixed thrust changes ideal hover power by...

   1/√2 = 0.707 times.

4. A jet of water (ρ = 1000) with A = 0.01 m² and V = 10 m/s hits a wall head-on. Find the force.

   F = ρAV² = 1000 N.

Common misconceptions

Myth: Momentum conservation ignores pressure.

Actually: Pressure forces on the control-volume surface must be included.

Myth: A bigger rotor needs more power to hover.

Actually: At the same thrust, a bigger disk needs less induced power.

Myth: Steady flow means the fluid is not accelerating.

Actually: Particles can accelerate (convective acceleration) even when the field does not change in time.

Flashcards

Mass flow

ṁ = ρAV.

Continuity

∇·V = 0 (incompressible).

Euler equation

ρDV/Dt = −∇p + ρg.

Wake drag

D' = ρ∫u(V∞ − u)dy.

Induced velocity

v = √(T/2ρA).

Hover power

T^(3/2)/√(2ρA).

Self-check quiz

1. For incompressible flow, continuity becomes...

$\nabla \cdot V = 0$

$\nabla \times V = 0$

$\rho = 0$

$p = \text{const}$

$\nabla \cdot V = 0$. Density is constant.

2. Drag from a wake survey comes from...

the energy in the wake

the momentum deficit

temperature

lift

the momentum deficit. Momentum balance.

3. Ideal hover power scales with thrust as...

T

$T^{\wedge}1.5$

T^2

$\sqrt{T}$

$T^{\wedge}1.5$. $P = T^{\wedge}(3/2)/\sqrt{(2\rho A)}$.

4. Euler's equation is Newton's law for...

a viscous fluid

an inviscid fluid particle

a rigid body

a shock

an inviscid fluid particle. Pressure and gravity only.

Try this quiz again

How this helps save the planet

Momentum theory shows why Ingenuity was a feat. NASA lists its mass as 1.8 kg, its rotors as about 1.2 m tip to tip, and its average flight power as about 350 W, from a solar panel charging lithium-ion batteries for one 90-second flight per Martian day. With Mars gravity (3.71 m/s^2, a textbook value), the thrust needed is 6.68 N.

Taking a sample Mars density of 0.0115 kg/m^3 (within NASA's "less than 1 percent" of Earth's) and one 1.2 m disk, momentum theory gives an ideal hover power of about 107 W; the 350 W NASA reports also covers real rotor losses (profile drag, tip losses, non-uniform inflow), climbing and onboard systems. The same equation sizes rotors for electric air taxis and drones on Earth, and it is behind this site's rotor hover power calculator.

On Saving Planets: Rotor Hover Power Calculator · Dynamics

Sources: NASA Science: Ingenuity Mars Helicopter (https://science.nasa.gov/mission/mars-2020-perseverance/ingenuity-mars-helicopter/) · MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/)

Free learning resources

- NASA Glenn: Conservation of mass (https://www.grc.nasa.gov/www/k-12/airplane/mass.html): Free NASA page.
- OpenStax UP1, 14.5 Fluid dynamics (https://openstax.org/books/university-physics-volume-1/pages/14-5-fluid-dynamics): Flow rate and continuity.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Control volumes and momentum theory.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Conservation laws in differential form.

Watch the lecture

Work-Energy Theorem and Law of Conservation of Energy (https://oyc.yale.edu/physics/phys-200/lecture-5) (in English)

Ramamurti Shankar, PHYS 200 Fundamentals of Physics I, Open Yale Courses. License: CC BY-NC-SA 3.0 US (https://creativecommons.org/licenses/by-nc-sa/3.0/us/) · Terms of use (https://oyc.yale.edu/terms). Linked for free study only; Saving Planets is not affiliated with or endorsed by this university.

- [ ] **I finished this unit**

Potential Flow and the Kutta–Joukowski Theorem

Vorticity and circulation, irrotational flow, the velocity potential and stream function, Laplace's equation and superposition, uniform flow, sources, sinks, doublets and vortices, flow over a cylinder with and without circulation, d'Alembert's paradox, and the Kutta–Joukowski lift theorem.

Learning objectives

By the end of this unit you can:

- Relate vorticity, circulation and irrotational flow.
- Build flows by superposing elementary solutions of Laplace's equation.
- Find the pressure distribution on a cylinder.
- Apply L' = ρ∞V∞Γ.

Key concepts

- Vorticity ω = ∇ × V is twice the local rotation rate. Circulation Γ = −∮V·ds around a closed curve equals the vorticity flux through it (Stokes' theorem; aerodynamics uses the clockwise-positive sign).
- If ω = 0 the flow is irrotational and V = ∇φ. With ∇·V = 0, φ satisfies Laplace's equation ∇²φ = 0, which is linear, so solutions add (superposition). In 2D the stream function ψ has u = ∂ψ/∂y, v = −∂ψ/∂x, and lines of constant ψ are streamlines.
- Elementary flows: uniform flow φ = V∞x; source of strength Λ, V_r = Λ/(2πr); doublet, the limit of a source–sink pair; point vortex, V_θ = −Γ/(2πr).
- Uniform flow plus a doublet gives flow over a cylinder of radius R: surface speed V_θ = −2V∞ sin θ, so C_p = 1 − 4 sin²θ. The pressure is symmetric, so there is no drag (d'Alembert's paradox) and no lift.
- Adding a vortex makes the flow asymmetric and produces lift per unit span L' = ρ∞V∞Γ, the Kutta–Joukowski theorem. It holds for any 2D body: lift is always tied to circulation.

Key equations

$$\omega = \nabla \times \mathbf{V}, \;\; \Gamma = -\oint \mathbf{V} \cdot d\mathbf{s}$$

ω=× V,\ \ =- V· d s

Vorticity and circulation

$$\nabla^2 \phi = 0$$

Laplace's equation

$$C_p = 1 - 4\sin^2\theta$$

C_p=1-4^2θ

Surface pressure on a non-lifting cylinder

$$L' = \rho_\infty V_\infty \Gamma$$

L'=ρ_∞ V_∞

Kutta–Joukowski theorem

Worked examples, step by step

Example 4.1 Find the maximum surface speed and the minimum C_p on a non-lifting cylinder in a 20 m/s stream.

1. |V_θ| = 2V∞|sin θ| is largest at θ = 90°.
2. C_p = 1 − 4 × 1.

Answer: V_max = 40 m/s; C_p,min = −3.

Example 4.2 A spinning cylinder at sea level in a 50 m/s stream has circulation Γ = 20 m²/s. Find the lift per metre of span.

1. L' = 1.225 × 50 × 20.

Answer: L' = 1225 N/m.

Example 4.3 Find the speed induced by a point vortex of Γ = 20 m²/s at a distance of 1 m.

1. V_θ = Γ/(2πr) = 20/(2π).

Answer: 3.183 m/s.

Practice problems

Work each on paper first, then check.

1. Where on a non-lifting cylinder is C_p = 0?

$\sin^2\theta = 0.25$, so $\theta = 30°, 150°, 210°, 330°$.

2. Is $\varphi = x^2 - y^2$ a valid potential?

   Yes: $\nabla^2\varphi = 2 - 2 = 0$.

3. What lift does a 2 m span spinning cylinder of the worked example make?

   2450 N.

4. What drag does potential flow predict on a cylinder?

   Zero, d'Alembert's paradox; real drag comes from viscosity.

Common misconceptions

Myth: Potential flow is useless because it predicts no drag.
Actually: It predicts lift and pressure well wherever the boundary layer stays attached.

Myth: Circulation means air is going around the wing in circles.
Actually: It is a net value of the line integral; particles do not orbit the wing.

Myth: Superposition works for any flow equation.
Actually: It works because Laplace's equation is linear.

Flashcards

Vorticity

$\omega = \nabla \times V$.

Circulation

$\Gamma = -\oint V \cdot ds$.

Laplace

$\nabla^2\varphi = 0$.

Point vortex

$V_\theta = \Gamma/2\pi r$.

Cylinder C_p

$1 - 4\sin^2\theta$.

Kutta–Joukowski

$L' = \rho V \Gamma$.

Self-check quiz

1. Irrotational flow means...

$\omega = 0$

$\Gamma = 0$ everywhere

$\nabla \cdot V = 0$

no viscosity

$\omega = 0$. Zero vorticity.

2. Lift per unit span equals...

$\rho V^2 \Gamma$

$\rho V \Gamma$

$\frac{1}{2}\rho V^2$

$\Gamma / \rho V$

$\rho V \Gamma$. Kutta–Joukowski.

3. Minimum C_p on a non-lifting cylinder is...

-1

-3

0

-4

$-3.1 - 4$.

4. Uniform flow plus a doublet models...

an airfoil

a cylinder

a sphere

a vortex

a cylinder. Non-lifting cylinder.

Try this quiz again

How this helps save the planet

The U.S. Department of Energy explains that wind turbine blades work much like an airplane wing or helicopter rotor blade: air pressure on one side of the blade drops, and the pressure difference across the blade creates both lift and drag. Because the lift is stronger than the drag, the rotor spins, turning a generator.

DOE notes that land-based turbines range from 100 kilowatts to several megawatts and that offshore turbines are typically larger, taller than the Statue of Liberty. Blade designers use the circulation and lift ideas from this unit, with blade-element momentum theory built on Unit 3, to shape each section of a blade so it extracts as much power from the wind as possible.

On Saving Planets: Solutions · Physics II

Sources: U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work)

Free learning resources

- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Potential flow lectures.
- NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html): See circulation and lift.
- NASA Glenn: Lift from flow turning (https://www.grc.nasa.gov/www/k-12/airplane/foil2.html): Why the flow turns.
- NASA Glenn: Incorrect lift theory (equal transit) (https://www.grc.nasa.gov/www/k-12/airplane/foil3.html): Common misconceptions.

☐ **I finished this unit**

Airfoils, Lift, Drag and Moment Coefficients

Airfoil geometry and NACA designations, angle of attack, the lift, drag and moment coefficients, the lift curve and stall, center of pressure and aerodynamic center, the Kutta condition, and how pressure distributions make lift.

Learning objectives

By the end of this unit you can:

- Describe airfoil geometry: chord, camber and thickness.
- Compute forces from L = qSC_L and D = qSC_D.
- Read a lift curve: slope, zero-lift angle and stall.
- Find the stall speed and the required lift coefficient.

Key concepts

- An airfoil has a chord c from leading to trailing edge, a mean camber line and a thickness distribution. In a NACA 4-digit name such as 2412, the first digit is max camber (2% of chord), the second is its location (40% of chord) and the last two are thickness (12%).
- Forces are made dimensionless: C_L = L/(qS), C_D = D/(qS), C_M = M/(qSc). For a 2D section use lowercase c_l, c_d, c_m per unit span.
- The lift curve c_l(α) is nearly a straight line up to stall, where the boundary layer separates from the upper surface and lift falls. Cambered airfoils make lift at zero angle of attack, so their zero-lift angle α_L=0 is negative.
- The Kutta condition says the flow leaves the sharp trailing edge smoothly; it fixes the circulation and hence the lift.
- The center of pressure moves with α; the aerodynamic center (near quarter chord at low speed) is the point where the moment does not change with α, which makes it useful for stability.

Key equations

$$L = \tfrac{1}{2}\rho V^2 S C_L$$

`L=12ρ V^2SC_L`

Lift equation

$$D = \tfrac{1}{2}\rho V^2 S C_D$$

`D=12ρ V^2SC_D`

Drag equation

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L,max}}}$$

`V_stall=2Wρ SC_L,max`

Stall speed

$$C_M = \frac{M}{qSc}$$

`C_M=M/qSc`

Moment coefficient

Worked examples, step by step

Example 5.1 A wing of S = 16 m² flies at 60 m/s at sea level with C_L = 0.5. Find the lift.

1. q = ½ × 1.225 × 60² = 2205 Pa.
2. L = 2205 × 16 × 0.5.

Answer: L = 17,640 N.

Example 5.2 A 1000 kg airplane with S = 16 m² flies level at 50 m/s at sea level. Find the C_L needed.

1. W = 9810 N; q = 1531.25 Pa.
2. C_L = 9810/(1531.25 × 16).

Answer: C_L = 0.400.

Example 5.3 The same airplane has C_L,max = 1.6. Find the sea-level stall speed.

1. V = √(2 × 9810/(1.225 × 16 × 1.6)).

Answer: V_stall = 25.0 m/s (about 48.6 knots).

Practice problems

Work each on paper first, then check.

1. What thickness does a NACA 0012 have?

12% of chord, symmetric (no camber).

2. Lift at 1.5× the speed, same C_L?

   2.25 times.

3. Stall speed of the worked example at 11 km (ρ = 0.3652)?

   25.0 × √(1.225/0.3652) = 45.8 m/s.

4. What happens to lift past stall?

   It drops as the flow separates; drag rises sharply.

Common misconceptions

Myth: Higher angle of attack always gives more lift.
Actually: Only up to stall.

Myth: Symmetric airfoils cannot make lift.
Actually: They make lift at positive angle of attack.

Myth: The center of pressure is a fixed point.
Actually: It moves with α; the aerodynamic center does not.

Flashcards

Lift

L = qSC_L.

Drag

D = qSC_D.

Stall speed

√(2W/ρSC_Lmax).

NACA 4-digit

Camber, location, thickness.

Kutta condition

Smooth trailing-edge flow.

Aerodynamic center

Moment independent of α.

Self-check quiz

1. C_L is defined as...

L/q

L/(qS)

qS/L

L/S

L/(qS). Lift over q times area.

2. NACA 2412 has max camber of...

2%

4%

12%

24%

2%. First digit.

3. Stall is caused by...

shock waves

flow separation

low density

high Re

flow separation. Boundary layer separates.

4. The Kutta condition fixes...

drag

circulation

Mach number

thickness

circulation. Smooth flow off the trailing edge.

Try this quiz again

How this helps save the planet

NASA tells how its engineer Richard Whitcomb created the supercritical airfoil: nearly flat on top, rounded on the bottom, thicker, with a blunt leading edge. Near the speed of sound, around 660 mph at cruise altitude, a conventional wing forms a standing shock wave that adds drag; the new shape minimized that shock.

NASA reports that supercritical wings increased efficiency by as much as 15%, also work better at slower speeds, and allow a lighter wing. They became the industry standard, and NASA says they save billions of dollars in fuel each year. Shaping a section's pressure distribution, as in this unit, is how fuel and emissions are cut.

On Saving Planets: Intro to Aerospace Engineering · Solutions

Sources: NASA: How a NASA engineer created the modern airplane wing (2016) (https://www.nasa.gov/aeronautics/how-a-nasa-engineer-created-the-modern-airplane-wing/)

Free learning resources

- NASA Glenn: The lift equation (https://www.grc.nasa.gov/www/k-12/airplane/lifteq.html): Free NASA page.
- NASA Glenn: The drag equation (https://www.grc.nasa.gov/www/k-12/airplane/drageq.html): Free NASA page.
- NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html): Explore lift curves.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Airfoil lectures.

☐ **I finished this unit**

Thin Airfoil Theory

Vortex sheets, modeling the camber line with a vortex sheet, the Kutta condition, the Fourier-series solution, the lift slope of 2π per radian, the zero-lift angle of cambered airfoils, the quarter-chord aerodynamic center, and panel methods.

Learning objectives

By the end of this unit you can:

- Model an airfoil by a vortex sheet on its camber line.
- Derive c_l = 2πα for a symmetric thin airfoil.
- Find the lift coefficient of a cambered airfoil from α and α_L=0.
- Explain why the aerodynamic center is at quarter chord.

Key concepts

- A vortex sheet of strength γ(s) per unit length creates a jump in tangential velocity across it. Placing one on the camber line and requiring the camber line to be a streamline gives an integral equation for γ.
- With the Kutta condition γ(TE) = 0 and a Fourier series in θ (x = c(1 − cos θ)/2), the solution gives c_l = 2π(α − α_L=0): the lift slope is 2π per radian (about 0.11 per degree) for every thin airfoil.
- For a symmetric airfoil α_L=0 = 0, c_m,c/4 = 0 and c_m,LE = −c_l/4, so the center of pressure is at quarter chord. Camber shifts α_L=0 negative and adds a constant nose-down c_m,c/4, but the aerodynamic center stays at quarter chord.
- Real airfoils of moderate thickness have lift slopes close to 2π until the boundary layer begins to separate; thin airfoil theory does not predict stall or drag.
- Panel methods cover the real surface with many source and vortex panels and solve for their strengths numerically; they are how designers compute pressure on any shape, from blades to wings.

Key equations

$$c_l = 2\pi(\alpha - \alpha_{L=0})$$

`c_l=2π(α-α_L=0)`

Thin airfoil lift

$$c_{m,c/4} = 0 \ (\text{symmetric})$$

`c_m,c/4=0\ (symmetric)`

Quarter-chord moment

$$c_{m,LE} = -\frac{c_l}{4}$$

`c_m,LE=-(c_l)/(4)`

Leading-edge moment (symmetric)

$$x = \frac{c}{2}(1 - \cos\theta)$$

`x= c2(1-θ)`

Glauert transformation

Worked examples, step by step

Example 6.1 Find c_l for a symmetric thin airfoil at α = 5°.

1. α = 0.08727 rad.
2. c_l = 2π × 0.08727.

Answer: c_l = 0.548.

Example 6.2 A cambered airfoil has α_L=0 = −2° (sample value). Find c_l at α = 4°.

1. α − α_L=0 = 6° = 0.10472 rad.
2. c_l = 2π × 0.10472.

Answer: c_l = 0.658.

Example 6.3 For the symmetric airfoil at α = 5°, find c_m about the leading edge and the center of pressure.

1. c_m,LE = −c_l/4 = −0.548/4.
2. x_cp = −c_m,LE c/c_l = c/4.

Answer: c_m,LE = −0.137; x_cp = 0.25c.

Practice problems

Work each on paper first, then check.

1. What angle gives c_l = 0.5 on a symmetric thin airfoil?

0.5/(2π) = 0.0796 rad = 4.56°.

2. Lift slope per degree?

   2π × π/180 = 0.1097 per degree.

3. Does thin airfoil theory predict stall?

   No; it is inviscid and linear.

4. Where is the aerodynamic center of a cambered thin airfoil?

   At quarter chord.

Common misconceptions

Myth: Camber changes the lift slope.
Actually: In thin airfoil theory it shifts the curve; the slope stays 2π.

Myth: The center of pressure is always at quarter chord.
Actually: Only for a symmetric airfoil; cambered airfoils have a moving c.p.

Myth: 2π is per degree.
Actually: It is per radian.

Flashcards

Vortex sheet

Jump in tangential velocity.

Lift

$c_l = 2\pi(\alpha - \alpha_{L0})$.

Symmetric moment

$c_{m,c/4} = 0$.

Leading-edge moment

$-c_l/4$.

Glauert

$x = c(1 - \cos\theta)/2$.

Panel method

Numerical source/vortex panels.

Self-check quiz

1. Thin airfoil lift slope is...

2π per degree

2π per radian

π per radian

4π

2π per radian. $c_l = 2\pi\alpha$.

2. Camber makes $\alpha_{L=0}$...

positive

negative

zero

undefined

negative. Lift at zero angle.

3. The aerodynamic center of a thin airfoil is at...

leading edge

quarter chord

half chord

trailing edge

quarter chord. Theory result.

4. Thin airfoil theory models the airfoil with...

sources only

a vortex sheet

shocks

boundary layers

a vortex sheet. On the camber line.

Try this quiz again

How this helps save the planet

DOE explains that a turbine blade, like an airplane wing, makes lift from a pressure difference, and that lift stronger than drag is what turns the rotor. Each blade section sees a local angle of attack set by the wind speed and its own rotational speed, so blades are twisted so that every section works near its best lift-to-drag angle.

Thin airfoil theory gives the first estimate of that section lift: $c_l = 2\pi(\alpha - \alpha_{L=0})$. Panel methods and wind-tunnel data refine it. DOE notes that land-based turbines run from 100 kilowatts to several megawatts, so a few percent better section lift-to-drag ratio means a lot of extra clean energy over a turbine's life.

On Saving Planets: Solutions · Linear Algebra

Sources: U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work)

Free learning resources

- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Thin airfoil theory lectures.
- NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html): Compare theory with the simulator.
- NASA Glenn: Lift from flow turning (https://www.grc.nasa.gov/www/k-12/airplane/foil2.html): Free NASA page.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Unified Engineering fluids.

☐ **I finished this unit**

Finite Wings and Induced Drag

Wingtip vortices and downwash, the induced angle of attack, Prandtl's lifting-line theory, elliptic lift distribution, span efficiency, induced drag $C_{Di} = C_L^2/(\pi e AR)$, the finite-wing lift slope, aspect ratio, winglets and truss-braced wings.

Learning objectives

By the end of this unit you can:

- Explain how tip vortices create downwash and induced drag.
- Compute induced drag with the span efficiency factor.
- Find the finite-wing lift slope.
- Explain how aspect ratio and winglets reduce induced drag.

Key concepts

- On a finite wing, high-pressure air below curls around the tips to the low-pressure top, forming trailing vortices. They induce a downward velocity (downwash) at the wing, tilting the local flow by the induced angle α_i.
- Tilting the lift vector back by α_i creates a drag component: induced drag, the price of making lift with a finite span. It is largest at low speed and high C_L.
- Prandtl's lifting-line theory models the wing as a bound vortex with a sheet of trailing vortices. An elliptic lift distribution gives uniform downwash, $\alpha_i = C_L/(\pi AR)$, and the minimum induced drag $C_{Di} = C_L^2/(\pi AR)$.
- Real wings have span efficiency $e < 1$: $C_{Di} = C_L^2/(\pi e AR)$. Aspect ratio $AR = b^2/S$ is the key lever: doubling AR halves induced drag.
- The finite-wing lift slope is lower than 2π: $a = a_0/(1 + a_0/(\pi e AR))$. Winglets and long, thin truss-braced wings both cut induced drag by weakening or spreading the tip vortices.

Key equations

$$AR = \frac{b^2}{S}$$

`AR=(b^2)/(S)`

Aspect ratio

$$\alpha_i = \frac{C_L}{\pi AR}$$

`α_i=(C_L)/(π AR)`

Induced angle (elliptic)

$$C_{D_i} = \frac{C_L^2}{\pi e AR}$$

`C_D_i=(C_L^2)/(π eAR)`

Induced drag

$$a = \frac{a_0}{1 + a_0/(\pi e AR)}$$

`a=(a_0)/(1+a_0/(π eAR))`

Finite-wing lift slope

Worked examples, step by step

Example 7.1 A wing with AR = 8 and e = 0.9 flies at C_L = 0.5. Find C_Di.

1. C_Di = 0.25/(π × 0.9 × 8).

Answer: C_Di = 0.01105.

Example 7.2 Find the lift slope of that wing if a_0 = 2π per radian.

1. a = 2π/(1 + 2π/(π × 0.9 × 8)) = 6.2832/1.2778.

Answer: a = 4.917 per radian (0.0858 per degree).

Example 7.3 Find the induced angle of an elliptic wing (AR = 8) at C_L = 0.5.

1. α_i = 0.5/(π × 8) rad.

Answer: α_i = 0.01989 rad = 1.14°.

Practice problems

Work each on paper first, then check.

1. A wing has b = 30 m and S = 100 m². Find AR.

   900/100 = 9.

2. Raise AR from 8 to 12 at the same e and C_L. New C_Di?

$0.01105 \times 8/12 = 0.00737$.

3. When is induced drag largest?

   At low speed and high C_L, e.g. take-off and climb.

4. What e does an elliptic wing have?

   $e = 1$.

Common misconceptions

Myth: Induced drag comes from skin friction.
Actually: It is the backward tilt of lift from downwash, an inviscid effect.

Myth: A finite wing has the same lift slope as its airfoil.
Actually: It is lower because of downwash.

Myth: Winglets work by adding wing area.
Actually: They mainly weaken tip vortices and their downwash.

Flashcards

Downwash

Induced by trailing vortices.

Aspect ratio

b^2/S.

Induced drag

$C_L^2/(\pi e AR)$.

Induced angle

$C_L/(\pi AR)$.

Span efficiency

$e \le 1$.

Finite-wing slope

$a_0/(1 + a_0/\pi e AR)$.

Self-check quiz

1. Induced drag coefficient is...

$C_L/(\pi e AR)$

$C_L^2/(\pi e AR)$

πeAR/C_L^2

C_L^2AR

C_L^2/(πeAR). Lifting-line result.

2. Doubling aspect ratio changes induced drag by…

×2

×½

×4

none

×½. $C_{Di} \propto 1/AR$.

3. Minimum induced drag comes from a…

rectangular lift distribution

elliptic lift distribution

triangular

tip-loaded

elliptic lift distribution. Uniform downwash.

4. Aspect ratio is…

S/b

b^2/S

c/b

b/c^2

b^2/S. Span squared over area.

Try this quiz again

How this helps save the planet

NASA Armstrong recounts that winglets first flew on a KC-135 on July 24, 1979; over 48 test flights they raised fuel efficiency by 6 to 7 percent and reduced drag by as much as 7 percent. NASA says winglets are now on roughly 10,000 jets, saving more than 10 billion gallons of jet fuel and over 130 million tons of carbon dioxide.

The next step pushes aspect ratio further. NASA's 2023 Sustainable Flight Demonstrator award (NASA $425 million over seven years, partners about $725 million) is for a transonic truss-braced wing: extra-long, thin wings braced by struts, which NASA says means less drag and, combined with other advances, up to 30% less fuel use and emissions. That is $C_{Di} = C_L^2/(\pi e AR)$ at work.

On Saving Planets: Solutions · Statics

Sources: NASA Armstrong: On the winglets of innovation (2023) (https://www.nasa.gov/centers-and-facilities/armstrong/on-the-winglets-of-innovation-with-nasa-armstrong-2/) · NASA: NASA issues award for greener, more fuel-efficient airliner of future (Jan. 18, 2023) (https://www.nasa.gov/news-release/nasa-issues-award-for-greener-more-fuel-efficient-airliner-of-future/)

Free learning resources

- NASA Glenn: Induced drag (https://www.grc.nasa.gov/www/k-12/airplane/induced.html): Free NASA page.
- NASA Glenn: Lift to drag ratio (https://www.grc.nasa.gov/www/k-12/airplane/ldrat.html): Free NASA page.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Lifting-line lectures.
- NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html): Simulator.

☐ **I finished this unit**

Viscous Flow, Reynolds Number and Boundary Layers

The no-slip condition, the Reynolds number and similarity, the Navier–Stokes equations in words, laminar and turbulent boundary layers, the Blasius flat-plate solution, skin friction, transition, separation and pressure drag, and the drag breakdown of an aircraft.

Learning objectives

By the end of this unit you can:

- Compute the Reynolds number and use it for similarity.
- Find boundary-layer thickness and skin-friction drag on a flat plate.
- Distinguish laminar and turbulent boundary layers.
- Explain separation, pressure drag and the total drag breakdown.

Key concepts

- Viscous fluid sticks to a surface (no slip), so a thin boundary layer forms where speed rises from zero to the outer flow. Outside it, the inviscid theory of Units 4–7 works well.
- The Reynolds number Re = ρVL/μ compares inertial to viscous forces. Two geometrically similar flows with equal Re (and Mach number) have the same coefficients; that is why wind-tunnel tests work.
- The Navier–Stokes equations add viscous stresses to Euler’s equation. Prandtl’s boundary-layer simplification and the Blasius solution give, for a laminar flat plate, δ = 5.0x/√Re_x and C_f = 1.328/√Re_L.
- Above a transition Reynolds number (about 5 × 10⁵ on a smooth flat plate) the layer becomes turbulent: thicker, with more skin friction, C_f ≈ 0.074/Re_L^0.2, but more resistant to separation.
- In an adverse pressure gradient the boundary layer can separate, leaving a wide wake and pressure drag. Total drag = skin friction + pressure (form) drag + induced drag + wave drag (Unit 9).

Key equations

$$Re = \frac{\rho V L}{\mu}$$

`Re=(ρ VL)/(μ)`

Reynolds number

$$\delta = \frac{5.0x}{\sqrt{Re_x}}$$

`=5.0x_x`

Laminar boundary-layer thickness

$$C_f = \frac{1.328}{\sqrt{Re_L}}$$

`C_f=1.328_L`

Laminar skin friction (Blasius)

$$C_f = \frac{0.074}{Re_L^{0.2}}$$

`C_f=0.074Re_L^0.2`

Turbulent skin friction (flat plate)

Worked examples, step by step

Example 8.1 Find the Reynolds number of a wing with chord 1.5 m at 70 m/s at sea level.

1. Re = 1.225 × 70 × 1.5/1.789 × 10^{-5}.

Answer: Re = 7.19 × 10^{6} (turbulent over most of the chord).

Example 8.2 Compare laminar and turbulent skin-friction coefficients at Re_L = 10^{6}, and the boundary-layer thickness at x = 1 m.

1. Laminar C_f = 1.328/1000; turbulent C_f = 0.074/15.85.
2. δ_lam = 5.0/1000; δ_turb = 0.37/15.85.

Answer: C_f = 0.00133 vs 0.00467; δ = 5.0 mm vs 23.3 mm.

Example 8.3 A 1 m × 1 m flat plate at 30 m/s at sea level stays laminar (Re = 2.05 × 10^{6}, assumed). Find the skin-friction drag on one side.

1. q = ½ × 1.225 × 900 = 551.25 Pa.
2. C_f = 1.328/√(2.054 × 10^{6}) = 0.000927.
3. D = qSC_f.

Answer: D = 0.511 N.

Practice problems

Work each on paper first, then check.

1. A model at 1/10 scale must match Re in the same air. What speed?

   10 times the full-scale speed (often impossible, so pressurized or cryogenic tunnels are used).

2. Does C_f grow or shrink as Re rises?

   It shrinks for both laminar and turbulent layers.

3. Why do golf balls have dimples?

   A turbulent boundary layer separates later, cutting pressure drag.

4. Name four drag components.

   Skin friction, pressure, induced, wave.

Common misconceptions

Myth: Turbulent flow always means more drag.
Actually: More skin friction, but often less pressure drag if separation is delayed.

Myth: The boundary layer is thick.
Actually: It is millimetres to centimetres on a wing.

Myth: Reynolds number depends only on speed.
Actually: It depends on density, speed, length and viscosity.

Flashcards

No slip

Zero velocity at the wall.

Reynolds number

ρVL/μ.

Laminar δ

5.0x/√Re_x.

Laminar C_f

1.328/√Re_L.

Turbulent C_f

0.074/Re_L^0.2.

Separation

Adverse gradient, wake, pressure drag.

Self-check quiz

1. Reynolds number compares...

- inertia to viscosity
- pressure to gravity
- speed to sound
- lift to drag

inertia to viscosity. Re = ρVL/μ.

2. Laminar flat-plate C_f is...

- 0.074/Re^0.2
- 1.328/√Re
- 5/√Re
- 2π

1.328/√Re. Blasius.

3. Separation is promoted by...

- favorable pressure gradient
- adverse pressure gradient
- high density
- shocks only

adverse pressure gradient. Rising pressure slows the layer.

4. No-slip means...

- air speed at the wall is zero
- no friction

inviscid flow

no pressure

air speed at the wall is zero. Fluid sticks to the wall.

Try this quiz again

How this helps save the planet

Skin friction is a large share of an airliner's cruise drag, which is why engineers work so hard on smooth surfaces and on keeping boundary layers laminar longer. Every unit of drag removed is fuel and carbon not burned.

EPA reports that the aircraft covered by its greenhouse gas standards account for 10 percent of U.S. transportation greenhouse gas emissions and 3 percent of total U.S. emissions, and that its 2020 standards align with the International Civil Aviation Organization's CO_2 standards. Boundary-layer engineering, from riblets to laminar-flow wings, is one route to meeting those rules.

On Saving Planets: Solutions · Differential Equations

Sources: U.S. EPA: Regulations for greenhouse gas emissions from aircraft (https://www.epa.gov/regulations-emissions-vehicles-and-engines/regulations-greenhouse-gas-emissions-aircraft) · NASA Glenn: Boundary layer (https://www.grc.nasa.gov/www/k-12/airplane/boundlay.html)

Free learning resources

- NASA Glenn: Boundary layer (https://www.grc.nasa.gov/www/k-12/airplane/boundlay.html): Free NASA page.
- NASA Glenn: Reynolds number (https://www.grc.nasa.gov/www/k-12/airplane/reynolds.html): Free NASA page.
- OpenStax UP1, 14.7 Viscosity and turbulence (https://openstax.org/books/university-physics-volume-1/pages/14-7-viscosity-and-turbulence): Free textbook section.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Viscous flow lectures.

☐ **I finished this unit**

Compressible Flow: Isentropic Relations and Shock Waves

When compressibility matters, stagnation properties, isentropic relations, area–velocity relation and nozzles, normal shocks, oblique shocks and expansion waves in brief, the critical Mach number, drag divergence, wave drag and supercritical airfoils.

Learning objectives

By the end of this unit you can:

- Use isentropic relations to find stagnation properties.
- Explain the area–velocity relation and choked nozzles.
- Compute properties across a normal shock.
- Explain critical Mach number and why supercritical airfoils delay drag rise.

Key concepts

- Below about M = 0.3 density changes are under 5% and flow can be treated as incompressible. Above that, energy and density changes must be included.
- Stagnation properties for adiabatic, isentropic flow: $T_0/T = 1 + (\gamma - 1)M^2/2$, $p_0/p = (T_0/T)^{(\gamma/(\gamma-1))}$, $\rho_0/\rho = (T_0/T)^{(1/(\gamma-1))}$.
- Area–velocity: $dA/A = (M^2 - 1)dV/V$. Subsonic flow speeds up in a converging duct; supersonic flow speeds up in a diverging one. Sonic flow occurs at a throat; A/A* sets the Mach number.
- A normal shock is a thin, irreversible jump: supersonic flow becomes subsonic, and pressure, density and temperature jump while total pressure falls. For $\gamma = 1.4$ and $M_1 = 2$, $M_2 = 0.577$ and $p_2/p_1 = 4.5$.
- On a wing at high subsonic speed, the flow over the top reaches M = 1 first, at the critical Mach number. Beyond it a shock forms and drag rises sharply (drag divergence). Supercritical airfoils and wing sweep push this rise to higher speeds.

Key equations

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

`(T_0)/(T)=1+(γ-1)/(2)M^2`

Stagnation temperature

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

`(p_0)/(p)=((T_0)/(T))^γ/(γ-1)`

Isentropic pressure ratio

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

`M_2^2=1+(γ-1)/(2)M_1^2γ M_1^2-(γ-1)/(2)`

Normal-shock Mach number

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

`(p_2)/(p_1)=1+(2γ)/(γ+1)(M_1^2-1)`

Normal-shock pressure ratio

Worked examples, step by step

Example 9.1 An airliner flies at M = 0.85. Find T_0/T and p_0/p.

1. $T_0/T = 1 + 0.2 \times 0.7225 = 1.1445$.
2. $p_0/p = 1.1445^{3.5}$.

Answer: $T_0/T = 1.1445$; $p_0/p = 1.604$.

Example 9.2 Find M_2, p_2/p_1, ρ_2/ρ_1 and T_2/T_1 across a normal shock with $M_1 = 2$.

1. $M_2 = \sqrt{(1.8/5.4)}$.
2. $p_2/p_1 = 1 + (2.8/2.4)(3)$.
3. $\rho_2/\rho_1 = 2.4 \times 4/(2 + 0.4 \times 4) = 2.667$.
4. $T_2/T_1 = (p_2/p_1)/(\rho_2/\rho_1)$.

Answer: $M_2 = 0.577$, $p_2/p_1 = 4.5$, $\rho_2/\rho_1 = 2.667$, $T_2/T_1 = 1.688$.

Example 9.3 NASA puts the speed of sound at cruise altitude at around 660 mph. Convert it to m/s and compare with the standard-atmosphere value at 11 km (295.1 m/s).

1. $660 \times 0.44704 = 295.0$ m/s.

Answer: 295.0 m/s, within 0.1% of the standard-atmosphere value.

Practice problems

Work each on paper first, then check.

1. Find p_0/p at M = 2.

   1.8^3.5 = 7.82.

2. Find A/A* at M = 2.

   1.688.

3. Below what Mach number can density changes usually be ignored?

   About 0.3.

4. Across a normal shock, does total pressure rise or fall?

   It falls (entropy rises).

Common misconceptions

Myth: A shock speeds up the flow.
Actually: A normal shock makes supersonic flow subsonic.

Myth: Wave drag only matters above M = 1.
Actually: It starts above the critical Mach number, below M = 1 on a wing.

Myth: A diverging duct always slows the flow.
Actually: It speeds up supersonic flow.

Flashcards

Incompressible limit

M < 0.3.

T_0/T

$1 + 0.2M^2$.

p_0/p

(T_0/T)^3.5.

Area–velocity

$dA/A = (M^2 - 1)dV/V$.

Normal shock M=2

$M_2 = 0.577$, $p_2/p_1 = 4.5$.

Critical Mach

Local M = 1 on the wing.

Self-check quiz

1. T_0/T at M = 1 is…

1.0

1.2

1.4

2.0

1.2. 1 + 0.2.

2. Supersonic flow speeds up in a…

converging duct

diverging duct

constant duct

no duct

diverging duct. $dA/A = (M^2 - 1)dV/V$.

3. p_2/p_1 across a normal shock at $M_1 = 2$ is…

2

4.5

7.8

1.7

4.5. Normal-shock relation.

4. Supercritical airfoils…

raise the drag-divergence Mach number

eliminate lift

add camber only at the nose

work only at Mach 2

raise the drag-divergence Mach number. Weaker shock.

Try this quiz again

How this helps save the planet

NASA explains that as airliners approach the speed of sound, about 660 mph at cruise altitude, a standing shock wave forms on a conventional wing and adds drag. Whitcomb's supercritical airfoil, flatter on top and rounded on the bottom, minimized that shock, raising efficiency by as much as 15% according to NASA.

That is the critical-Mach and drag-divergence story from this unit. NASA says the design became the industry standard and saves billions of dollars in fuel a year; less fuel burned at cruise means less CO_2 from the flights that make up a large share of aviation emissions.

On Saving Planets: Intro to Aerospace Engineering · Space Weather

Sources: NASA: How a NASA engineer created the modern airplane wing (2016) (https://www.nasa.gov/aeronautics/how-a-nasa-engineer-created-the-modern-airplane-wing/) · NASA Glenn: Normal shock wave (https://www.grc.nasa.gov/www/k-12/airplane/normal.html)

Free learning resources

- NASA Glenn: Isentropic flow (https://www.grc.nasa.gov/www/k-12/airplane/isentrop.html): Free NASA page.
- NASA Glenn: Normal shock wave (https://www.grc.nasa.gov/www/k-12/airplane/normal.html): Free NASA page.
- NASA Glenn: Mach number (https://www.grc.nasa.gov/www/k-12/airplane/mach.html): Free NASA page.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Compressible flow lectures.

☐ **I finished this unit**

Aircraft Performance: Drag Polar, Glide, Range and Endurance

The drag polar C_D = C_D0 + KC_L², thrust and power required, minimum-drag speed, maximum lift-to-drag ratio, gliding flight, the Breguet range and endurance equations, and how aerodynamic efficiency translates into fuel and emissions.

Learning objectives

By the end of this unit you can:

- Build a drag polar and find (L/D)max.
- Find the minimum-drag speed and glide distance.
- Use the Breguet range equation.
- Connect drag reductions to fuel burn.

Key concepts

- The drag polar C_D = C_D0 + KC_L² with K = 1/(πeAR) combines zero-lift (parasite) drag and induced drag.
- In level flight L = W and T = D. Drag is lowest where parasite and induced drag are equal: C_L* = √(C_D0/K), giving (L/D)max = 1/(2√(KC_D0)).
- Gliding: with no thrust, the glide ratio equals L/D, so the best glide distance from height h is h(L/D)max.
- Breguet range for a jet: R = (V/c)(L/D) ln(W_i/W_f), with c the thrust-specific fuel consumption. Range grows directly with L/D, so aerodynamic efficiency multiplies with engine efficiency and weight.
- For a fixed trip, raising L/D lowers the fuel needed; that is why winglets, supercritical airfoils and high-aspect-ratio wings all cut CO_2.

Key equations

$$C_D = C_{D0} + K C_L^2,\ K = \frac{1}{\pi e A R}$$

```
C_D=C_D0+KC_L^2,\ K=(1)/(π eAR)
```

Drag polar

$$\left(\frac{L}{D}\right)_{max} = \frac{1}{2\sqrt{K C_{D0}}}$$

```
( LD)_max=12_D0
```

Maximum lift-to-drag ratio

$$C_L^* = \sqrt{C_{D0}/K}$$

C_L^*=_D0/K

Lift coefficient for minimum drag

$$R = \frac{V}{c}\frac{L}{D}\ln\frac{W_i}{W_f}$$

R=V/cL/D(W_i)/(W_f)

Breguet range (jet)

Worked examples, step by step

Example 10.1 A wing has AR = 8, e = 0.9 and C_D0 = 0.02. Find K, (L/D)max and C_L*.

1. K = 1/(π × 0.9 × 8) = 0.04421.
2. (L/D)max = 1/(2√(0.04421 × 0.02)).
3. C_L* = √(0.02/0.04421).

Answer: K = 0.04421, (L/D)max = 16.81, C_L* = 0.673.

Example 10.2 For that airplane (W = 9810 N, S = 16 m², sea level), find the minimum-drag speed and the best glide distance from 1000 m.

1. V = √(2W/(ρSC_L*)) = √(19,620/(1.225 × 16 × 0.6726)).
2. Glide = 1000 × 16.81.

Answer: V = 38.6 m/s; glide distance 16.8 km.

Example 10.3 A jet (sample values) cruises at V = 230 m/s with c = 1.6 × 10⁻⁴ s⁻¹, L/D = 17 and W_i/W_f = 1.25. Find the range.

1. R = (230/1.6 × 10⁻⁴) × 17 × ln 1.25.

Answer: R = 5453 km.

Practice problems

Work each on paper first, then check.

1. Raise L/D from 17 to 18 in the Breguet example. New range?

   5453 × 18/17 = 5774 km.

2. What C_D does the polar give at C_L^*?

   $2C_{D0} = 0.04$.

3. What is L/D at $C_L = 1.0$ for the worked polar?

   1.0/0.06421 = 15.57.

4. A glider has L/D = 40. How far from 2 km?

   80 km.

Common misconceptions

Myth: The best glide is at the slowest speed.
Actually: It is at the minimum-drag speed, faster than stall.

Myth: Range depends only on fuel carried.
Actually: It depends on L/D, engine efficiency and the weight ratio.

Myth: Parasite drag is always the largest part.
Actually: At low speed induced drag dominates.

Flashcards

Drag polar

$C_{D0} + KC_L^2$.

K

$1/(\pi eAR)$.

(L/D)max

$1/(2\sqrt{(KC_{D0})})$.

C_L^*

$\sqrt{(C_{D0}/K)}$.

Glide

Distance = h · L/D.

Breguet

$R = (V/c)(L/D)\ \ln(W_i/W_f)$.

Self-check quiz

1. At minimum drag…

parasite = induced

induced = 0

C_L = 0

V = stall

parasite = induced. Derivative of D is zero.

2. Glide ratio equals…

L/D

D/L

C_L

W/S

L/D. No thrust.

3. Breguet range is proportional to…

D/L

L/D

C_D0

K

L/D. R ∝ L/D.

4. K in the polar is…

1/(πeAR)

πeAR

C_{D0}

2π

$1/(\pi e AR)$. Induced-drag factor.

Try this quiz again

How this helps save the planet

NASA's 2023 release on the Sustainable Flight Demonstrator says single-aisle aircraft account for nearly half of worldwide aviation emissions, and that the truss-braced-wing design, combined with other advances, could reduce fuel consumption and emissions by up to 30%. It ties this to the goal of net-zero aviation emissions by 2050.

The Breguet equation shows why: range and fuel efficiency scale with L/D. EPA notes that covered aircraft are 10 percent of U.S. transportation greenhouse gas emissions, so a few points of L/D, multiplied across thousands of flights a day, matter for the climate.

On Saving Planets: Solutions · Calculus I

Sources: NASA: NASA issues award for greener, more fuel-efficient airliner of future (Jan. 18, 2023) (https://www.nasa.gov/news-release/nasa-issues-award-for-greener-more-fuel-efficient-airliner-of-future/) · U.S. EPA: Regulations for greenhouse gas emissions from aircraft (https://www.epa.gov/regulations-emissions-vehicles-and-engines/regulations-greenhouse-gas-emissions-aircraft)

Free learning resources

- NASA Glenn: Lift to drag ratio (https://www.grc.nasa.gov/www/k-12/airplane/ldrat.html): Free NASA page.
- NASA Glenn: Induced drag (https://www.grc.nasa.gov/www/k-12/airplane/induced.html): Free NASA page.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Performance lectures.
- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): MIT aerodynamics.

Watch the lecture

Lecture 12: Aircraft Performance (https://ocw.mit.edu/courses/16-687-private-pilot-ground-school-january-iap-2019/resources/lecture-12-aircraft-performance/) (in English)

MIT 16.687 Private Pilot Ground School (IAP 2019), MIT OpenCourseWare. License: CC BY-NC-SA 4.0 (https://creativecommons.org/licenses/by-nc-sa/4.0/) · Terms of use (https://ocw.mit.edu/pages/privacy-and-terms-of-use/). Linked for free study only; Saving Planets is not affiliated with or endorsed by this university.

☐ **I finished this unit**

Problem Sets

These practice problems live inside each unit. Work on paper first, then open the answer.

1. Fluid Properties and the Standard Atmosphere
   4 problems
2. Fluid Statics, Bernoulli's Equation and Airspeed Measurement
   4 problems
3. Conservation Laws: Mass, Momentum and Energy; Control Volumes and Momentum Theory
   4 problems
4. Potential Flow and the Kutta–Joukowski Theorem
   4 problems
5. Airfoils, Lift, Drag and Moment Coefficients
   4 problems
6. Thin Airfoil Theory
   4 problems
7. Finite Wings and Induced Drag
   4 problems
8. Viscous Flow, Reynolds Number and Boundary Layers
   4 problems
9. Compressible Flow: Isentropic Relations and Shock Waves
   4 problems
10. Aircraft Performance: Drag Polar, Glide, Range and Endurance
    4 problems

The lab (Aerodynamics lab)

The lab is the hands-on part of the course. Here it is free and for practice only, with no credit.

Every lab can be done in a free simulation or with paper and household items. Throw gliders only in open spaces, never toward people, and keep away from roads and power lines; use a sturdy step stool, not a chair, for drop tests.

Safety reference: OSHA: Fall protection (https://www.osha.gov/fall-protection)

Lab A: The standard atmosphere spreadsheet

WEEK 1

Goal: Build NASA Glenn's standard atmosphere model in a spreadsheet and see how temperature, pressure and density fall with altitude.

$$T = 15.04 - 0.00649h, \;\; p = 101.29\left[\frac{T+273.1}{288.08}\right]^{5.256}, \;\; \rho = \frac{p}{0.2869(T+273.1)}$$

`T=15.04-0.00649h,\ \ p=101.29[(T+273.1)/(288.08)]^5.256,\ \ ρ=(p)/(0.2869(T+273.1))`

NASA Glenn troposphere model (h in m, T in °C, p in kPa)

Procedure

1. Make a column of altitudes from 0 to 11,000 m in 500 m steps.
2. Add columns for T, p and ρ using NASA's metric formulas.
3. Add the speed of sound a = √(1.4 × 287 × (T + 273.15)).
4. Plot ρ/ρ_0 against altitude and compare with FoilSim's altitude setting.

Data table

h (m)	T (°C)	p (kPa)	ρ (kg/m³)
0	15.04	101.40	1.2266
5,000	−17.41	54.11	0.7377
11,000	−56.35	22.71	0.3652

Values from NASA Glenn's curve fits; textbook standard-atmosphere tables differ slightly (for example 1.225 kg/m³ at sea level).

Sample calculation

1. ρ(11 km)/ρ(0) = 0.3652/1.2266 = 0.298.
2. a(11 km) = √(1.4 × 287 × 216.8) = 295.1 m/s.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any free spreadsheet works.

NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html) · NASA Glenn: Earth atmosphere model (metric units) (https://www.grc.nasa.gov/www/k-12/airplane/atmosmet.html)

Planet link: Cruise altitude sits near the top of this table, where low density cuts drag; NASA's model is the one used in FoilSim. NASA Glenn: Earth atmosphere model (metric units) (https://www.grc.nasa.gov/www/k-12/airplane/atmosmet.html)

- [ ] **I completed this lab (wet or virtual)**

Lab B: Bernoulli, the venturi and the pitot tube

WEEK 2

Goal: See pressure fall where flow speeds up, and turn a pitot-static pressure reading into airspeed.

$$V = \sqrt{2(p_0 - p)/\rho}$$

V=2(p_0-p)/ρ

Pitot-static airspeed

Procedure

1. Open PhET Fluid Pressure and Flow, Flow tab; narrow the pipe and read pressure and speed at both sections.
2. Check continuity $A_1V_1 = A_2V_2$ and Bernoulli's pressure drop.
3. Home: hold a strip of paper below your lips and blow across the top; it rises. Hang two sheets of paper 3 cm apart and blow between them; they move together.
4. Fill the table for a pitot tube at sea level.

Data table

$p_0 - p$ (Pa)	V (m/s)
100	12.8
500	28.6
1500	49.5

Computed for ρ = 1.225 kg/m³; the paper demos show the effect, not exact numbers (the strip also rises because the jet follows the curved paper).

Sample calculation

1. V = √(2 × 500/1.225) = 28.6 m/s.
2. Doubling V needs 4 times the pressure difference.

> **Safety**
>
> - Blow gently; stop if you feel dizzy.

Home or virtual alternative

- The paper demos are the home lab.

PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow) · PhET: Under Pressure (https://phet.colorado.edu/en/simulations/under-pressure)

Planet link: NASA notes Mars air is less than 1% as dense as Earth's, so the same pitot reading on Mars would mean about 10 times the speed. NASA Science: Ingenuity Mars Helicopter (https://science.nasa.gov/mission/mars-2020-perseverance/ingenuity-mars-helicopter/)

- [ ] **I completed this lab (wet or virtual)**

Lab C: The lift curve in NASA FoilSim

WEEK 8

Goal: Measure lift coefficient against angle of attack, find the lift slope and zero-lift angle, and compare with thin airfoil theory.

$$c_l = 2\pi(\alpha - \alpha_{L=0})$$

`c_l=2π(α-α_L=0)`

Thin airfoil theory

Procedure

1. Open NASA FoilSim, choose a symmetric airfoil, set speed and altitude.
2. Record lift at α = 0°, 2°, 4°, 6°, … up to 10°, and convert to c_l = L/(qS).
3. Repeat with camber; find α_L=0.
4. Plot both curves and compare with the theory table.

Data table

α (°)	c_l theory (symmetric)
0	0
2	0.219
4	0.439
6	0.658

Theory values; FoilSim includes thickness and other corrections, so your slope may differ by a few percent.

Sample calculation

1. Slope = 0.658/6 = 0.110 per degree = 2π per radian.
2. c_l at 5° = 2π × 0.08727 = 0.548.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- FoilSim runs in a browser.

NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html)

Planet link: DOE says turbine blades make lift like wings; blade designers use lift curves like this to set each section's angle. U.S. DOE: How do wind turbines work? (https://www.energy.gov/eere/wind/how-do-wind-turbines-work)

- [ ] **I completed this lab (wet or virtual)**

Lab D: Paper gliders, aspect ratio and glide ratio

WEEK 10

Goal: Measure the glide ratio (L/D) of paper gliders with different aspect ratios and see induced drag in action.

$$\frac{L}{D} = \frac{\text{distance}}{\text{height}}, \quad AR = \frac{b^2}{S}$$

LD=,\ \ AR=(b^2)/(S)

Glide ratio and aspect ratio

Procedure

1. Build two simple gliders from printer paper with the same wing area: one short and wide, one long and narrow. Add a paper clip at the nose for balance.
2. Launch gently, level, from a fixed height (for example 1.5 m) in a large room or a calm outdoor space.
3. Measure horizontal distance for 5 flights each and average.
4. Compute AR and L/D for each and compare.

Data table

Glider	b (m)	S (m²)	AR	Distance from 1.5 m (m)	L/D
Short wing	0.20	0.010	4	6.0	4.0
Long wing	0.30	0.010	9	9.0	6.0

Sample values to show the method; your results depend on folding, balance and launch.

Sample calculation

1. AR = $0.30^2/0.010 = 9$.
2. L/D = 9.0/1.5 = 6.0.

> **Safety**
>
> - Never throw toward people or faces; clear the area first.
> - Launch from the floor or a sturdy step stool, not a chair (OSHA fall-protection guidance).

Home or virtual alternative

- This is a home lab.

NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html) · NASA Glenn: Induced drag (https://www.grc.nasa.gov/www/k-12/airplane/induced.html)

Planet link: NASA's truss-braced wing uses extra-long, thin wings for the same reason the long glider flies farther: less drag. NASA: NASA issues award for greener, more fuel-efficient airliner of future (Jan. 18, 2023) (https://www.nasa.gov/news-release/nasa-issues-award-for-greener-more-fuel-efficient-airliner-of-future/)

- [ ] **I completed this lab (wet or virtual)**

Lab E: Coffee-filter drag and terminal velocity

WEEK 11

Goal: Measure terminal velocity, find a drag coefficient, and show that drag grows with the square of speed.

$$mg = \tfrac{1}{2}\rho V_t^2 C_D A$$

`mg=12ρ V_t^2C_DA`

Drag equals weight at terminal velocity

Procedure

1. Drop one paper coffee filter from about 2 m and film it in slow motion next to a tape measure.
2. Find the terminal speed from the last metre of fall.
3. Stack 2 and then 4 filters (same shape, more weight) and repeat.
4. Compute C_D and check that V_t grows as √m.

Data table

Filters	m (g)	V_t (m/s)	C_D
1	1.0	1.20	0.98
2	2.0	1.70	0.98

Sample values with a 12 cm diameter filter; weigh your own filters on a kitchen scale.

Sample calculation

1. C_D = 2 × 0.00981/(1.225 × 1.20² × π × 0.06²) = 0.98.
2. V_t(2 filters) = 1.20 × √2 = 1.70 m/s.

> **Safety**
>
> - Drop from standing height or a sturdy step stool only.
> - Keep the area clear of tripping hazards.

Home or virtual alternative

- This is a home lab.

NASA Glenn: The drag equation (https://www.grc.nasa.gov/www/k-12/airplane/drageq.html) · PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow)

Planet link: EPA notes covered aircraft are 3% of total U.S. greenhouse gas emissions; drag is what their engines burn fuel to overcome. U.S. EPA: Regulations for greenhouse gas emissions from aircraft (https://www.epa.gov/regulations-emissions-vehicles-and-engines/regulations-greenhouse-gas-emissions-aircraft)

- [ ] **I completed this lab (wet or virtual)**

Lab F: Drag polar and range spreadsheet

WEEK 15

Goal: Build a drag polar, find (L/D)max, and use the Breguet equation to see how drag reductions change range and fuel.

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$

`C_D=C_D0+(C_L^2)/(π eAR)`

Drag polar

Procedure

1. Set C_D0 = 0.02, e = 0.9 and AR = 8 (sample values).
2. Compute C_D and L/D for C_L from 0.1 to 1.2.
3. Find the maximum L/D and compare with 1/(2√(KC_D0)).
4. Change AR to 12, and total drag by −7% (NASA's winglet drag figure), and see how range changes.

Data table

C_L	C_D	L/D
0.2	0.0218	9.19
0.673	0.0400	16.81
1.0	0.0642	15.57

Computed from the sample polar.

Sample calculation

1. K = 0.04421; (L/D)max = 16.81 at C_L = 0.673.
2. Breguet range ∝ L/D, so +10% L/D gives +10% range.

> **Safety**
>
> - Computer only.

Home or virtual alternative

- Any spreadsheet works.

NASA Glenn: Lift to drag ratio (https://www.grc.nasa.gov/www/k-12/airplane/ldrat.html) · NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html)

Planet link: NASA says winglets cut drag by as much as 7% and are now on about 10,000 jets, saving over 10 billion gallons of fuel. NASA Armstrong: On the winglets of innovation (2023) (https://www.nasa.gov/centers-and-facilities/armstrong/on-the-winglets-of-innovation-with-nasa-armstrong-2/)

- [ ] **I completed this lab (wet or virtual)**

Exam prep practice sets

Time yourself: about 10 minutes per problem. Then compare with the full solutions.

Midterm practice set (Units 1–5)

1. Find the density of air at 60 kPa and 250 K.

    1. ρ = 60,000/(287 × 250).

    Answer: 0.836 kg/m³.

2. A pitot-static tube reads 800 Pa where ρ = 0.9 kg/m³. Find the true airspeed.

    1. V = √(2 × 800/0.9).

    Answer: 42.2 m/s.

3. Find the ideal hover power of a 500 kg helicopter with an 8 m rotor at sea level.

    1. T = 4905 N; A = 50.27 m².
    2. P = T^1.5/√(2ρA).

    Answer: 31.0 kW.

4. Find C_p at θ = 45° on a non-lifting cylinder.

    1. 1 − 4 × 0.5.

    Answer: −1.

5. A body at sea level in a 40 m/s stream has Γ = 15 m²/s. Find L'.

    1. 1.225 × 40 × 15.

    Answer: 735 N/m.

Final practice set (all units)

1. Find c_l for a symmetric thin airfoil at 3°.
    1. 2π × 0.05236.

    Answer: 0.329.

2. Find C_Di for C_L = 0.6, AR = 10, e = 0.85.
    1. 0.36/(π × 8.5).

    Answer: 0.0135.

3. Find Re for a 2 m chord at 50 m/s at sea level.
    1. 1.225 × 50 × 2/1.789 × 10^{-5}.

    Answer: 6.85 × 10^{6}.

4. Find M_2 and p_2/p_1 across a normal shock at M_1 = 3.
    1. M_2 = √(2.8/12.4); p_2/p_1 = 1 + (2.8/2.4) × 8.

    Answer: M_2 = 0.475, p_2/p_1 = 10.33.

5. Find T_0/T and p_0/p at M = 0.8.
    1. 1 + 0.2 × 0.64 = 1.128; 1.128^3.5.

    Answer: 1.128 and 1.524.

6. Find (L/D)max for C_D0 = 0.025, AR = 10, e = 0.85.
    1. K = 0.03745.

    Answer: 16.34.

Formula sheet

$$p = \rho R T,\ a = \sqrt{\gamma R T},\ M = V/a$$

$$p + \tfrac{1}{2}\rho V^2 = p_0,\ q = \tfrac{1}{2}\rho V^2$$

$$C_p = 1 - (V/V_\infty)^2$$

$$\dot{m} = \rho A V,\ \nabla \cdot \mathbf{V} = 0$$

$$P_{ideal} = T^{3/2}/\sqrt{2\rho A}$$

$$L' = \rho_\infty V_\infty \Gamma,\ C_p = 1 - 4\sin^2\theta$$

$$L = qSC_L,\ D = qSC_D$$

$$c_l = 2\pi(\alpha - \alpha_{L=0})$$

$$C_{D_i} = C_L^2/(\pi e AR),\ AR = b^2/S \qquad Re = \rho V L/\mu,\ C_f = 1.328/\sqrt{Re}$$

$$T_0/T = 1 + \tfrac{\gamma-1}{2}M^2 \qquad p_2/p_1 = 1 + \tfrac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

$$(L/D)_{max} = 1/(2\sqrt{KC_{D0}}),\ R = \tfrac{V}{c}\tfrac{L}{D}\ln\tfrac{W_i}{W_f}$$

Constants

Constant	Value
Sea-level standard air (textbook values)	p = 101,325 Pa, T = 288.15 K, ρ = 1.225 kg/m³, μ = 1.789 × 10⁻⁵ kg/(m·s), a = 340.3 m/s
Gas constant for air	R = 287 J/(kg·K); γ = 1.4
1 knot	0.514444 m/s
1 foot	0.3048 m
1 mile per hour	0.44704 m/s
Gravity on Mars (textbook value)	g = 3.71 m/s²

Source: NIST Guide to the SI, Appendix B.9: conversion factors (https://www.nist.gov/pml/special-publication-811/nist-guide-si-appendix-b-conversion-factors/nist-guide-si-appendix-b9)

Free texts and simulations

Use these free lectures, NASA pages and simulations alongside the course.

- MIT OpenCourseWare 16.100 Aerodynamics (Fall 2005) (https://ocw.mit.edu/courses/16-100-aerodynamics-fall-2005/): Free MIT lecture notes and problem sets.
- MIT OpenCourseWare 16.01–16.04 Unified Engineering (2005–2006) (https://ocw.mit.edu/courses/16-01-unified-engineering-i-ii-iii-iv-fall-2005-spring-2006/): Free MIT fluids and thermodynamics notes.
- NASA Glenn: Beginner's Guide to Aeronautics (https://www.grc.nasa.gov/www/k-12/airplane/short.html): Hundreds of free NASA pages on lift, drag and compressible flow.
- NASA Glenn: FoilSim interactive airfoil simulator (https://www.grc.nasa.gov/www/k-12/FoilSim/index.html): Change airfoil shape, speed and altitude and see lift and drag.
- OpenStax University Physics Volume 1, 14.6 Bernoulli's equation (https://openstax.org/books/university-physics-volume-1/pages/14-6-bernoullis-equation): Free textbook section.
- PhET: Fluid Pressure and Flow (https://phet.colorado.edu/en/simulations/fluid-pressure-and-flow): Pressure, flow and Bernoulli.

Glossary

Angle of attack
Angle between the chord and the oncoming flow.

Aspect ratio
b^2/S, span squared over wing area.

Boundary layer
Thin layer near a surface where viscosity slows the flow.

Camber
Curvature of an airfoil's mean line.

Circulation
Line integral of velocity around a closed curve; tied to lift.

Critical Mach number
Free-stream Mach at which local flow first reaches $M = 1$.

Downwash
Downward flow induced by trailing vortices.

Drag polar
C_D as a function of C_L.

Dynamic pressure
$q = \frac{1}{2}\rho V^2$.

Induced drag
Drag from tilting lift by downwash.

Kutta condition
Flow leaves a sharp trailing edge smoothly.

Mach number
Flow speed over the speed of sound.

Normal shock
Thin jump that turns supersonic flow subsonic.

Reynolds number
$\rho VL/\mu$, inertia over viscosity.

Separation
Boundary layer leaving the surface.

Stagnation pressure
Pressure if flow is brought to rest isentropically.

Stall
Loss of lift when flow separates at high angle of attack.

Supercritical airfoil
Flat-topped airfoil that weakens shocks near $M = 1$.

Vorticity
$\nabla \times V$, twice the local rotation rate.

Winglet
Near-vertical tip surface that reduces induced drag.

Self-assessment rubric

No grade or credit is given. Use this to judge honestly where you are in each unit.

Level	What it looks like
4 · Mastery	Identifies the flow regime, chooses the right model (Bernoulli, momentum, potential flow, lifting line, boundary layer, isentropic, shock), checks units and limits, and runs labs safely.
3 · Proficient	Solves the practice problems with occasional unit or density slips.
2 · Developing	Follows worked examples but mixes up 2D and 3D coefficients, or degrees and radians.
1 · Beginning	Plugs numbers into formulas without a sketch; needs to revisit the unit.

Suggested weighting if you want a score

Part	Weight
Practice problems	20%
Labs	25%
Midterm practice set	25%
Final practice set	30%

Aim for level 3 or better in every unit before moving to the next course.

Related courses

- Circuits ENGR 214 + ENGR 214L
- Thermodynamics ENGR 320
- Fluid Mechanics ENGR 330
- All courses